

Electron Tracking with SBI and PPL in Toy Cerenkov Detector

CIDeR-ML

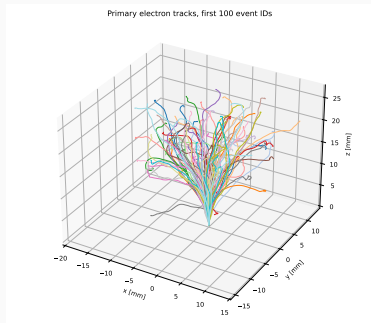
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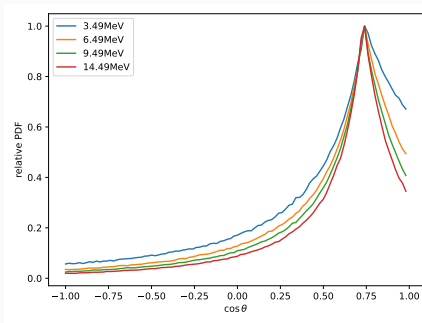


Motivation

MeV electron direction is limited by multiple scattering



5 MeV Electron track in water.



Marginalized Cerenkov photon polar-angle PDF $p_1(\cos \theta)$

- $\cos \theta$ marginal distribution loses information about individual tracks
- Photons share a scattered track; their directions are not iid draws from the marginal.
- Angular resolution improvement can bring a large boost in solar *hep* neutrino sensitivity.

Information beyond the marginal Cherenkov profile

Photon direction set relative to initial e^-

$$S_{\hat{\mathbf{u}}} = \{\Omega_j(\hat{\mathbf{u}})\}_{j=1}^N, \quad \Omega_j = (x_j, \phi_j), \quad x_j = \hat{\mathbf{q}}_j \cdot \hat{\mathbf{u}}.$$

Initial e^- direction

Photon direction

Likelihood using marginal template

$$p_0(S_{\hat{\mathbf{u}}} | \text{hatsu}, N) = \prod_{j=1}^N \frac{\hat{p}_1(x_j)}{2\pi}.$$

Single-photon marginal polar-angle PDF

Covariance over latent tracks

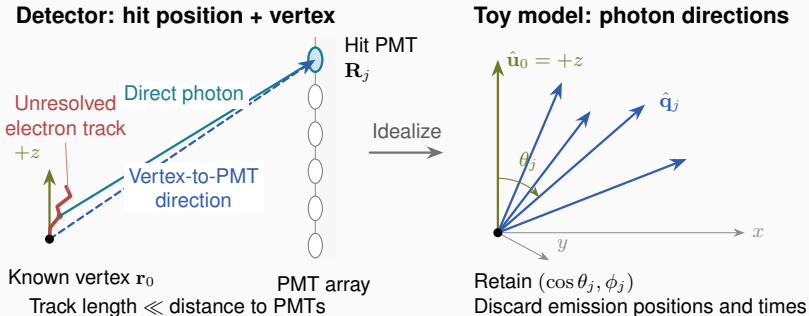
$$K_2(\Omega_1, \Omega_2) = \text{Cov}_{\Gamma} [\lambda_{\Gamma}(\Omega_1), \lambda_{\Gamma}(\Omega_2)].$$

Photon intensity on sphere for one track

SBI: learn a likelihood-ratio correction from photon sets.

PPL: write an explicit latent electron transport model.

Toy detector model

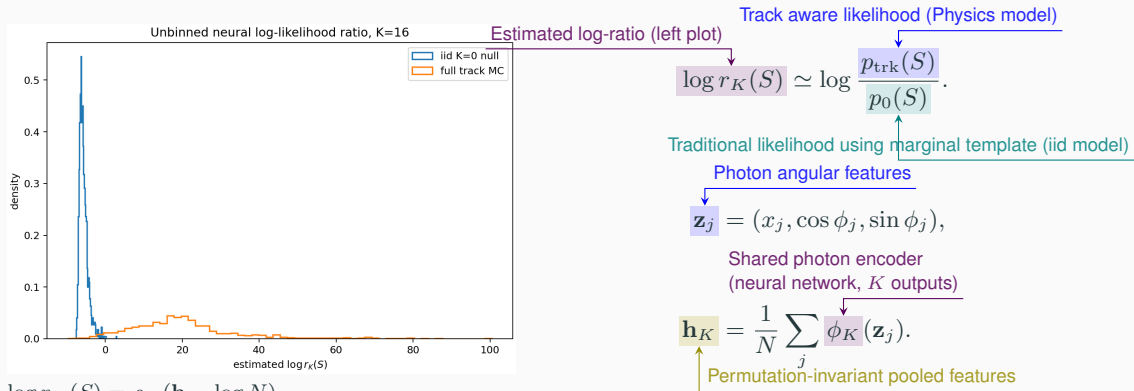


Schematic projection; track displacement enlarged. Direct-light approximation: $\hat{\mathbf{q}}_j \simeq (\mathbf{R}_j - \mathbf{r}_0)/|\mathbf{R}_j - \mathbf{r}_0|$.

- PMT timing locates the vertex; individual emission points and times remain unresolved.
- Idealized input: known vertex at $(0, 0, 0)$ and truth photon directions $(\cos \theta_j, \phi_j) \equiv (x_j, \phi_j)$; electrons start along $+z$.

Simulation-Based Inference

DeepSet likelihood-ratio diagnostic



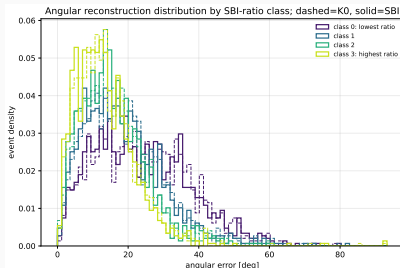
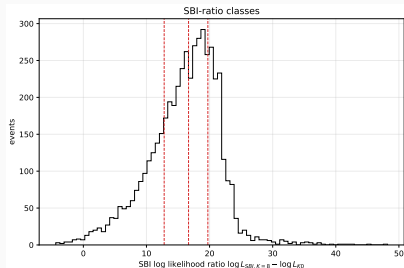
$$\log r_K(S) = \rho_K(\mathbf{h}_K, \log N).$$

ρ_K : event-level readout neural network.

- 5 MeV Geant4 single scattering vs iid null; $K = 16$, 512 photons/event, 25 epochs.
- Test AUC: 0.995; iid-closure AUC: 0.487.

Small overall angular resolution gain, but SBI finds event classes

- The best tested correction changed 68% quantile (angular resolution) from 21.764° to 21.102° .



- The learned ratio separates event classes with different angular resolution. Similar effect seen in multiple-scattering-goodness used in SK.
- Interpretation: higher-order angular information is measurable, but this first correction is not yet a strong direction estimator.

Probabilistic Program

PPL forward model: compact Geant4 translation

Latent positions, directions, steps, energies, times

$$\Gamma = \{(\mathbf{r}_s, \hat{\mathbf{v}}_s, \Delta s_s, T_s, t_s)\}_{s=1}^S.$$

Segment photon PDF

$$p(S|\Gamma, \hat{\mathbf{u}}_0, N) = \prod_{j=1}^N \sum_s \pi_s f_s(\hat{\mathbf{q}}_j),$$

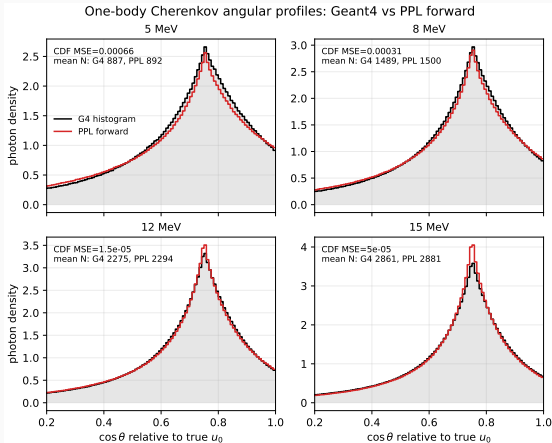
$$\pi_s = \frac{w_s}{\sum_r w_r}, \quad c_s = \frac{1}{\beta(T_s)n}.$$

Emission weight

Cone cosine

- Energy loss and CSDA range tables.
- Moller-like delta electrons.
- Geant4-Tsai bremsstrahlung surrogate.
- Explicit screened single Coulomb scatters on H and O.
- Cherenkov emission from charged segments.
- Variable physical step lengths with fixed dimension.

Universal cross-energy calibration



Mean across energies

$$\mathcal{L}_{\text{calib}} = \frac{1}{4} \sum_E \mathcal{L}_E$$

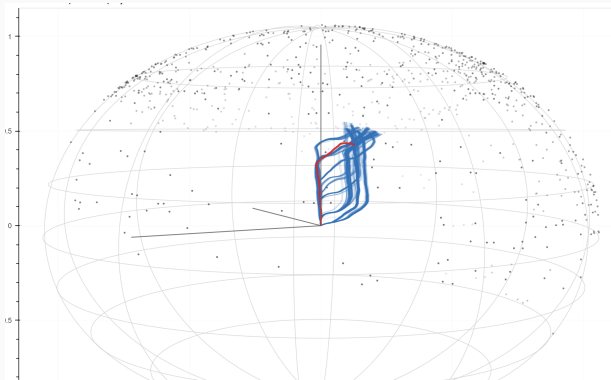
$$+ \max_E \mathcal{L}_E + 10^{-4} \mathcal{L}_{\text{prior}}$$

Worst energy

Prior penalty

- Fit one shared physics parameter vector to 5, 8, 12, and 15 MeV.
- Each $\mathcal{L}_E$ compares Geant4 and PPL marginal $\cos \theta$ profiles.
- Terms: CDF, JS, stabilized binned residuals, KS, photon yield, and peak height.

Latent-track posterior exposes the hard part



Joint posterior

$$p(\hat{\mathbf{u}}_0, \Gamma | S) \propto p(S | \Gamma, \hat{\mathbf{u}}_0)$$

Photon likelihood

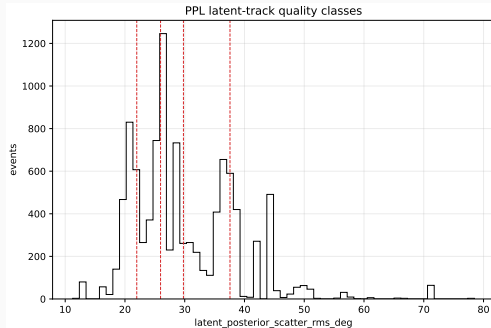
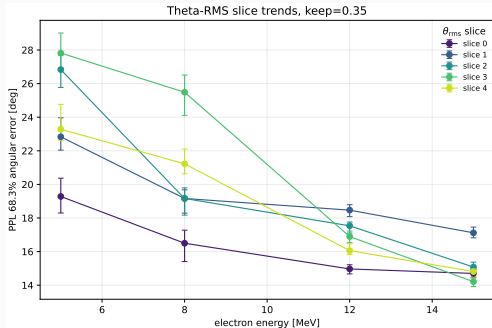
Track prior

Direction prior

$$\times p(\Gamma | \hat{\mathbf{u}}_0) p(\hat{\mathbf{u}}_0).$$

- Track posterior: high-dimensional and multimodal.
- Different bent tracks can support similar photon directions.
- Thus, no improvement on initial e^- direction inference.

PPL scattering-severity classes

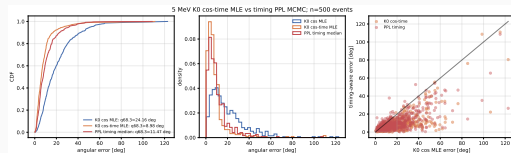


- Posterior theta-RMS slices sort events by angular response, similar concept as multiple-scattering-goodness
- Inclusive PPL direction estimates remain comparable to, or worse than, the marginal template in the tested angular-only studies.

Timing

Timing is an identifiability lever

Method, 500 events	68.3%
$K = 0, p_0(\cos \theta)$	24.158°
$K = 0, p_0(\cos \theta, t)$	8.982°
Latent PPL, directions	26.757°
Latent PPL, directions+times	11.468°



- These are PPL-forward 5 MeV closure tests with truth emission time.
- The fair timing baseline is the single-photon marginal joint template $p_0(\cos \theta, t)$.
- Timing helps strongly; the current latent-track sampler is still slower and less stable.
- However, this requires PMT TTS to be $< 100\text{ps}$.

Outlook

Current picture

- Photon directions contain measurable non-iid structure beyond the marginal Cherenkov angular profile.
- SBI gives a surrogate track-aware likelihood using classifier approximated likelihood-ratio, but only a small tested inclusive direction gain.
- PPL gives a physics-coded latent electron track and calibrated forward closure, but angular-only latent-track inference is multimodal, explaining why it is difficult to achieve angular resolution boost from track-aware reconstruction.
- SBI score (likelihood-ratio) or posterior scattering metrics contain information that measures electron scattering severity (same as MSG variable in SK LowE analysis chain), which can classify events with different angular resolution.
- Additional timing information provides much stronger constraint on electron initial direction, although $< 100\text{ps}$ TTS is required to resolve 5 MeV electron track.

Future thoughts for PPL

- Keep cross-energy calibration universal where the parameter has physics meaning.
- Train SBI classifiers as diagnostics for PPL-vs-Geant4 mismatch and eventually for adversarial calibration.

backup

Cerenkov angular distribution for segment

$$f_s(\hat{\mathbf{q}}) = \frac{\exp[-(\hat{\mathbf{q}} \cdot \hat{\mathbf{v}}_s - c_s)^2 / (2\sigma_x^2)]}{2\pi \int_{-1}^1 \exp[-(x - c_s)^2 / (2\sigma_x^2)] dx}.$$