Electric Dipole Moments and Sources of CP violation

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Goals

- **Goal 1:** A crash course in Electric Dipole Moments
  - What are EDMs and why do people bother to find them?
  - Overview of EDM theory and experimental landscape (I’m a theorist…)

- **Goal 2:** Put EDMs in a broader context (LHC/flavor/…)
  - How do EDM measurements complement other searches?

Excellent reviews: Pospelov/Ritz ‘05, Engel/van Kolck/Ramsey Musolf ’13
Outline

- **Part I:** What are EDMs and why do (should) we care?

- **Part II:** Sources of CP violation: an EFT approach

- **Part III:** Classes of EDM searches + theoretical interpretation
Magnetic dipole moments

- Let’s remind ourselves about magnetic dipole moments
- Particle at rest with spin (i.e. electron) in magnetic field is described by

\[ H = -\mu (\vec{S} \cdot \vec{B}) \]

\[ \vec{S} = \vec{\sigma} / 2 \mu = \frac{e\sigma}{2m} \]

- The B-field puts a torque on the system \(\rightarrow\) spin precession

\[ \omega = 2\mu B \sin \theta \]

- Dirac equation predicts tree-level magnetic moment
**Magnetic dipole moments**

\[ e\bar{u}(p')\gamma^\mu u(p)A_\mu \rightarrow e \left[ (\chi^\dagger\chi)A_0 - \frac{1}{2m}\chi^\dagger\sigma \cdot B\chi \right] \]

- Dirac predicts: \[ H = -\frac{\mu}{2}(\vec{\sigma} \cdot \vec{B}) \quad \mu = \frac{eg}{2m} \quad g = 2 \]

- \( g \) is the gyromagnetic ratio (classically \( g=1 \)), triumph of QM!
- Electron magnetic moment \( \sim e/m_e \sim 10^{-11} \text{ e cm} \sim 100 \text{ e fm} \)

- Measurements in 1940’s: \( g_e = 2*(1.00118+0.00003) \)…..
- **Enter Quantum Field Theory**

\[ a_e = \frac{\alpha_{em}}{2\pi} + \mathcal{O}(\alpha_{em}^2) \]
Magnetic Electric dipole moments

• Let’s remind ourselves about electric dipole moments
• A particle with spin (i.e. neutron) in an electric field is described by

\[ H = -d (\vec{S} \cdot \vec{E}) \]

\[ \vec{S} = \vec{\sigma} / 2 \]

• The E-field puts a torque on the system \( \rightarrow \) spin precession

\[ \omega = 2 \mu B \sin \theta \]
\[ \omega = 2 dE \sin \theta \]

• How large is the electric dipole moment?
Electric dipole moments

- We already exhausted the Dirac equation… No EDM there?
- Can we understand this?

\[ H = -\mu (\vec{S} \cdot \vec{B}) - d (\vec{S} \cdot \vec{E}) \]

- Perform a Time-reversal (T) transformation

  \( \vec{S} \rightarrow -\vec{S} \)
  \( \vec{B} \rightarrow -\vec{B} \)
  \( \vec{E} \rightarrow \vec{E} \)

  Spin like angular momentum \( \sim \vec{L} \sim \vec{r} \times \vec{p} \)

  \( \vec{B} \propto \epsilon^{ijk} F_{jk} \sim \partial_j A_k \rightarrow (+\partial_j)(-A_k) \)
  \( \vec{E} \propto F_{0i} \sim \partial_0 A_i \rightarrow (-\partial_0)(-A_i) \)

\[ H = -\mu (\vec{S} \cdot \vec{B}) + d (\vec{S} \cdot \vec{E}) \]
**CP violation in the Standard Model**

- We can try to calculate EDMs from SM CKM phase

\[ L_{\text{dip}} = -\frac{1}{2} \bar{\Psi} \sigma^{\mu\nu} (\mu + i\gamma^5 d) \Psi F_{\mu\nu} \]

\[ \mu = \frac{e}{2m_e} \frac{\alpha_{em}}{2\pi} \quad d = 0 \]

\[ \mu \propto \frac{m_d}{m_W^2} \frac{\alpha_{\text{weak}}}{2\pi} (V_{qd} V_{qd}^*) \]

\[ d_d \sim \text{Im} \left[ V_{qd} V_{qd}^* \right] = 0 \]
SM CKM EDMs (too many acronyms!)

- At two loops: individual diagrams contribute but sum vanishes
- \( d_q \) (2 loops) = 0, somewhat unexpected!

\[
d_{d} \simeq m_{d} \frac{m_{c}^{2} \alpha_{s} G_{F}^{2}}{108 \pi^{5}} J_{CP} \simeq 10^{-21} \text{ e fm}
\]

\[
J_{CP} = c_{12} c_{23} c_{13}^{2} s_{12} s_{23} s_{13} \sin \delta \simeq 3 \cdot 10^{-5}
\]

- At three loops:

- Electron EDM even at 4 loops !!! \( d_{e} \simeq 10^{-31} \text{ e fm} \)
- Compare with magnetic moment: \( \mu_{e} \simeq 100 \text{ e fm} \)
- CKM EDMs are really very very very small
- Nucleon EDMs more complicated but also small
Upperbound $n_{\text{EDM}} 10^5$ [e cm] 5 to 6 orders below upper bound Out of reach!

With linear extrapolation: CKM neutron EDM in 2075….

Note: actual size of SM $n_{\text{EDM}}$ has large uncertainty
Strong CP violation

• Second source: QCD theta-term. (Later more detail).
• Due to complicated vacuum structure of QCD

Causes a ‘new’ CP-violating interaction with coupling constant $\theta$

$$\theta \varepsilon^{\mu \nu \alpha \beta} G_{\mu \nu} G_{\alpha \beta}$$

(in QED $\sim \vec{E} \cdot \vec{B}$)

• Size of $\theta$ is unknown, one of the SM parameters
The strong CP problem

Sets $\theta$ upper bound: $\theta < 10^{-10}$

If $\theta \sim 1$

More details on calculation later

Is there a reason for this suppression? Axions?
Is $\theta=0$ exact, or merely very small?
Measurement of a nonzero EDM

Standard Model: $\theta$-term

BSM sources of CP-violation
SUSY, Left-Right, 2HDM,…

For the foreseeable future: EDMs are ‘background-free’ searches for new physics
Measurement of a nonzero EDM

Standard Model: $\theta$-term

BSM sources of CP-violation
SUSY, Left-Right, 2HDM,…

Collider/flavor input

Matter/Antimatter asymmetry (electroweak baryogenesis)
### Very active experimental field

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<td>muon</td>
<td>E821 BNL g–2</td>
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+ new electron, muon, neutron, proton, Xe, Ra, Rn ..... experiments

- Why do experiments on so many different systems?
- How do the different limits compare?
- How to interpret the limits or a future signal?
Measuring EDMs

- General idea now. Later more.

\[ \omega_1 = \frac{2 \mu B + 2dE}{\hbar} \]

\[ \omega_2 = \frac{2 \mu B - 2dE}{\hbar} \]

\[ \omega_1 - \omega_2 = \frac{4dE}{\hbar} \]

- Note

\[ d = 10^{-28} \text{ e cm} \quad E = 100 \text{ kV/cm} \]

\[ \delta \omega \sim 10^{-7} \text{ rad/s} \sim 1 \text{ rad/year} \]
CP violation beyond the SM

• We saw that CKM EDMs are too small to be measured
• We have the theta term, but that seems small for some reason…
• **Why bother with measuring further?**

• EDMs can be much larger in models of new physics, even at high scales
• CKM EDMs are small because of the peculiar SM flavor structure
• Need at least 3 loops to get an EDM (big suppression)

• **NOT GENERAL** at all. In many BSM models, EDMs at tree- or one-loop level. Much larger EDMs!
Example: the MSSM

The MSSM can contain many new sources of CP violation

**Higgsino and Higgs masses**
- 2 phases

**Squark and slepton masses**
- 15 phases

**Gaugino masses**
- 3 phases

**Trilinear couplings**
- 27 phases

\[ \mu \tilde{H}_u \tilde{H}_d + B \mu H_u H_d + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 \]

\[ m_Q^2 \tilde{Q}_L \tilde{Q}_L + m_U^2 \tilde{U}_R \tilde{U}_R + m_D^2 \tilde{D}_R \tilde{D}_R \]
\[ + m_L^2 \tilde{L}_L \tilde{L}_L + m_E^2 \tilde{E}_R \tilde{E}_R \]

\[ m_1 \tilde{B} \tilde{B} + m_2 \tilde{W} \tilde{W} + m_3 \tilde{g} \tilde{g} \]

\[ A_u H_u \tilde{Q}_L \tilde{U}_R + A_d H_d \tilde{Q}_L \tilde{D}_R + A_\ell H_d \tilde{L}_L \tilde{E}_R \]

*not all phases are physical! (like in the case of the CKM matrix)*

2 phases can be rotated away...
The SUSY/LR CP problem

• CPV phase already at one-loop!
• Typical size of EDM

\[ d_e \sim \left( \frac{\alpha_{em}}{\pi} \right)^n \frac{m_e}{\Lambda^2} \sin \phi \]

If phase = O(1): \( \Lambda > 30 \text{ TeV} \) (n=1)

There are clever ways to avoid this!

• In left-right symmetric models

\[
L = i \Xi (\bar{u}_R \gamma \mu d_R)(\bar{u}_L \gamma \mu d_L) + \text{h.c.}
\]

\[ \Xi \sim \sin \alpha / \Lambda^2 \]

• Tree-level CP violation, EDMs probe \( \sim 50 \text{ TeV} \) scale
The EDM metromap
Outline

- **Part I:** What are EDMs and why do (should) we care?

- **Part II:** Sources of CP violation: an EFT approach

- **Part III:** Classes of EDM searches + theoretical interpretation
Heavy BSM physics and the SM EFT

- Assume BSM fields exist but are heavy → Integrate them out

Fermi’s theory:

- We don’t need ‘high-energy details’, the W boson, at low energies!

Energy

\[ \Lambda \]

\[ E_{\text{exp}} << \Lambda \]

\[ G_F \sim \frac{g^2}{M_W^2} \]

Effective operators

\[ \sim \frac{1}{\Lambda^n} \]
Standard Model EFT

• To stay **model independent**: add all EFT operators (infinite…)

1) Degrees of freedom: Full SM field content
2) Symmetries: Lorentz, $\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)$

$$L_{\text{new}} = L_{\text{SM}} + \frac{1}{\Lambda} L_5 + \frac{1}{\Lambda^2} L_6 + \cdots$$

• Low-energy effects suppressed by $(E/\Lambda)^n$

**Advantages:** General framework, no need to specify a BSM model
Keep constraints from gauge invariance and EWSB
Great tool to analyze experiments at different scales

**Disadvantages:** Many operators at relevant order
Might not be valid for LHC physics if scale too low
Not applicable for light BSM physics

Buchmuller & Wyler ‘86
Gradzkowski et al ’10
Many others
Typical Energy Scale

\[ \Lambda \sim ? \text{ TeV} \]

\[ M_{EW} \sim v \sim M_{Z,W,H,t} \]
100 GeV

\[ \Lambda_\chi \sim 2\pi F_\pi \sim M_N \]
1 GeV

\[ 100 \text{ MeV} - \text{ eV} \]

Beyond-the-SM physics

\[ L_{EFF} \]

RG flow

EFT operators

Integrate out heavy SM fields

Nonperturbative QCD regimes

Lattice QCD + chiral effective field theory

EDMs
Typical Energy Scale

\[ \Lambda \sim \text{? TeV} \]

\[ M_{EW} \sim v \sim M_{Z,W,H,t} \sim 100 \text{ GeV} \]

\[ \Lambda_X \sim 2\pi F_\pi \sim M_N \sim 1 \text{ GeV} \]

\[ 100 \text{ MeV} - \text{eV} \]

\[ \sim \]

\[ \sim \]

\[ L_{\text{EFF}} \]

\[ L'_{\text{EFF}} \]

\[ L''_{\text{EFF}} \]

Beyond-the-SM physics

Integrate out heavy SM fields

Nonperturbative QCD regimes

Lattice QCD + chiral effective field theory

EDMs + Kaons + beta + ….

EFT operators

LHC physics

B + D flavor physics

RG flow
Examples of EFT operators: dipoles

EDMs and MDMs appear in the SMEFT Lagrangian at dimension-six

\[ \frac{1}{\Lambda^2} \bar{\phi} \sigma^{\mu\nu} \psi_R X_{\mu\nu} + h.c. \rightarrow \frac{\nu}{\Lambda^2} \bar{\psi}_L \sigma^{\mu\nu} \psi_R X_{\mu\nu} + h.c. \]

- **Quark chromo-EDM**
- **Quark EDM**
- **Electron (muon, tau) EDM**

One-loop QCD mixing
Gluon chromo-EDM

\[ M_{CP} ? \text{TeV} \]

\[ d_w f^{abc} \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^b_{\mu\lambda} G^c_{\nu} \]

Weinberg operator

QCD RGE mixing

Weinberg PRL '89
Braaten et al PRL '90
Gluon chromo-EDM

\[ d_w f^{abc} \varepsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^b_{\mu\lambda} G^c_{\nu\lambda} \]

\[ C_G t_L \sigma^{\mu\nu} t_R G_{\mu\nu} \varphi + h. c. \]

Threshold contributions from heavy CEDMs

QCD RGE mixing

\[ M_{CP} \quad ? \quad \text{TeV} \]

\[ m_Q \]

1 GeV

Weinberg PRL ’89
Braaten et al PRL ’90
Modified Yukawa couplings

\[ C_Y \, y_t \, \bar{t}_L \, t_R \, \tilde{\phi} \left( \phi^\dagger \phi \right) + h.c. \]

\[ L = C_Y \, m_t \, \bar{t}_L \, t_R \left( v_h + h^2 + \cdots \right) + h.c. \]

Barr, Zee '90

Barr-Zee diagrams

QCD RGE mixing

\[ \Lambda \]

\[ m_t \]

\[ 1 \, \text{GeV} \]
Many more… But when the dust settles…..

- Any BSM model at high scales with CP-violation can be described at low energies by $O(10)$ CPV operators (integrated out high-energy details !)
- Different BSM model, different relevant operators

Few GeV

**QCD** ($\theta$-term)

Quark EDM

Quark C-EDM

Gluon C-EDM

Four-quark operators

**Electron EDM**

**$\gamma$-leptonic interactions**

- Ramsey-Musolf/Chupp ‘13

JdV et al ‘12 ‘13

*semi-*leptonic interactions
Intermediate summary

- Parametrized BSM CP violation in terms of \textbf{dim6} operators
- Evolved them to lower energies to \( \sim 1 \) GeV
- O(10) operators left: theta, (C)EDMs, Weinberg, Four-fermion
- \textbf{Important}: different BSM models \( \Rightarrow \) different EFT operators

1. Standard Model: only \textbf{theta} has a chance to be measured
2. 2-Higgs doublet model: \textbf{quark+electron EDM, CEDMs, Weinberg} (exact hierarchy depends on detail of models)
3. Split SUSY: only \textbf{electron + quark EDMs} (ratio fixed)
4. Left-right symmetric models: \textbf{FQ operators}, way smaller (C)EDMs
5. Leptoquark: \textbf{FQ + semi-leptonic} operators

- Now we must calculate EDMs. Not easy, since EDMs involve horrible objects (for a particle physicist) such as nucleons, atoms, and molecules
Outline

- **Part I**: What are EDMs and why do (should) we care?
- **Part II**: Sources of CP violation: an EFT approach
- **Part III**: Classes of EDM searches + theory interpretation
The ‘easy case’: (semi-)leptonic CP violation

Few GeV

(semi-)leptonic interactions

< GeV

Electron EDM

Electron EDM

Hadronization (known matrix elements)

N = nucleon (neutron, proton)
Probing semi-leptonic operators

- Why these complicated systems? Cannot use free electrons....
- Look at neutral systems with unpaired electron (paramagnetic)
- **Why not simply use Hydrogen?**

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Schiff's theorem: the electron cloud will rearrange so that the nucleus does not ‘feel’ an external $E$-field. The EDMs of the nucleus and electron are screened (classical argument but works in QM as well)

\[ \vec{E} \]
Probing the electron EDM

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**Schiff Theorem:** EDMs of charged constituents are screened in a neutral atom

- Assumption: non-relativistic constituents
- Invalid in heavy atoms/molecules

$$d_A(d_e) = K_A d_e$$  \quad $$K_A \propto Z^3 \alpha_{em}^2$$  \quad Sandars ‘65

Bound on atomic Tl EDM:  $$d_{^{205}}\text{Tl} < 9 \cdot 10^{-25} \text{e cm}$$  \quad Regan et al ’02

$$d_A(d_e) = K_A d_e$$  \quad $$K_{\text{Tl}} = -(570 \pm 20)$$  \quad Strong enhancement!

$$d_e < 1.6 \cdot 10^{-27} \text{e cm}$$
State-of-the-art: polar molecules

Liu, Kelly ‘92, Dzuba, ‘09, Porsev et al ‘12, Mayer ‘09

Polar molecules: Convert small external to huge internal field

\[ \Delta E_{YbF} = (15 \pm 2) \cdot GeV \left( \frac{d_e}{e \text{ cm}} \right) + O(C_S) \]

\[ \Delta E_{ThO} = (80 \pm 10) \cdot GeV \left( \frac{d_e}{e \text{ cm}} \right) + O(C_S) \]

Current world record \( d_e < 1.1 \cdot 10^{-29} \text{ e cm} \) \( \text{ACME ‘18} \)

Table-top experiment! \( O(10) \) people.

If EDMs at one loop \( \sim \) roughly 30 TeV

\( d_e \sim \left( \frac{\alpha_{em}}{\pi} \right) \frac{m_e}{\Lambda^2} \)

Fuyuto et al ‘18

\( C_s \) can be induced at tree-level and no \( m_e \) \( \Rightarrow \) roughly 10⁴ TeV
Constraining Higgs physics

\[ C_Y y_t \bar{t}_L t_R \tilde{\varphi} (\varphi^\dagger \varphi) + h.c. \]

- Electron EDM limit tells us imaginary part of Top Yukawa < 0.3 %
- Or new physics only enters there \( \Lambda > 7 \) TeV
- Note LHC constraints on real part \( \sim < 10\text{-}20\% \) (1 TeV roughly)
Onwards to hadronic CPV

Few GeV

QCD (θ-term) + Quark EDM + Quark chromo-EDM + Weinberg operator

Hadronic/Nuclear CP-violation

Theoretically more difficult

Goal: Electric dipole moments of nucleons, nuclei, and diamagnetic atoms
Onwards to hadronic CPV

Goal: Electric dipole moments of nucleons, nuclei, and diamagnetic atoms
Classes of hadronic EDM experiments

- **Class 1**: neutron EDM experiments
  (traditional, easiest to interpret, but running out of steam? )

- **Class 2**: Diamagnetic atoms
  (powerful but hard to understand, nuclear physics uugh)

- **Class 3**: Storage ring experiments (the future? )
  (very cool idea but expensive and perhaps science fiction)
Running out of steam?
Experiments suffer from lack of neutrons

Ongoing experiments at LANL/Oak Ridge/PSI (goal $10^{-27}$-$10^{-28}$ e cm)
Ideal to illustrate the ‘strong CP problem’
Strong CP violation

• Second SM source: QCD theta-term
• Unlike CKM-phase: many open questions
• Technical subject: here main idea.
Introductory remarks

- Start by considering the QED Lagrangian

\[ L = \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \]

\[ D_\mu = \partial_\mu - iQ_qA_\mu \]

- SM = all renormalizable terms that obey SU(3)xSU(2)xU(1) gauge invariance involving known degrees of freedom

- \( F_{\mu\nu} \) is a gauge-invariant quantity.... So we could have added a dim-4 term:

\[ \theta \frac{e^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta}F^{\mu\nu} \equiv \theta \frac{e^2}{32\pi^2} \tilde{F}_{\mu\nu}F^{\mu\nu} \]

P or T transformation

\[ \theta \frac{e^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta}F^{\mu\nu} \sim \vec{E} \cdot \vec{B} \quad \rightarrow \quad -\vec{E} \cdot \vec{B} \]

So it describes a CP-odd interaction!
Total derivatives

• But.... This term has no physical consequences! Why?

• Cause its a total derivative: \[ \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F^{\mu\nu} = \partial_\mu (\varepsilon^{\alpha\beta\mu\nu} A_\nu F_{\alpha\beta}) \]

• QCD is more complicated, non-Abelian group, but still total derivative

\[ \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} = \partial_\mu \varepsilon^{\alpha\beta\mu\nu} (A_\nu F_{\alpha\beta} + A_\alpha A_\beta A_\nu) \]
• But…. This term has no physical consequences! Why?

• Cause its a total derivative:

\[ \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} = \partial_{\mu} (\epsilon^{\alpha\beta\mu\nu} A_{\nu} F_{\alpha\beta}) \]

• QCD is more complicated but still total derivative

\[ \epsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} = \partial_{\mu} \epsilon^{\alpha\beta\mu\nu} (A_{\nu} F_{\alpha\beta} + A_{\alpha} A_{\beta} A_{\nu}) \]

• 't Hooft: for non-Abelian gauge groups, there are instanton solutions that still contribute to the action

• Lecture on its own, but theta has physical effects!
Complex masses

• Let us look at 2-flavor QCD with masses + theta term

\[ L = \sum_{u,d} \bar{q}(i\gamma^\mu D_\mu - m_q)q + \theta \frac{g_s^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \]

• In principle, the quark mass can be complex

• To make the masses real, we do a U_{A}(1) transformation \( q \rightarrow e^{i\alpha_5\gamma^5} q \)

• The axial U(1) transformation is a classical symmetry, but broken by QM

‘t Hooft ’76, ’78
Complex masses

• Let us look at 2-flavor QCD with masses + theta term

\[ L = \sum_{u,d} \bar{q}(i\gamma^\mu D_\mu - m_q)q + \theta \frac{g_s^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \]

• In principle, the quark mass can be complex
• To make the masses real, we do a \( U_A(1) \) transformation \( q \rightarrow e^{i\alpha_5 \gamma^5} q \)
• The axial U(1) transformation is a classical symmetry, but broken by QM
• Action (or Lagrangian) invariant but path-integral measure is not (anomaly)

\[ \int [d\psi][d\bar{\psi}] \, e^{iS[\psi,\bar{\psi}]} = \int [d\psi'][d\bar{\psi}'] \, e^{iS[\psi',\bar{\psi}']} \]

\[ S[\psi, \bar{\psi}] = S[\psi', \bar{\psi}'] \]

\[ \int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}'] \, \mathcal{I} \]

\( \mathcal{I} \neq 1 \)

Axial transformation induces a shift in the \( \theta \) term

\[ \log \mathcal{I} = -\alpha \int d^4x \, \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a} \]
Complex masses

\[ L = \sum_{u,d} \bar{q}(i\gamma^\mu D_\mu - m_q)q + \theta \frac{g_s^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \]

- In principle, the quark mass can be complex
- To make the masses real, we do a \( U_A(1) \) transformation
  \[ q \rightarrow e^{i\alpha_5 \gamma^5} q \]
- But this induces a theta term from the anomaly!
- So we can ‘trade’ the theta term for a complex mass or vice versa

\[ +\theta \frac{g_s^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} G_{\alpha\beta} G^{\mu\nu} \quad \leftrightarrow \quad -\left( \frac{m_u m_d}{m_u + m_d} \right) \theta \bar{q}i\gamma^5 q \]

- One ‘physical’ combination
  \[ \bar{\theta} = \theta + \text{arg} \det m_q \]
How do we measure the theta term?

• Electric dipole moments of hadrons and nuclei

• Problem: low-energy QCD is nonperturbative.

• How to calculate the nucleon EDM from CPV at quark-gluon level?

\[ L_{\text{dip}} = -\frac{d_n}{2} \bar{\Psi}_n \sigma^{\mu\nu} i \gamma^5 \Psi F_{\mu\nu} \quad \text{from} \quad -\left( \frac{m_u m_d}{m_u + m_d} \right) \bar{\theta} \bar{q} i \gamma^5 q = -(m^*) \bar{\theta} \bar{q} i \gamma^5 q \]

• Let’s guess something: \( d_n \) should be proportional to \( \sim (m^* \theta) \)

• There should be a coupling to a photon somewhere \( \sim e \)

• To get dimensions right we need \( 1/\text{mass}^2 \), let’s say nucleon mass....

\[ d_n \sim e \frac{m^*}{m_N^2} \bar{\theta} \sim e \frac{10 \text{ MeV}}{(1 \text{ GeV})^2} \bar{\theta} \sim 10^{-3} \bar{\theta} e \text{ fm} \]
Limiting theta

If $\theta \sim 1$, just like the CKM phase

Sets $\theta$ upper bound: $\theta < 10^{-10}$
Perhaps the estimate is stupid….

- We can do better: **chiral perturbation theory**

  \[ L_{QCD} \rightarrow L_{\text{chiPT}} = L_{\pi\pi} + L_{\pi N} + L_{NN} + \cdots \]

- Quark masses = 0 \(\Rightarrow\) QCD has \(\text{SU}(2)_L \times \text{SU}(2)_R\) symmetry
  - Spontaneously broken to \(\text{SU}(2)\)-isospin (pions are Goldstone)
  - Explicit breaking (quark mass) \(\Rightarrow\) pion mass

- ChPT gives systematic expansion in \(Q/\Lambda_{\chi} \sim m_{\pi}/\Lambda_{\chi}\) \(\Lambda_{\chi} \approx 1 \text{ GeV}\)
  - Form of interactions fixed by symmetries
  - Each interactions comes with an unknown constant (LEC)

- **Extended to include P and CP violation**  
  Review: JdV & Ulf-G. Meissner ‘15

Weinberg, Gasser, Leutwyler, and many many others
Example

\[ \bar{m} = \frac{m_u + m_d}{2} \quad \varepsilon = \frac{m_u - m_d}{m_u + m_d} \]

2 flavor QCD: \[ \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin}} - \bar{m} \bar{q} q - \varepsilon \bar{m} \bar{q} \tau^3 q \]

ChPT gives you the form of the effective low-energy terms

\[ N = (p \ n)^T \]

Zero quark mass: \[ \mathcal{L}_\chi = \mathcal{L}_{\text{kin}} - m_N \bar{N} N + \frac{g_A}{f_\pi} D_\mu \bar{\pi} \bar{N} \tau^\mu \gamma^\nu \gamma^5 N \]

Size \( m_N = 930 \text{ MeV}, g_A = 1.27 \) we must measure or use lattice QCD!

Turn on quark mass: \[ \mathcal{L}_\chi' = \mathcal{L}_\chi - \frac{m_\pi^2}{2} \pi^2 - \delta m_N \bar{N} \tau^3 N \]

Small quark masses \( \rightarrow \) small pion mass and nucleon mass splitting
Theta and chiral perturbation theory

What does the theta-induced complex quark mass do?

\[ \mathcal{L}_{QCD} = \mathcal{L}_{\text{kin}} - \bar{m} \bar{q} q - \varepsilon \bar{m} \bar{q} \tau^3 q + m_* \bar{\theta} \bar{q} i \gamma^5 q \]

\[ m_* = \frac{m_u m_d}{m_u + m_d} \]

\[ \mathcal{L}'_\chi = \mathcal{L}_\chi - \frac{m^2}{2} \pi^2 - \delta m_N \bar{N} \tau^3 N + \bar{g}_0 \bar{N} \tau \cdot \pi N \]

CP-odd pion-nucleon interaction
Theta and chiral perturbation theory

After axial U(1) and SU(2) rotations, **complex CP-odd quark mass**:

\[
\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin}} - \bar{m}\bar{q}q - \varepsilon\bar{m}\bar{q}\tau^3 q + m_* \bar{\theta}\bar{q}i\gamma^5 q
\]

Linked via \(\text{SU}_A(2)\) rotation

\[
\mathcal{L}' = \mathcal{L}_\chi - \frac{m^2}{2}\pi^2 - \delta m_N \bar{N}\tau^3 N + \bar{g}_0 \bar{N}\tau \cdot \pi N
\]

**Nucleon mass splitting** (strong part, no EM!)

\[
g_0 = \delta m_N \frac{1 - \varepsilon^2}{2\varepsilon} \bar{\theta} = (15.5 \pm 2.5) \cdot 10^{-3} \bar{\theta}
\]

**CP-odd pion-nucleon interaction**

Use **lattice** for mass splitting

Walker-Loud ‘14, Borsanyi ’14, Aoki (FLAG) ’13

JdV et al‘15
The Neutron EDM

Neutron EDM

\[ d_n = -\frac{eg_A g_0}{8\pi^2 f_\pi} \log \frac{m_\pi^2}{m_N^2} \]

\[ d_n \simeq -2.5 \times 10^{-16} \bar{\theta} \text{ cm} \]

- Very close to the naïve estimate
- Recent developments. Use lattice-QCD (difficult!)

Lattice

\[ d_n = -(1.5 \pm 0.7) \times 10^{-16} \bar{\theta} \text{ cm} \]

- So everything agrees it seems

\[ \bar{\theta} < 10^{-10} \]

- Why is theta term so small? ‘Strong-CP Problem’
- Note other small numbers in the SM: \( y_e, V_{cb} \)
- No anthropic argument for small theta (as far as I know)
- Most popular solution: Axions (not in this lecture)
And dim-6 sources?

- Quark EDM accurately determined

\[
d_n = -(0.22 \pm 0.03) d_u + (0.74 \pm 0.07) d_d + (0.008 \pm 0.01) d_s
\]

- Other dim-6 operators no accurate numbers available. Model calculations or estimates only.

- For example: Weinberg operator

\[
d_n \sim d_p \sim \pm (50 \pm 40) \text{MeV \, } ed_W
\]

- If Weinberg induced at 1 (2) loop: nEDM probes 20 (2) TeV

- Uncertainties dilute EDM constraining/discriminating power

- A lot of recent lattice-QCD interest
Classes of hadronic EDM experiments

- **Class 1:** neutron EDM experiments
  (traditional, easiest to interpret, but running out of steam?)

- **Class 2:** Diamagnetic atoms
  (powerful but hard to understand, nuclear physics uugh)

- **Class 3:** Storage ring experiments (the future?)
  (very cool idea but expensive and perhaps science fiction)
Neutrons are hard ....

- Would be easier to use something stable
- Protons are charged and would fly out of the experimental setup
- Diamagnetic atoms, closed electron shell and nonzero nuclear spin.

Schiff Theorem: EDMs of charged constituents are screened in a neutral atom
Neutrons are hard ....

- Would be easier to use something stable
- Protons are charged and would fly out of the experimental setup
- Diamagnetic atoms, closed electron shell and nonzero nuclear spin.

Schiff Theorem: EDMs of charged constituents are screened in a neutral atom

- Assumption: non-relativistic constituents (nucleus is non-rel...)
- Assumption: point particles! (nucleus has finite size!)

Typical suppression: \( \frac{d_{\text{Atom}}}{d_{\text{nucleus}}} \propto 10Z^2 \left( \frac{R_N}{R_A} \right)^2 \approx 10^{-3} \)

- Atomic part well under control

\[ d_{199\text{Hg}} = (2.8 \pm 0.6) \cdot 10^{-4} \, S_{\text{Hg}} \, e \, fm^2 \]

- So the atomic limit gets screened by a factor 1000 roughly

Dzuba et al, '02, '09
Sing et al, '15
Huge collaboration.....

The Team
Graduate Students
Jennie Chen
Brent Graner*
Scientific Glassblower
Eric Lindahl
Faculty
B. R. Heckel
Primary support from NSF
* Supported by DOE Office of Nuclear Science

Best EDM limit in the world!

$$d_{199\text{Hg}} < 8.7 \cdot 10^{-30} \text{ e cm}$$
Theory for nuclear CP violation

- New contribution from CP-odd pion exchange: no loop suppression
- Nuclear CP-violation typically dominated by CP-odd nuclear force

- Theory input 1: $g_{0,1}$ in terms of theta, qCEDM, Weinberg, etc
- Theory input 2: Nuclear Schiff moment in terms of $g_{0,1}$
- Lots of progress on both theoretical aspects in recent years.
Back to pion-nucleon couplings

- 2 CP-odd structures

\[ L = g_0 \bar{N} \pi \cdot \tau N + g_1 \bar{N} \pi^0 N \]

**Key idea**

- The theta-term and dim-6 operators have different chiral properties
- Different models \( \rightarrow \) Different \( g_0/g_1 \) ratios

<table>
<thead>
<tr>
<th>Theta</th>
<th>2HDM</th>
<th>mLRSM</th>
<th>JdV et al ’12 ‘14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theta term</td>
<td>Quark CEDMs</td>
<td>FQLR</td>
<td>Quark EDM and Weinberg</td>
</tr>
<tr>
<td>[ \frac{\bar{g}_1}{\bar{g}_0} ]</td>
<td>(-0.2)</td>
<td>(\approx 1)</td>
<td>(+50)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Both couplings are suppressed !</td>
</tr>
</tbody>
</table>

- But how to experimentally probe these ratios?
Nuclear theory.... Uuugh...

- Very complicated many-body calculation
- Use nuclear model and mean-field theory

\[ S = g(a_0 \bar{g}_0 + a_1 \bar{g}_1) \text{ fm}^3 \quad g = 13.5 \]

<table>
<thead>
<tr>
<th></th>
<th>( a_0 ) range (best)</th>
<th>( a_1 ) range (best)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{199}\text{Hg} )</td>
<td>0.03±0.025 (0.01)</td>
<td>0.030±0.060 (±0.02)</td>
</tr>
<tr>
<td>( ^{225}\text{Ra} )</td>
<td>-3.5±2.5 (-1.5)</td>
<td>14±10 (6)</td>
</tr>
<tr>
<td>( ^{129}\text{Xe} )</td>
<td>-0.03±0.025 (-0.008)</td>
<td>-0.03±0.025 (-0.009)</td>
</tr>
</tbody>
</table>

Based on calculations from various groups
- Hg & Xe: spread >100% (unclear why, difficult ‘soft’ nuclei)
- **Difficult** to interpret the limits on BSM parameters.

Flambaum, de Jesus, Engel, Dobaczewski,....

table from review: Engel et al, ‘13
There has been some tension between lattice-QCD predictions for CP-violation in Kaon decays and experiments. (probably just a fluke).

See e.g. Buras '17 '18

This tension easily explained by a right-handed charged currents at $\sim 100$ TeV

But the extra CP violation also generates EDMs

Using central values of matrix element, dHg rules out this explanation

But matrix elements uncertainty makes this difficult

Lectures by Ulrich Nierste tomorrow
Link to flavor

- Certain models that for the RK(*) and RD(*) also affected by EDM constraints

Probably covered extensively at this school

- Model based on a leptoquark solution to flavor puzzles
- This particular model in principle ruled out by Hg EDM
- Uncertainties are too big to be sure ....
- Future nEDM would test the model.
Classes of hadronic EDM experiments

- **Class 1:** neutron EDM experiments
  (traditional, easiest to interpret, but running out of steam?)

- **Class 2:** Diamagnetic atoms
  (powerful but hard to understand, nuclear physics uugh)

- **Class 3:** Storage ring experiments (the future?)
  (very cool idea but expensive and perhaps science fiction)
EDMs of light nuclei

\[
\frac{d\vec{S}}{dt} = \vec{S} \times \vec{\Omega}
\]

Anomalous magnetic moment

\[
\vec{\Omega} = \frac{q}{m} \left[ a\vec{B} + \left( \frac{1}{v^2} - a \right) \vec{v} \times \vec{E} \right] + 2d \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]

Electric dipole moment

All-purpose ring \((^1\text{H}, ^2\text{H}, ^3\text{He}, \ldots) \sim 10^{-28.29} \text{ e cm}

100-1000 x current neutron EDM sensitivity! (takes a while tough....)

Already used for muon EDM

\[
d_\mu \leq 1.8 \cdot 10^{-19} \text{ e cm } \text{ (95% C.L.)}
\]

Bennett et al (BNL g-2) PRL ‘09

Major progress in:

JEDI collaboration,

PRL ’15, ’16 ‘18
Why EDMs of light nuclei

- Several advantages of using light instead of heavy systems

- **No Schiff screening!** No suppression associated to Hg/Ra/Xe
- This means a measurement at $10^{-26}$ level would be world-leading
- Theory is well under control. Few-body equations are ‘easy’

$$d_D = 0.9(d_n + d_p) + [(0.18 \pm 0.02) \bar{g}_1 + (0.0028 \pm 0.0003) \bar{g}_0] \text{e fm}$$

$$d_{3\text{He}} = 0.9 \, d_n - 0.05 \, d_p + [(0.14 \pm 0.04) \bar{g}_1 + (0.10 \pm 0.03) \bar{g}_0] \text{e fm} + \ldots$$

- Disadvantage: Expensive, requires a big storage ring

- Under development in Germany/Korea/CERN? but funding not guaranteed
The chiral filter

- The theoretical accuracy can be used to isolate the source of CP violation

\[ d_D = 0.9(d_n + d_p) + [(0.18 \pm 0.02) \bar{g}_1 + (0.0028 \pm 0.0003) \bar{g}_0] \text{ e fm} \]

<table>
<thead>
<tr>
<th></th>
<th>Theta term</th>
<th>Quark CEDMs</th>
<th>Four-quark operator</th>
<th>Quark EDM and Weinberg</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{d_D - d_n - d_p}{d_n} ]</td>
<td>0.5 \pm 0.2</td>
<td>7 \pm 4</td>
<td>14 \pm 7</td>
<td>\approx 0</td>
</tr>
</tbody>
</table>

- EDM ratio hint towards **underlying CP-odd operator**!
- Separate theta term from BSM physics, unravel different BSM models!

Dekens, JdV et al, ’14 ‘Unraveling models of CP violations with EDMs’
Complementary measurements

- Class 1: Paramagnetic atoms and molecules
  - Very sensitive to electron EDM, but not to hadronic sources
  - No SM background, not even theta.
  - Atomic theory is under control

- Class 2: Neutron EDM experiments
  - Theta + BSM CP violation involving quarks and gluons
  - Difficult to improve a lot

- Class 3: Diagmagnetic atoms
  - Sensitive to nuclear CPV and thus complements nEDM
  - Very good measurements but Schiff screening and theory…

- Class 4: Light nuclear EDMs
  - New idea, no Schiff screening and good theory
  - Expensive and requires new technology
The EDM metromap
Conclusion/Summary/Outlook

**EDMs**

- CP-violation but no CKM contribution
- Probe of strong CP violation: QCD theta term
- Very powerful search for BSM physics (probe the highest scales)
- Heroic experimental effort and great outlook

**Classes of EDM experiments**

- Basically 4 ‘classes: neutron, paramagnetic, diamagnetic, storage rings
- Probe complementary sources of CP-violation
- Measurements in several classes $\rightarrow$ unravel CPV source

**EFT framework**

- Framework exists for CP-violation (EDMs) from 1st principles
- Keep track of symmetries (gauge/CP/chiral) from multi-Tev to atomic scales
- Can be combined to LHC and flavor experiments! Very complementary!
- Still need much better theory control of hadronic and nuclear theory
Backup
Other electric dipole moments

- Take a classical dipole configuration
- Electric dipole $\sim d \sim q \, r$
- Does not violate anything

- So we mean with an EDM: the coupling of spin and the E-field.

- For electron, neutron, atom, the only quantity available is the spin. So there is no ‘r’ around

- So where does the non-CPV EDM of molecules come from?
Double-well potential

• Analogy take a double-well potential

• If $V_0$ is very small, get usual solutions

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi x}{a} \right) & \text{if } 0 < x < a, \\ 0 & \text{otherwise,} \end{cases}$$
Double-well potential

- Analogy take a double-well potential

- If $V_0$ is very small, get usual solutions

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin \left( \frac{n\pi x}{a} \right) & \text{if } 0 < x < a, \\ 0 & \text{otherwise}, \end{cases}$$

- With nonzero $V_0$, two solutions appear with different parity and a small energy difference (tunneling effect!). $E_+ - E_- \sim b$

- A molecule like water has indeed a nearly-degenerate ground state with opposite parity
Fake EDMs

- So we have 2 states which we call \( |\pm\rangle \)
- Turn on Electric Field \( E \) (mixing of states)

\[
H = \begin{pmatrix}
    \mathcal{E}^+ & 0 \\
    0 & \mathcal{E}^-
\end{pmatrix}
+ \begin{pmatrix}
    0 & Eb \\
    Eb & 0
\end{pmatrix}
\]

- Diagonalize matrix to get energy eigenvalues

\[
\mathcal{E}_{1,2} = \frac{1}{2} (\mathcal{E}_+ + \mathcal{E}_-) \pm \sqrt{(\mathcal{E}_+ - \mathcal{E}_-)^2/4 + E^2b^2}
\]
Fake EDMs

• So we have 2 states which we call $|\pm\rangle$

• Turn on Electric field $E$ (mixing of states)

$$H = \begin{pmatrix} \mathcal{E}^+ & 0 \\ 0 & \mathcal{E}^- \end{pmatrix} + \begin{pmatrix} 0 & Eb \\ Eb & 0 \end{pmatrix}$$

• Diagonalize matrix to get energy eigenvalues

$$\mathcal{E}_{1,2} = \frac{1}{2} (\mathcal{E}_+ + \mathcal{E}_-) \pm \sqrt{(\mathcal{E}_+ - \mathcal{E}_-)^2/4 + E^2b^2}$$

• If the E field is smaller than the energy gap

$$\mathcal{E}_{1,2} = \frac{1}{2} (\mathcal{E}_+ + \mathcal{E}_-) \pm \frac{1}{2} (\mathcal{E}_+ - \mathcal{E}_-) \left( 1 + \frac{2E^2b^2}{(\mathcal{E}_+ - \mathcal{E}_-)^2} \right)$$

• The energy shift is quadratic in the E field !! So no P or T violation

• If the E field is larger than the gap: degenerate ground state

$$\mathcal{E}_{1,2} = \frac{1}{2} (\mathcal{E}_+ + \mathcal{E}_-) \pm Eb$$
But there is more than $d_e$

- Find a signal: what is responsible? eEDM or Cs?
- Need at least two measurements

\[ \Delta E = \alpha \, d_e + \beta \, C_s \]

<table>
<thead>
<tr>
<th></th>
<th>YbF</th>
<th>ThO</th>
<th>HfF$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha/\beta$</td>
<td>1.09</td>
<td>0.67</td>
<td>1.1</td>
</tr>
</tbody>
</table>

- Unfortunately: **Probing roughly same combination**

Dzuba et al ‘11
M. Jung ‘13
Chupp, Ramsey-Musolf ‘15
Cancellations?

\[ d_e < 8.7 \cdot 10^{-29} \text{ e cm} \]

\[ C_S < 5.9 \cdot 10^{-9} \]

Allow for cancellations

\[ d_e < 5.4 \cdot 10^{-27} \text{ e cm} \]

\[ C_S < 4.5 \cdot 10^{-7} \]

- Also more difficult to pinpoint \( C_S \) and \( d_e \) from 2 measurements!
- Difficult to disentangle different models
- 2017 HfF\(^{+}\) measurement helps

Fleig, Jung '18

Plots and numbers from Chupp, Ramsey-Musolf PRC '15
Including diamagnetic atoms

This would allow to separate $d_e$ from $C_s$

But... not realistic that Hg EDM solely from leptonic sources

Without cancellations: The $C_s$ limit roughly suggests $10^4$ TeV scales.

But there can be small Yukawa couplings.

Single source

\[ d_e < 8.7 \cdot 10^{-29} \text{ e cm} \]
\[ C_s < 5.9 \cdot 10^{-9} \]

Allow for cancellations

\[ d_e < 3.8 \cdot 10^{-28} \text{ e cm} \]
\[ C_s < 4.9 \cdot 10^{-8} \]