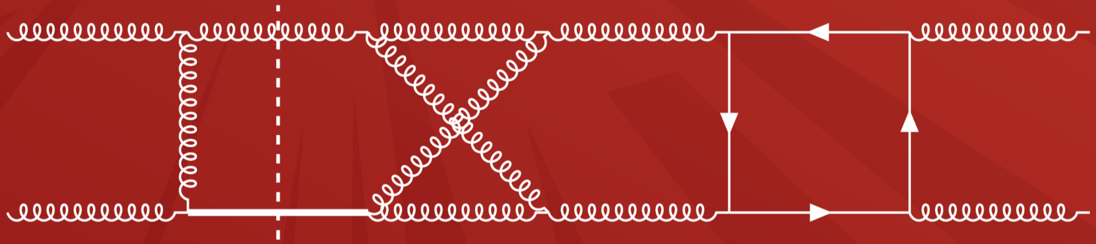
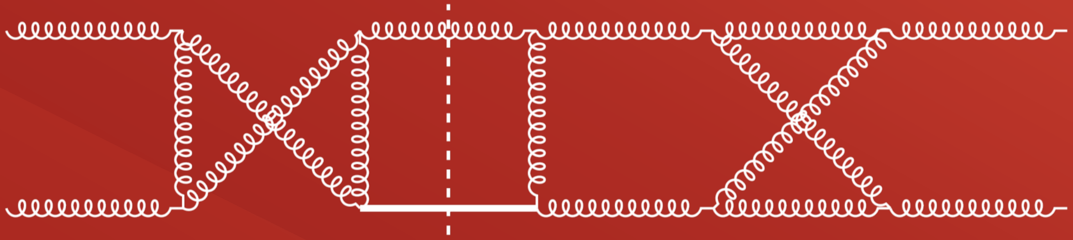
Taking the First Steps Toward the Higgs Production at N^4LO in QCD: the Single-real Ingredient

Adi Suresh & Bernhard Mistlberger

SLAC Theory Seminar (June 5th, 2026)



arXiv:2606.xxxxx



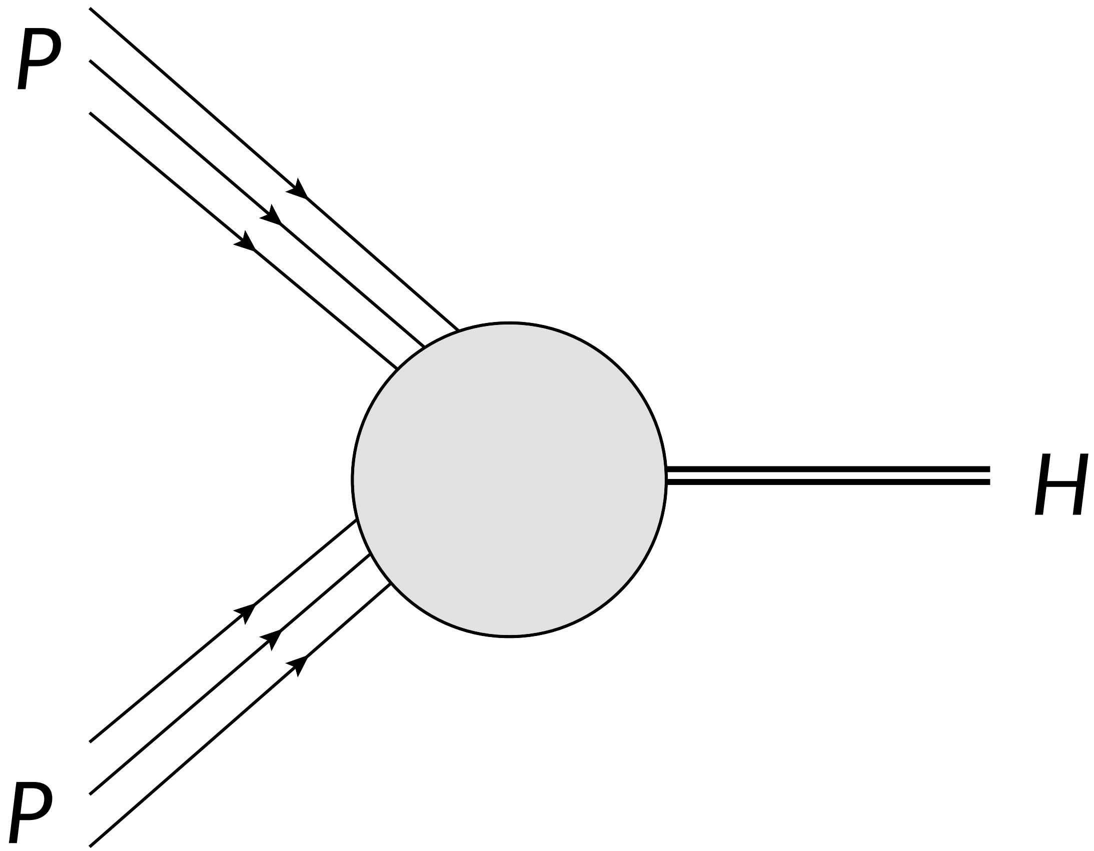
arXiv:2504.10574

SLAC NATIONAL ACCELERATOR LABORATORY

Stanford University

U.S. DEPARTMENT of ENERGY

How many Higgs bosons are produced at the LHC?



And how many did we expect…?

- i.e. what is the production rate?
- Signal strength:

$$\mu = \sigma_{\mathrm{obs}}/\sigma_{\mathrm{SM}}$$

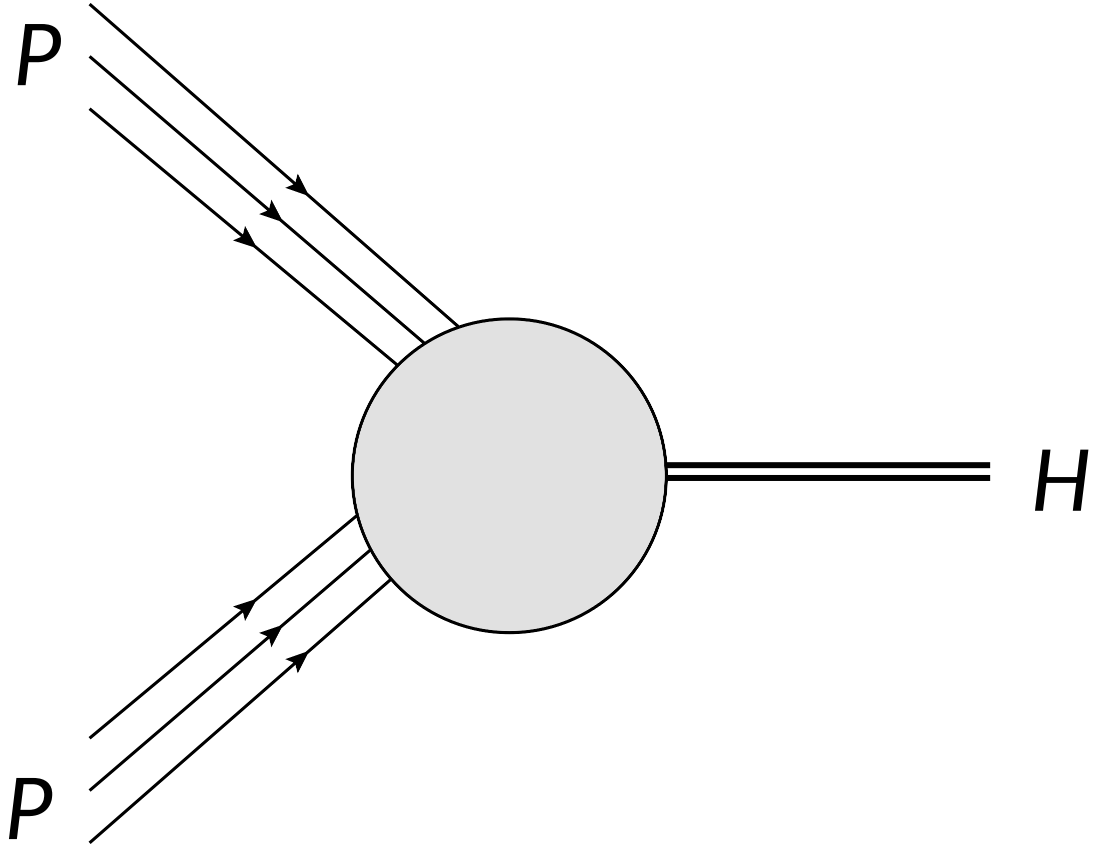
Observed cross section

Standard Model calculation

- ATLAS Run 2 (from *Nature* **607**, 52–59 (2022)):

$$\begin{aligned}\mu &= 1.05 \pm 0.06 \\ &= 1.05 \pm \boxed{0.03(\mathrm{stat.}) \pm 0.03(\mathrm{exp.})} \\ &\quad \boxed{\pm 0.04(\mathrm{sig.\ th.}) \pm 0.02(\mathrm{bkg.\ th})}\end{aligned}$$

How many Higgs bosons are produced at the LHC?



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Observed cross section

Standard Model calculation

- **ATLAS Run 2 Update** (from ATLAS-CONF-2025-006):

$$\begin{aligned}\mu &= 1.023^{+0.056}_{-0.053} \\ &= 1.023 \boxed{\pm 0.028(\text{stat.})\ ^{+0.026}_{-0.025}\,(\text{exp.})} \\ &\qquad \boxed{^{+0.039}_{-0.036}(\text{sig. th.}) \pm 0.012(\text{bkg. th})}\end{aligned}$$

In the foreseeable future:

LHC Era	(Projected) Integrated Luminosity (fb^{-1})
Run 2	140
Run 3 (2026)	300+
HL-LHC (2040)	**3000+**

We will be collecting a lot more data!

- Along with improvements in experimental uncertainty, **much better statistics!**
- The LHC will deliver measurements at **the percent level!**
- We will need to get more precise on theory side…

Outline

1. Higgs boson production at hadron colliders.
    - How and why do we count them?
    - How are they produced?
2. The journey to the precision frontier.
3. The single-real ingredient to the inclusive production cross section.

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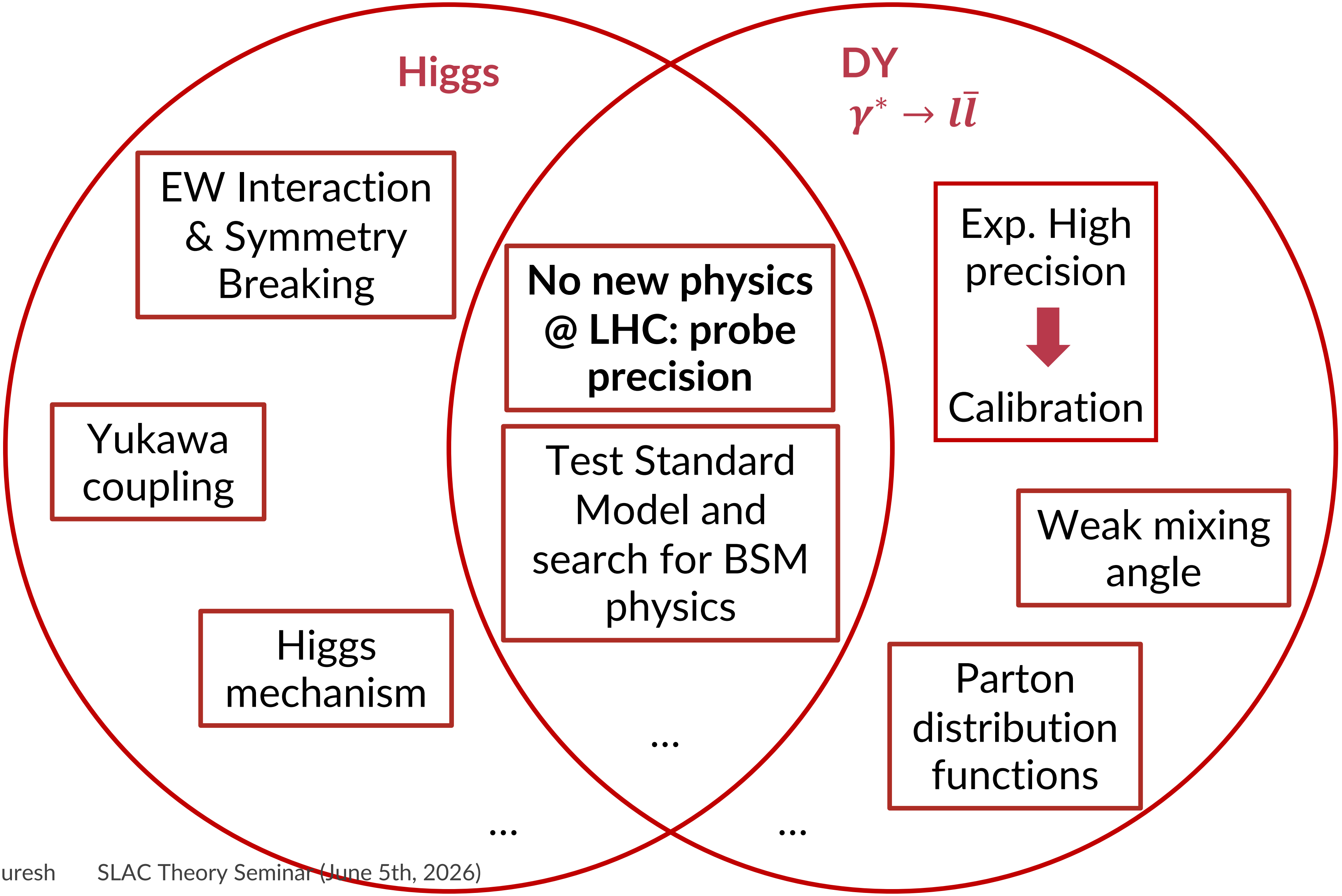
Why probe Higgs and Drell-Yan Production?



Higgs

EW Interaction & Symmetry Breaking

Yukawa coupling

Higgs mechanism

…

No new physics @ LHC: probe precision

Test Standard Model and search for BSM physics

…

DY

$\gamma^* \to l\bar{l}$

Exp. High precision ⬇ Calibration

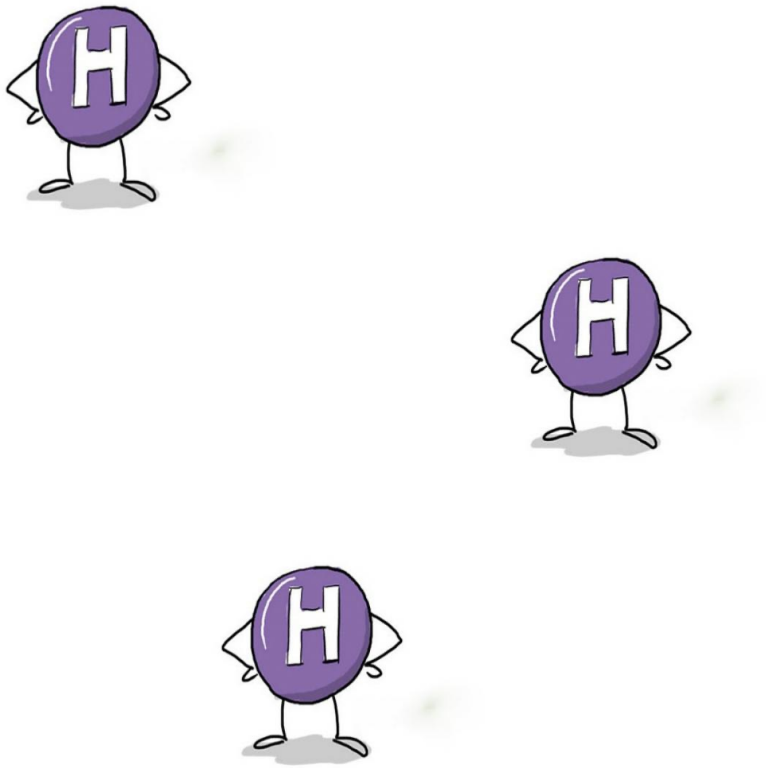
Weak mixing angle

Parton distribution functions

…

The Inclusive Cross Section

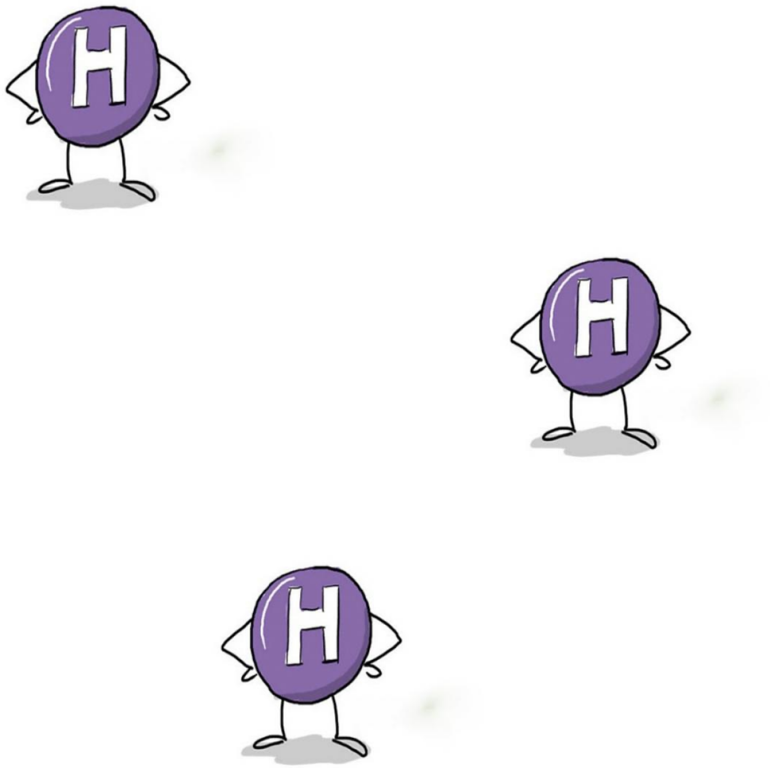
- The inclusive cross section serves as a fundamental benchmark for all other measurements
- Enters into global fits
  - Higgs couplings
  - PDF constraints
- Precise differential predictions require precise inclusive benchmarks.



From Counts to Cross Sections

How do we determine the cross section from what we see in the detector?

- $N_{PP\to H} = \mathcal{L}\,\sigma_{PP\to \mathrm{H+X}}\,\mathrm{BR}_{H\to \mathrm{Y}}\,A\,\epsilon - N_{\mathrm{bkg}}$
- We need to understand:
  - Luminosity
  - Branching Ratio
  - Acceptance and efficiency
  - Background



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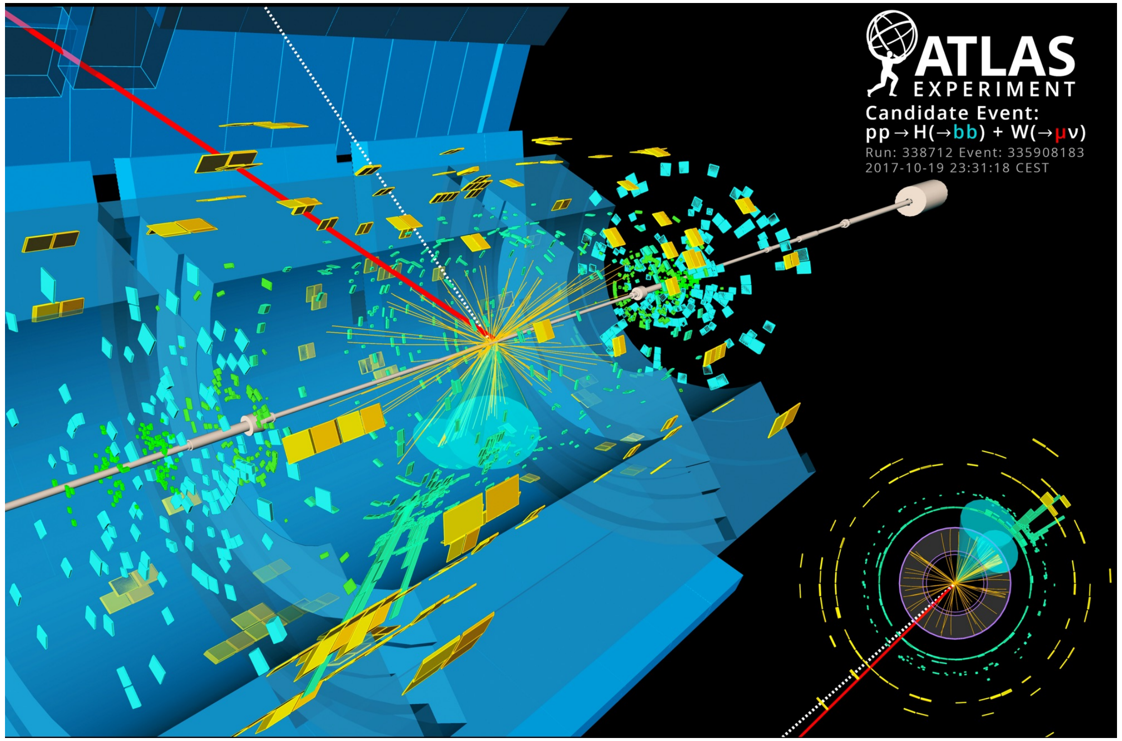
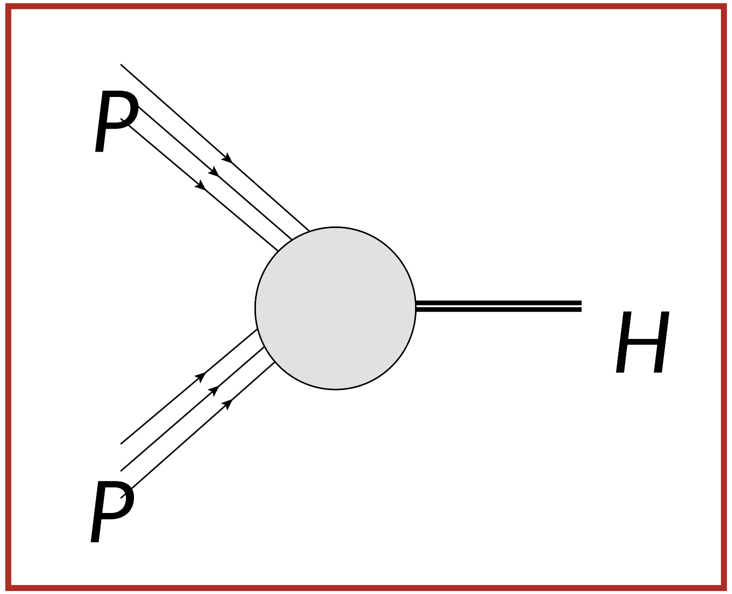


How are Higgs bosons produced at the LHC?

What we **want:** $\sigma_{PP\to H+X}$



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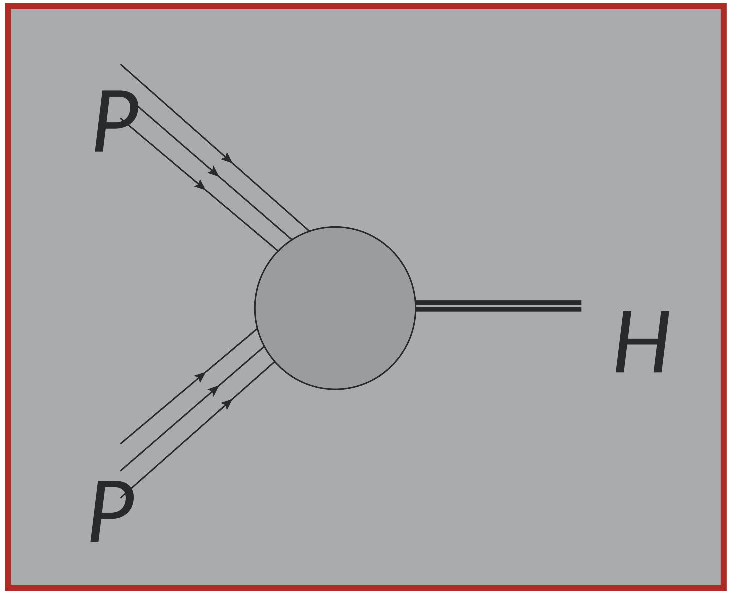
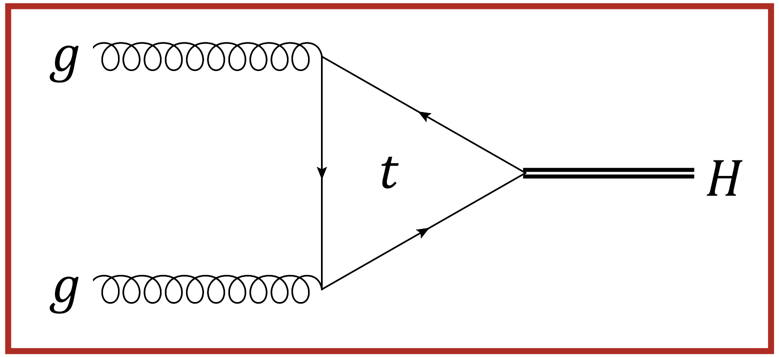


vs.

What we can **compute**: $\hat{\sigma}_{i\ j\to H+X}$



How are Higgs bosons produced at the LHC?

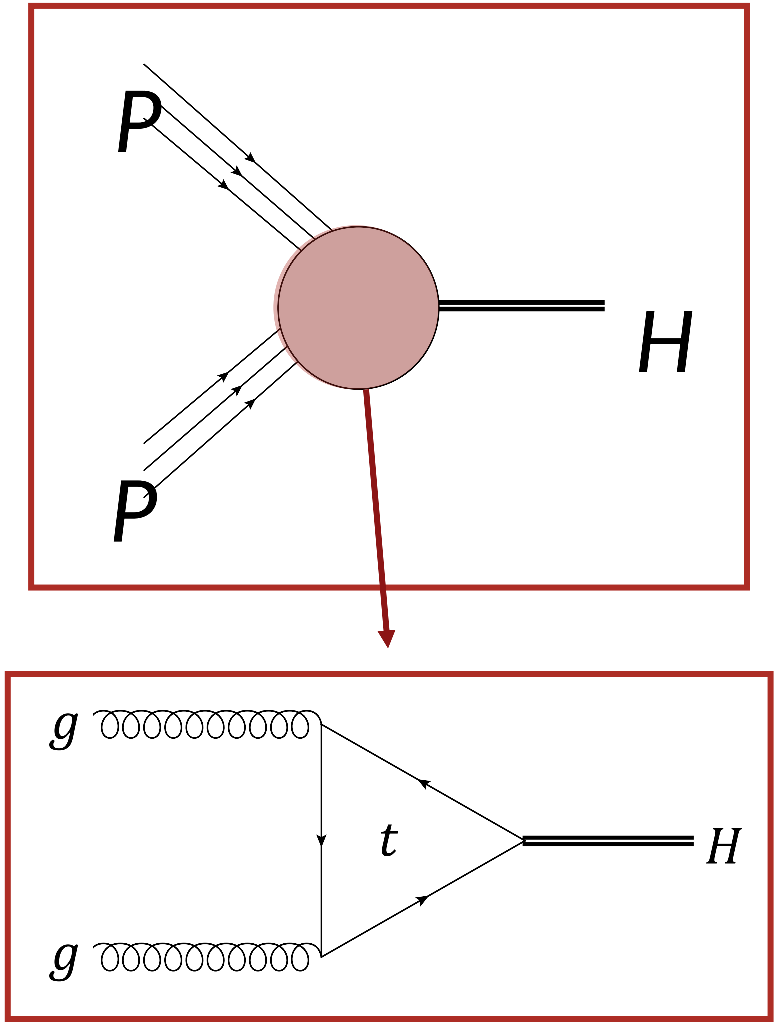
Factorization model:

$$\sigma = \sum_{i,j} f_i \otimes f_j \otimes \hat{\sigma}_{ij}$$

Hadronic process (long-range, nonperturbative)

PDFs (proton substructure)

Partonic process (short-range, perturbative!)

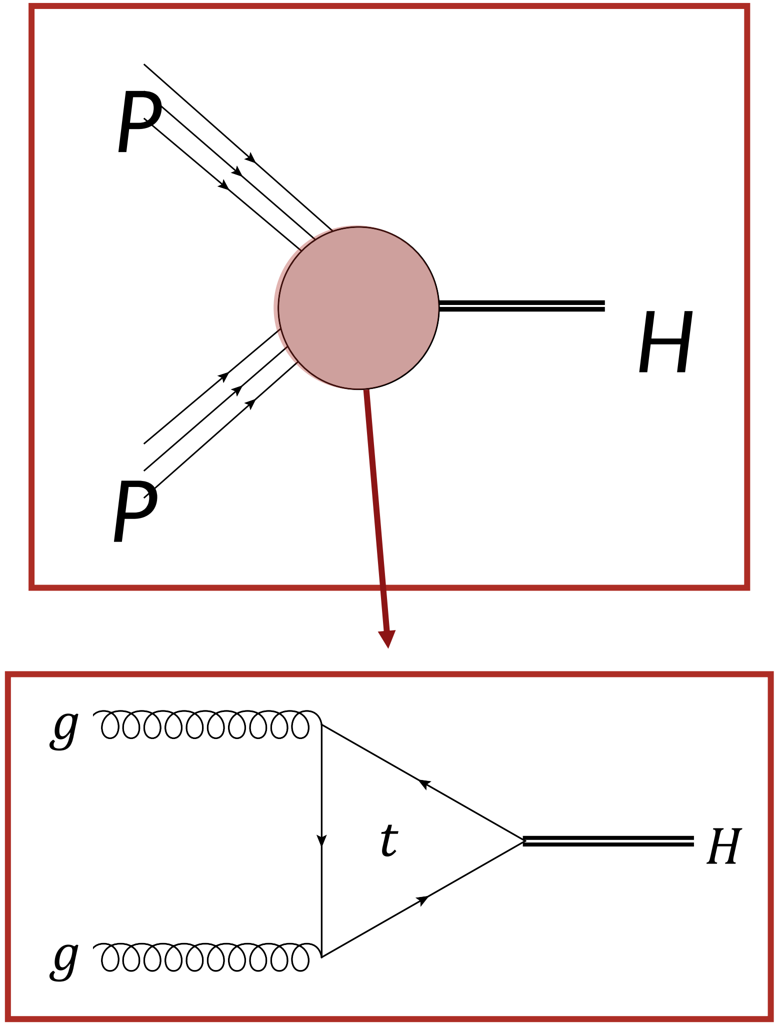


How are Higgs bosons produced at the LHC?

Factorization ~~model~~ theorem:

$$\sigma = \sum_{i,j} f_i \otimes f_j \otimes \hat{\sigma}_{ij}$$

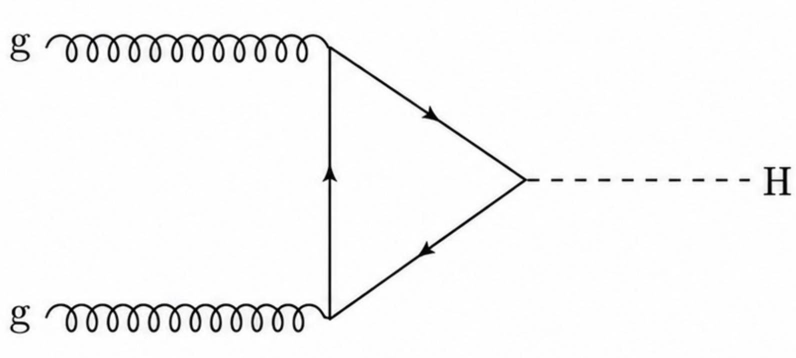
$$+\mathcal{O}\left(\Lambda^2_{QCD}/Q^2\right)$$



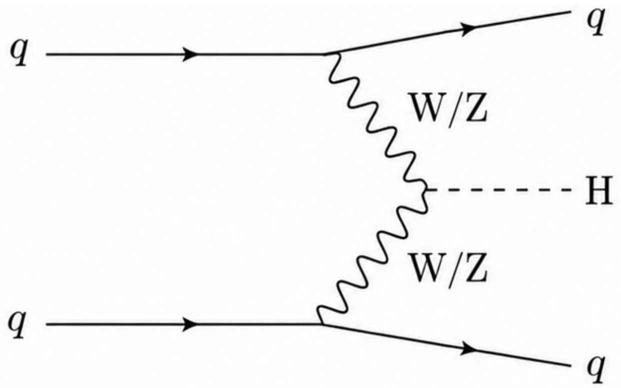
- Rigorously shown to be valid for inclusive color singlet production*

*[J. C. Collins, D. E. Soper, G. Sterman, Nucl. Phys. B261 (1985) 104; Nucl. Phys. B308 (1988) 833.]

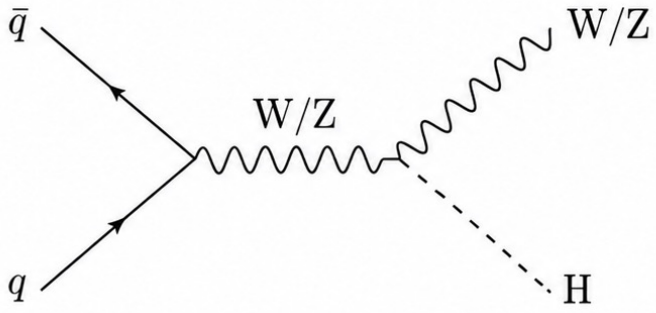
How are Higgs bosons produced at the LHC partonically?*



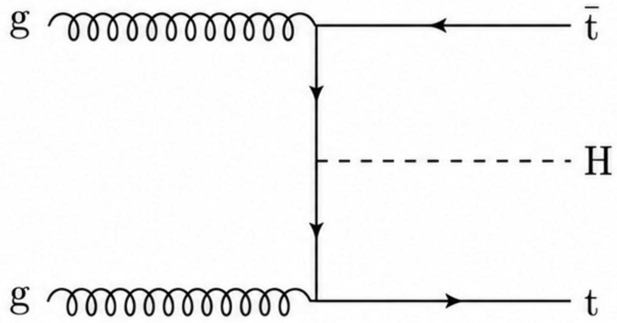
Gluon-fusion



Vector-boson fusion



W/Z Associated



$t\bar{t}$ associated

*[Higgs Cross Section Working Group; arXiv:1610.07922]

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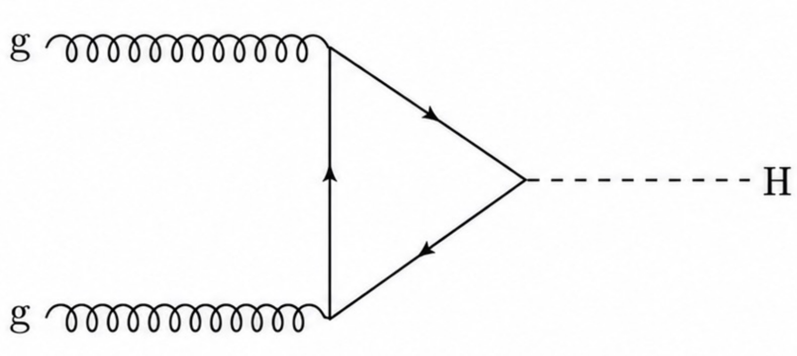
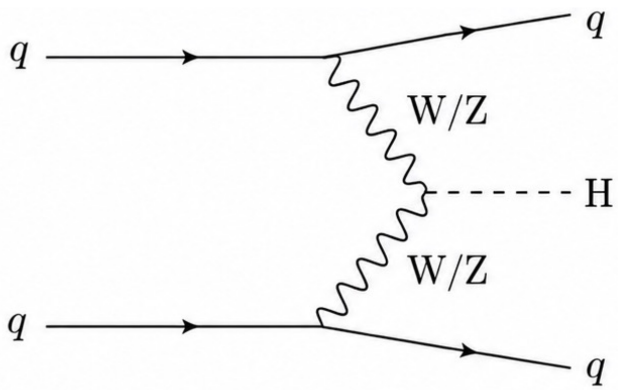
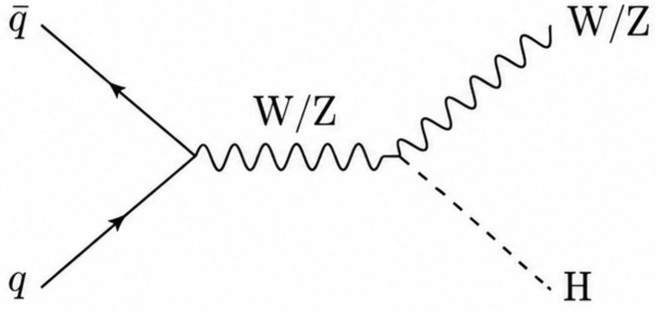


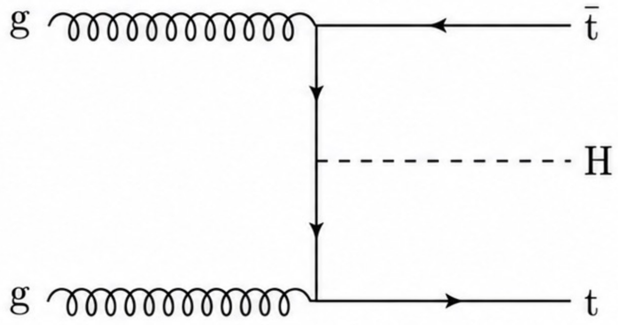
Gluon-fusion **88%**



Vector-boson fusion **4%**



W/Z Associated **7%**



$t\bar{t}$ associated **1%**

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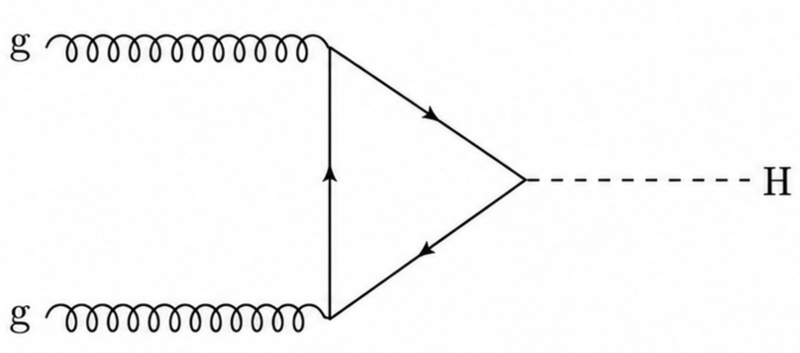

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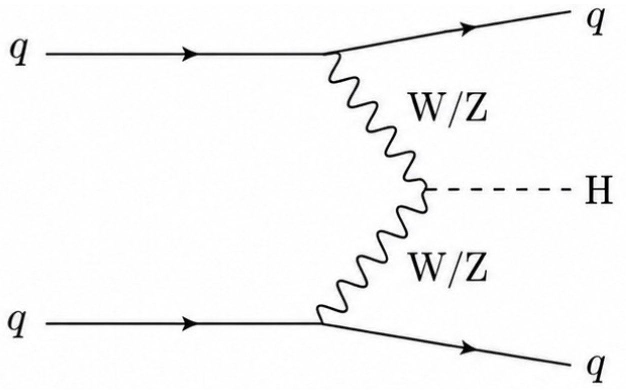

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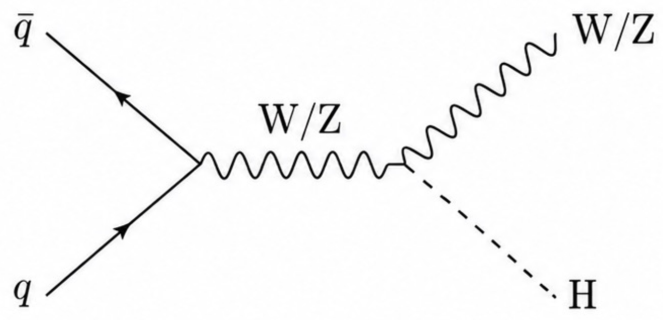

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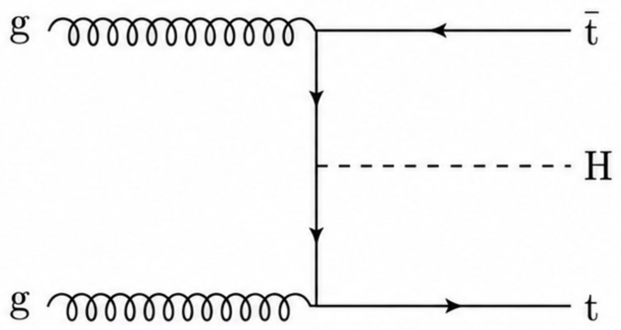

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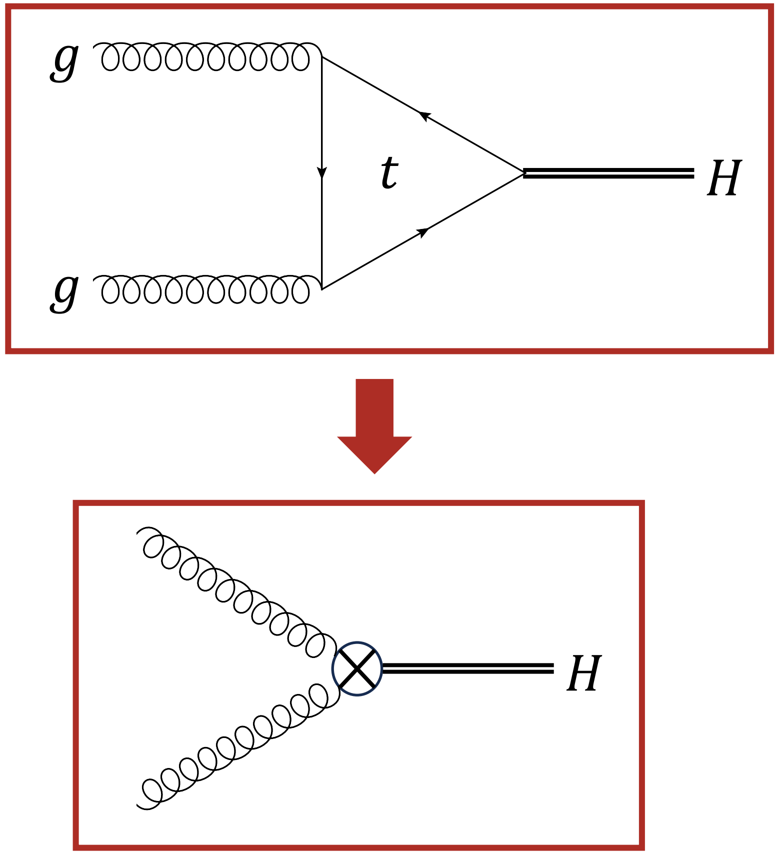

$$\mathcal{L} \supset -\frac{1}{4} C^0 H G^a_{\mu\nu} G^{\mu\nu}_a$$

About 90% are produced via gluon fusion!

- $gg \to H$ via top quark loop
- We can simplify:

  take "heavy-top" limit $m_{\text{top}} \to \infty$
  - i.e. leading term in m_H/m_{top} expansion
  - Higgs looks like it directly couples to gluons

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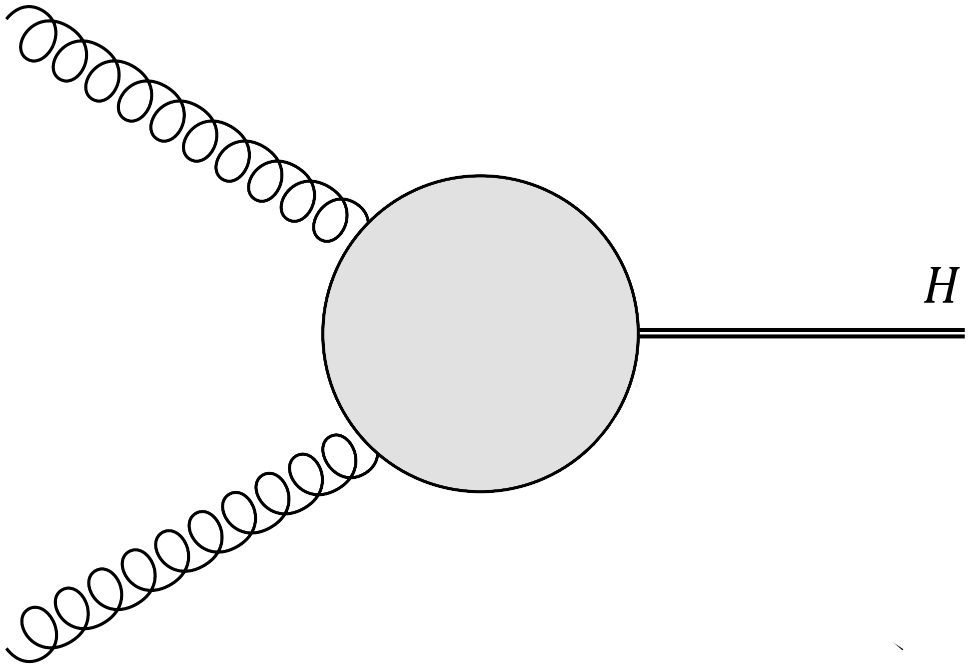
How do we compute how many Higgs bosons are produced?

To correctly compute an **inclusive** cross section:



- i.e. probability of Higgs produced

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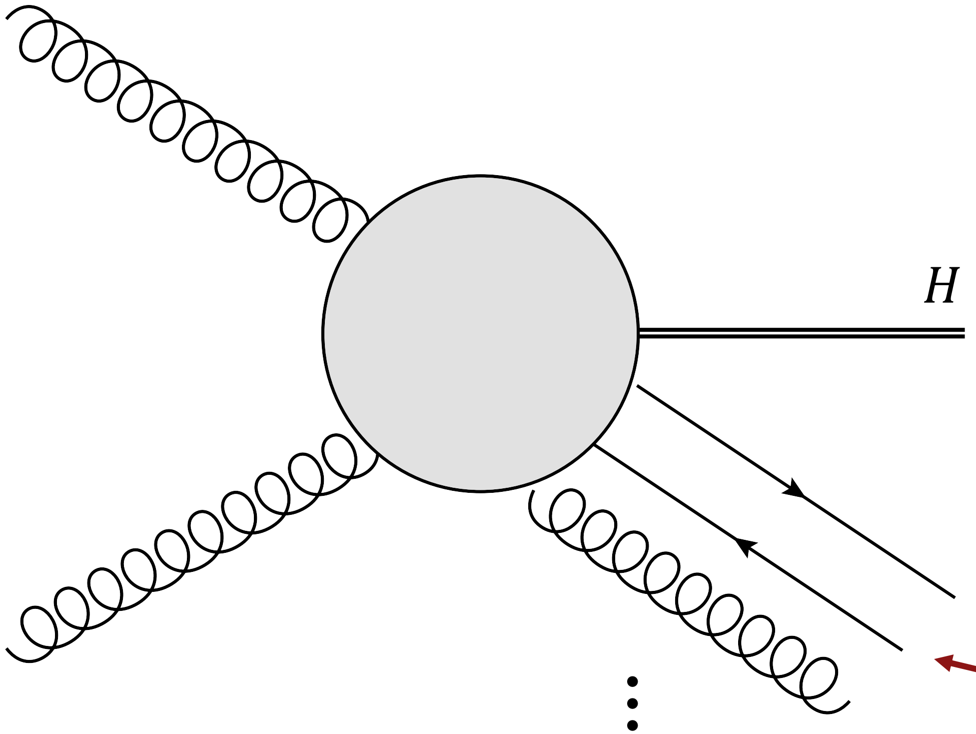

- i.e. probability of Higgs produced
- KLN theorem: Perturbative corrections must include:
  - loops &
  - **Radiation**

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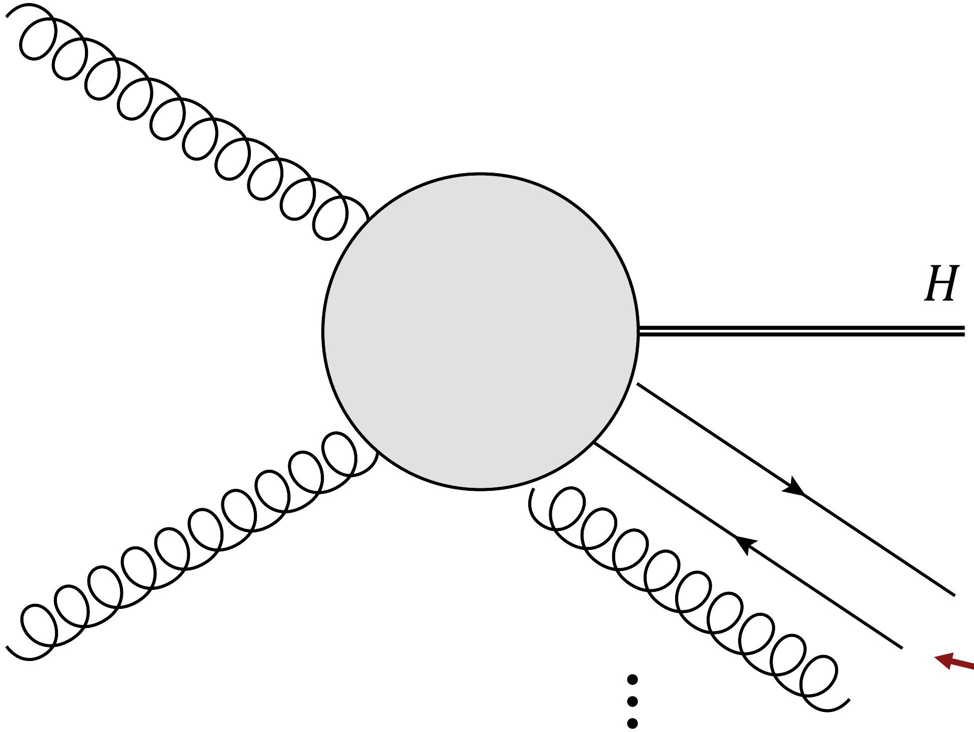
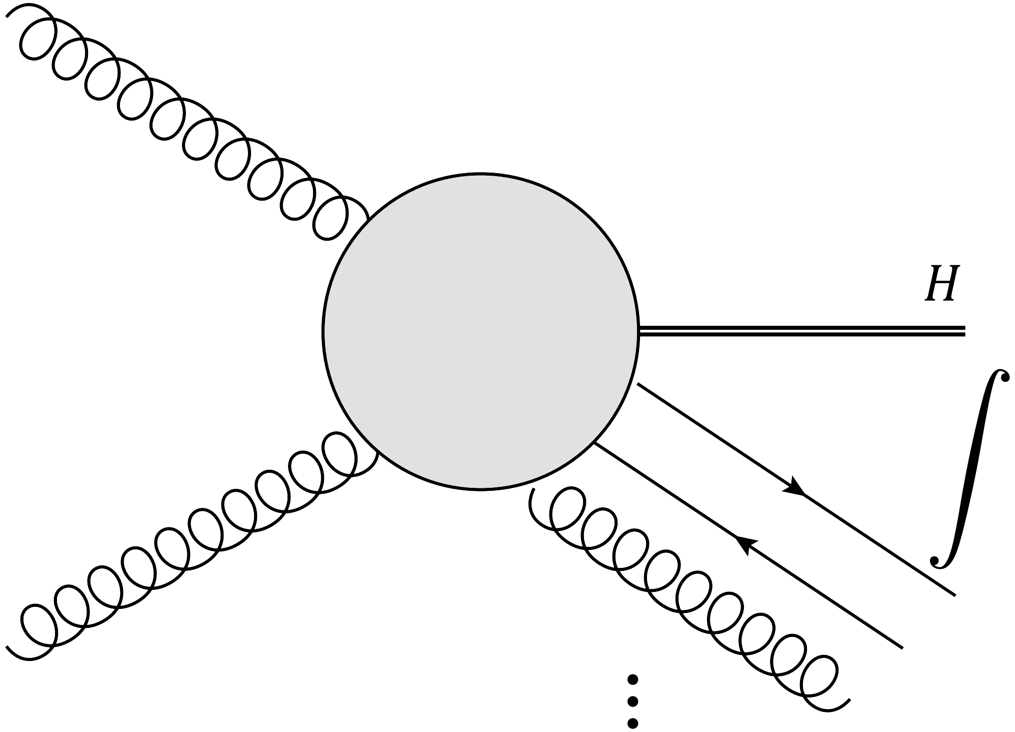


- i.e. probability of ***Higgs + X*** *produced*
- KLN theorem: Perturbative corrections must include:
  - loops &
  - **Radiation**

Building blocks at N^{k}LO:

To correctly compute an **inclusive** cross section:



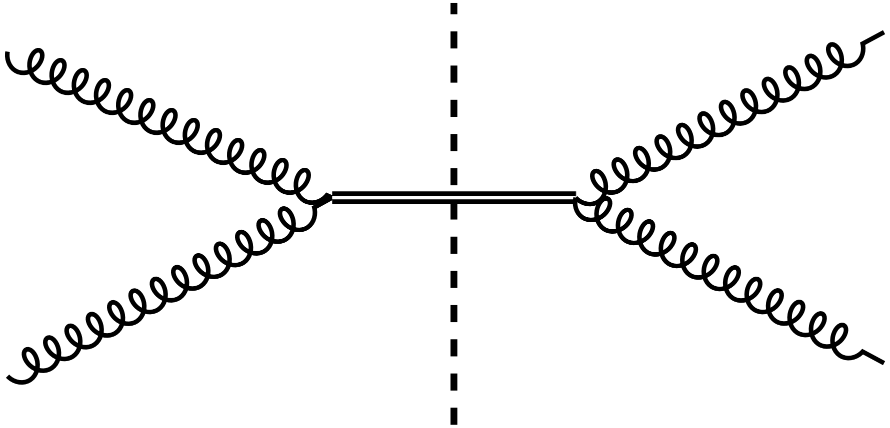
- Include up to k loops and k additional partons: $L + M = k$
- Integrate over $M + 1$ final state phase space
- Each piece is well-defined, gauge invariant
- Natural way to partition the computation

Theoretical Uncertainty for Gluon-fusion Higgs

Contribution* of the QCD perturbative expansion at each order (at 13 TeV):

*[Anastasiou, Duhr, Falko, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger; arXiv:1602.00695]

$$\begin{aligned}\hat{\sigma} = \hat{\sigma}_{\mathrm{LO}} &\\ +\hat{\sigma}_{\mathrm{NLO}} &\\ +\hat{\sigma}_{\mathrm{NNLO}} &\\ +\hat{\sigma}_{\mathrm{N3LO}} &\\ +\cdots &\end{aligned}$$



Born Process

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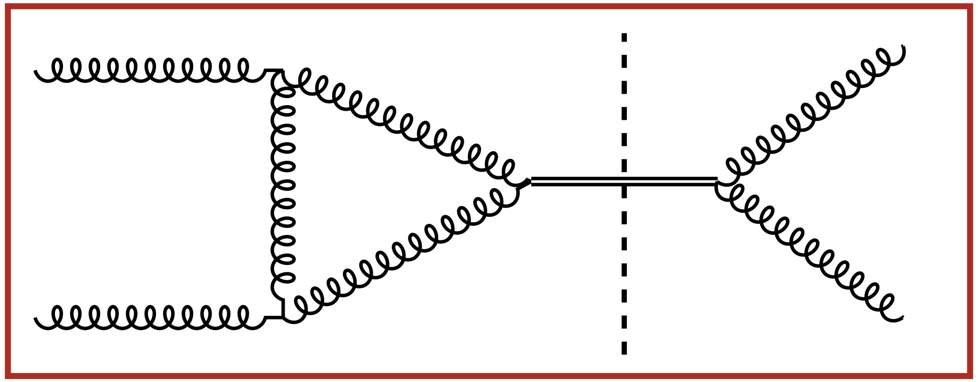
$$\boxed{\boldsymbol{\mathcal{O}(100\%)}}$$

**[Dawson; Nucl.Phys.B 359 (1991), 283-300]

The Gluon-fusion Higgs at NLO*

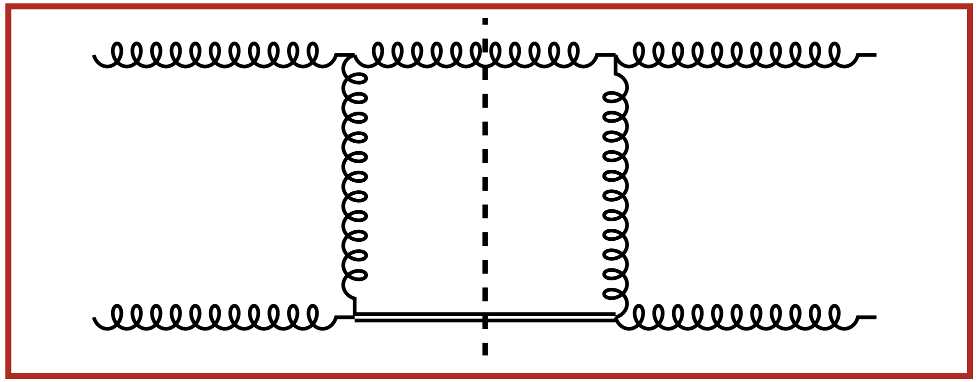
Ingredients:



V

Virtual Correction

+



R

Real Correction

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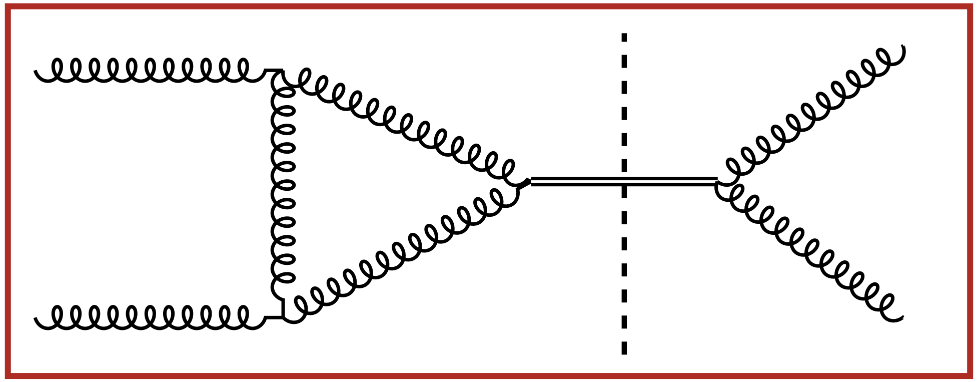

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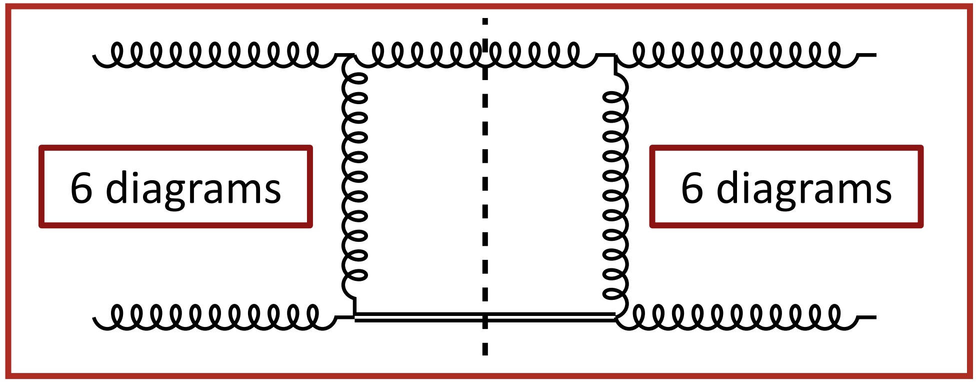

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The Gluon-fusion Higgs at NLO*

Result

$$\hat{\sigma}_{\mathrm{NLO,reg}} \sim \frac{11 n_C \bar{\mathrm{z}}^3}{6(\bar{z}-1)} + \frac{2 n_C \ \ (\bar{\mathrm{z}}^2 - \bar{z} + 1)^2 \log(1-\bar{z})}{(\bar{z}-1) \ \ \bar{z}} - \frac{4 n_C (\bar{\mathrm{z}}^3 - 2\bar{\mathrm{z}}^2 + 3\bar{z} - 1) \log \ \ \ (\bar{z})}{\bar{z}-1}$$

- $\bar{z} = 1 - \frac{m_B^2}{s}$
- Simple rational functions and logs!

*[Dawson; Nucl.Phys.B 359 (1991), 283-300]

Current Theoretical Uncertainty for Gluon-fusion Higgs

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$$\boxed{O(100\%)}$$

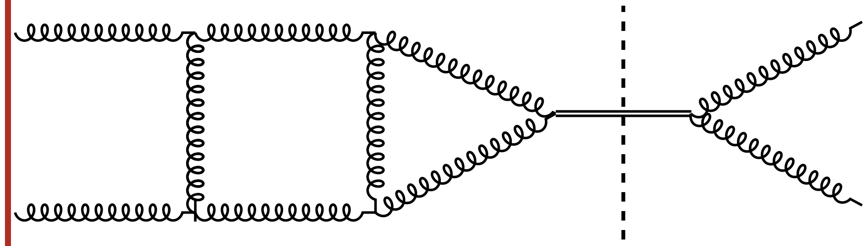
$$\boxed{\boldsymbol{O(25\%)}}$$

**[Dawson; Nucl.Phys.B 359 (1991), 283-300]

***[Harlander, Kilgore; arXiv:0201206], [Anastasiou, Melnikov; arXiv: 0207004]

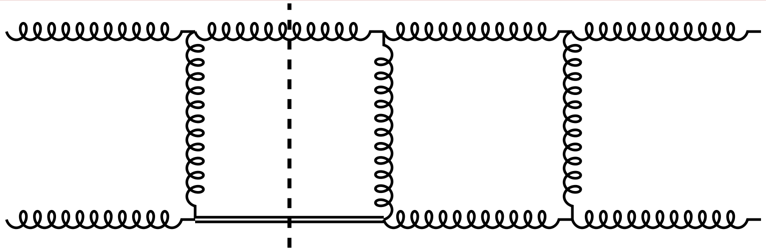
The Gluon-fusion Higgs at NNLO*

Ingredients:

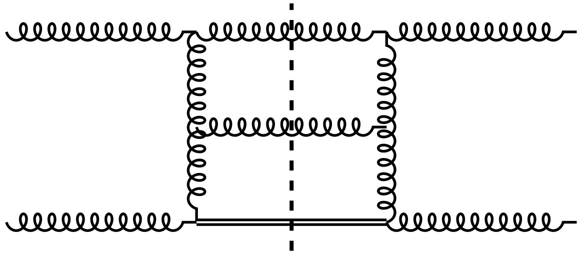


+ 1-loop self-interference

VV Double-Virtual Correction



RV Real-Virtual Correction



RR Double-Real Correction

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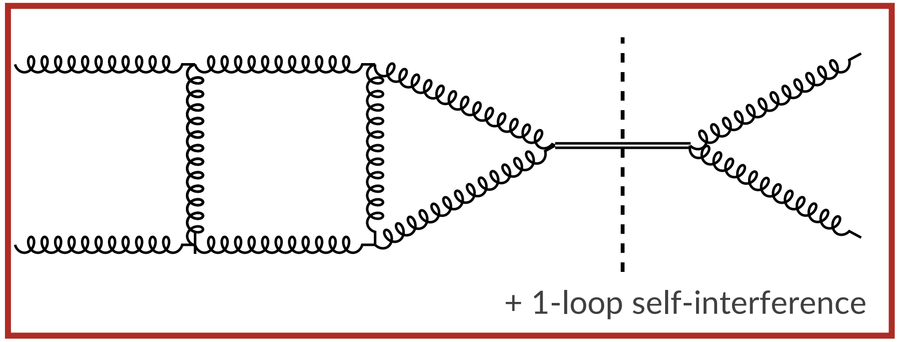
Ingredients:



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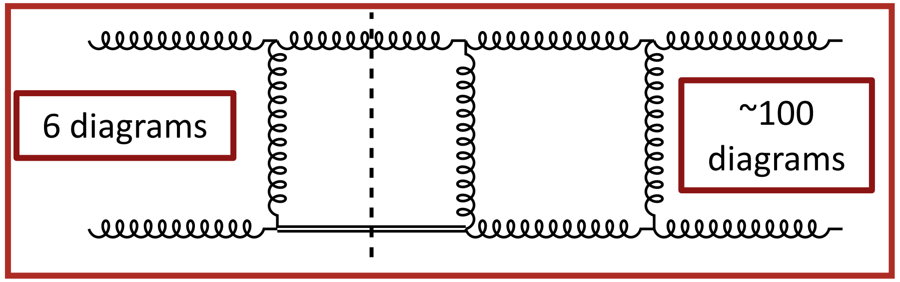
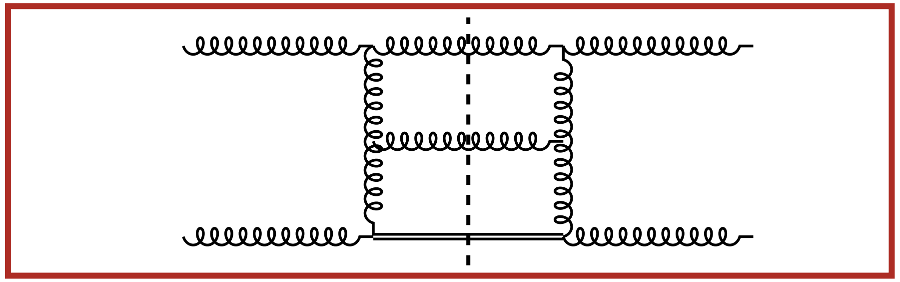


6 diagrams

~100 diagrams

RV

Real-Virtual Correction



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The Gluon-fusion Higgs at NNLO*

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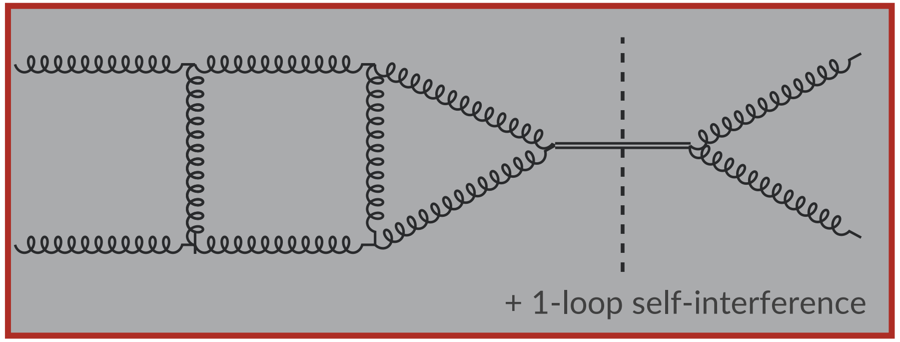

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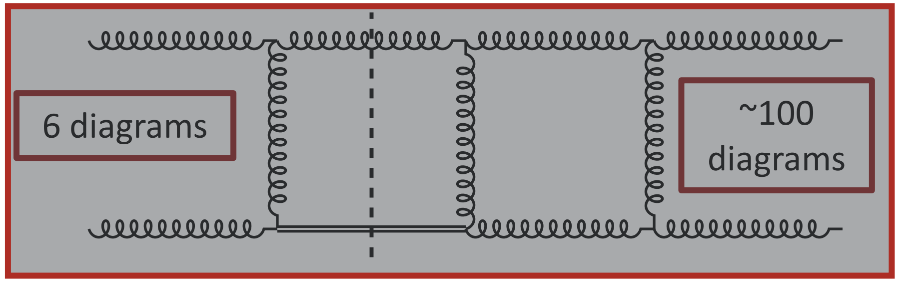
Double-Virtual Correction



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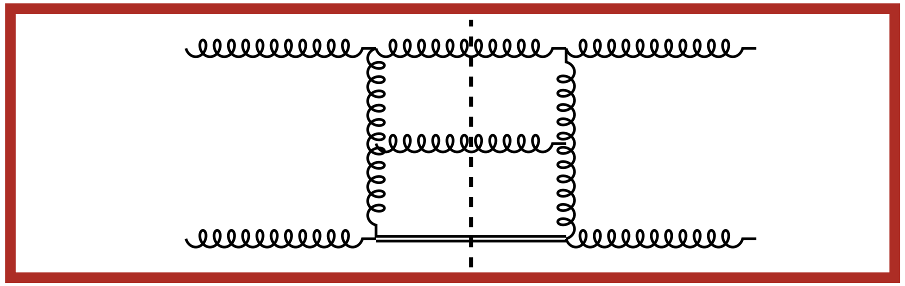

RR

Double-Real Correction

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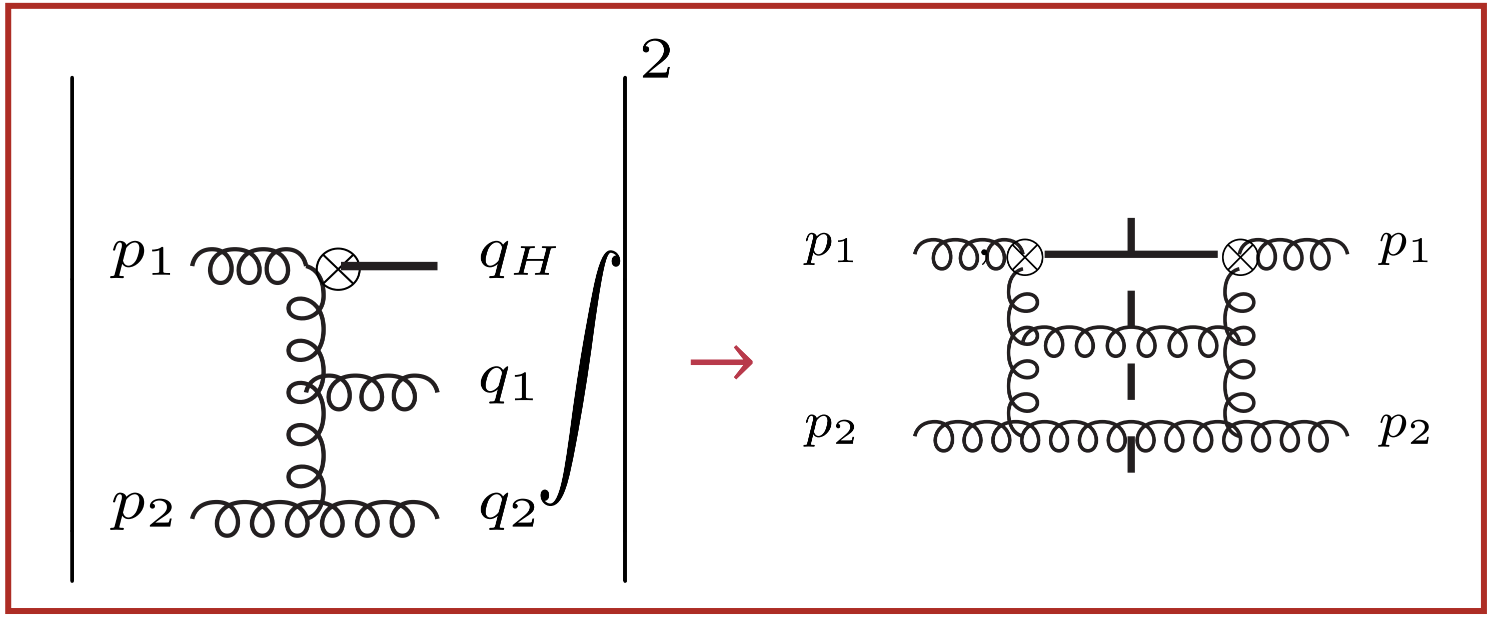
*[Harlander, Kilgore; arXiv:0201206], [Anastasiou, Melnikov; arXiv: 0207004]

The Gluon-fusion Higgs at NNLO*

The reverse unitarity approach



$$\delta(q^2 - m^2) \to \frac{i}{q^2 - m^2} - \text{c. c.}$$

- Phase space integration becomes loop integration…
- Can use loop integration technology!

*[Harlander, Kilgore; arXiv:0201206], [Anastasiou, Melnikov; arXiv: 0207004]

The Gluon-fusion Higgs at NNLO*

Result

$\hat{\sigma}_{\mathrm{NNLO,reg}} \sim$

$$
\begin{aligned}
&\frac{4\,G(2,1,0,\bar z)\,(\bar z^2-3\,\bar z+3)^2\,n_C^2}{(\bar z-2)\,(\bar z-1)}+\frac{31\,G(0,1,0,\bar z)\,(\bar z^2-\bar z+1)^2\,n_C^2}{(\bar z-1)\,\bar z}+\frac{3\,G(2,2,1,\bar z)\,(3\,\bar z^2-8\,\bar z+6)\,n_C^2}{(\bar z-2)\,(\bar z-1)}-\frac{48\,G(0,0,0,\bar z)\,(\bar z^3-2\,\bar z^2+3\,\bar z-1)\,n_C^2}{\bar z-1}+\frac{G(2,1,\bar z)\,(11\,\bar z^3-54\,\bar z^2+63\,\bar z-6)\,n_C^2}{6\,(\bar z-1)}-\\
&\frac{G(1,2,1,\bar z)\,(3\,\bar z^4-16\,\bar z^3+36\,\bar z^2-40\,\bar z+19)\,n_C^2}{(\bar z-2)\,(\bar z-1)}-\frac{G(2,1,1,\bar z)\,(4\,\bar z^4-24\,\bar z^3+75\,\bar z^2-112\,\bar z+66)\,n_C^2}{2\,(\bar z-2)\,(\bar z-1)}+G(1,1,1,\bar z)\left(\frac{(9\,\bar z^5-28\,\bar z^4+15\,\bar z^3+15\,\bar z^2-\bar z-14)\,n_C^2}{(\bar z-2)\,(\bar z-1)\,\bar z}-\frac{5}{4}\,(\bar z-2)\,n_f\,n_C+\frac{5\,(\bar z-2)\,n_f}{4\,n_C}\right)+\\
&G(1,0,1,\bar z)\left(\frac{(-37\,\bar z^5+98\,\bar z^4-10\,\bar z^3-100\,\bar z^2+11\,\bar z+78)\,n_C^2}{2\,(\bar z-2)\,(\bar z-1)\,\bar z}+\frac{3}{2}\,(\bar z-2)\,n_f\,n_C-\frac{3\,(\bar z-2)\,n_f}{2\,n_C}\right)+G(1,1,0,\bar z)\left(-\frac{2\,(9\,\bar z^5-26\,\bar z^4+11\,\bar z^3+14\,\bar z^2+\bar z-16)\,n_C^2}{(\bar z-2)\,(\bar z-1)\,\bar z}+2\,(\bar z-2)\,n_f\,n_C-\frac{2\,(\bar z-2)\,n_f}{n_C}\right)+\\
&\frac{G(0,0,1,\bar z)\,(2\,(59\,\bar z^4-120\,\bar z^3+183\,\bar z^2-124\,\bar z+61)\,n_C^3-\bar z\,(\bar z^2+1)\,n_f)}{4\,(\bar z-1)\,\bar z\,n_C}+\frac{G(0,1,1,\bar z)\,(\bar z\,(\bar z^2+1)\,n_f-8\,(6\,\bar z^4-13\,\bar z^3+21\,\bar z^2-15\,\bar z+7)\,n_C^3)}{4\,(\bar z-1)\,\bar z\,n_C}+\frac{G(1,0,0,\bar z)\,((31\,\bar z^4-30\,\bar z^3-3\,\bar z^2+2\,\bar z+31)\,n_C^3-2\,\bar z\,(\bar z^2-3\,\bar z+2)\,n_f\,n_C^2+2\,\bar z\,(\bar z^2-3\,\bar z+2)\,n_f)}{(\bar z-1)\,\bar z\,n_C}+\\
&\frac{G(0,0,\bar z)\,((220\,\bar z^3-428\,\bar z^2+450\,\bar z-22)\,n_C^3+(-8\,\bar z^3+23\,\bar z^2-27\,\bar z+4)\,n_f\,n_C^2+\bar z\,(4\,\bar z^2-15\,\bar z+15)\,n_f)}{3\,(\bar z-1)\,n_C}+\frac{G(0,\bar z)\,((-2295\,\bar z^3+4991\,\bar z^2-5256\,\bar z+60\,\pi^2\,(\bar z^3-2\,\bar z^2+3\,\bar z-1)+265)\,n_C^3+(182\,\bar z^3-533\,\bar z^2+534\,\bar z-37)\,n_f\,n_C^2-4\,\bar z\,(17\,\bar z^2-84\,\bar z+75)\,n_f)}{36\,(\bar z-1)\,n_C}+\\
&\frac{G(1,\bar z)\,((4054\,\bar z^4-9789\,\bar z^3+10\,698\,\bar z^2-3162\,\bar z-12\,\pi^2\,(9\,\bar z^4-2\,\bar z^3-21\,\bar z^2+14\,\bar z+9)+268)\,n_C^3+(-280\,\bar z^4+3\,(261+4\,\pi^2)\,\bar z^3-18\,(37+2\,\pi^2)\,\bar z^2+3\,(19+8\,\pi^2)\,\bar z-40)\,n_f\,n_C^2-4\,\bar z\,(-34\,\bar z^3+138\,\bar z^2-120\,\bar z+3\,\pi^2\,(\bar z^2-3\,\bar z+2)+8)\,n_f)}{72\,(\bar z-1)\,\bar z\,n_C}+\\
&\frac{G(1,1,\bar z)\,(2\,(374\,\bar z^4-669\,\bar z^3+462\,\bar z^2-24\,\bar z+11)\,n_C^3+(-32\,\bar z^4+45\,\bar z^3+33\,\bar z^2-58\,\bar z-4)\,n_f\,n_C^2+\bar z\,(24\,\bar z^3-51\,\bar z^2-9\,\bar z+44)\,n_f)}{24\,(\bar z-1)\,\bar z\,n_C}+\\
&\frac{G(1,0,\bar z)\,(-2\,(187\,\bar z^4-383\,\bar z^3+339\,\bar z^2-66\,\bar z+11)\,n_C^3+(12\,\bar z^4-19\,\bar z^3-6\,\bar z^2+17\,\bar z+4)\,n_f\,n_C^2+\bar z\,(-8\,\bar z^3+15\,\bar z^2+6\,\bar z-17)\,n_f)}{6\,(\bar z-1)\,\bar z\,n_C}+\\
&\frac{G(0,1,\bar z)\,((-605\,\bar z^4+1240\,\bar z^3-1317\,\bar z^2+96\,\bar z-55)\,n_C^3+(24\,\bar z^4-67\,\bar z^3+75\,\bar z^2-18\,\bar z+10)\,n_f\,n_C^2-3\,\bar z\,(4\,\bar z^3-16\,\bar z^2+15\,\bar z+1)\,n_f)}{12\,(\bar z-1)\,\bar z\,n_C}+\\
&\frac{1}{432\,(\bar z-1)\,n_C}\left(\left((21\,165-8424\,\zeta(3))\,\bar z^3+16\,(-2734+1053\,\zeta(3))\,\bar z^2-36\,(-1181+702\,\zeta(3))\,\bar z-6\,\pi^2\left(363\,\bar z^3-856\,\bar z^2+900\,\bar z-44\right)+8424\,\zeta(3)-1616\right)n_C^3+\right.\\
&\quad\left.\left(-2030\,\bar z^3+5941\,\bar z^2-5778\,\bar z+12\,\pi^2\left(8\,\bar z^3-23\,\bar z^2+27\,\bar z-4\right)+224\right)n_f\,n_C^2-4\,\bar z\left(-124\,\bar z^2+987\,\bar z+3\,\pi^2\left(4\,\bar z^2-15\,\bar z+15\right)-825\right)n_f\right)
\end{aligned}
$$

- *Not just* simple rational functions and logs!
- MPLs $\mathtt{G}(\ldots,\bar{\mathtt{z}})$ and transcendental constants enter

*[Harlander, Kilgore; arXiv:0201206], [Anastasiou, Melnikov; arXiv: 0207004]

Multiple polylogarithms (MPLs)*

Recursively defined iterated integrals:

$$\boxed{\begin{gathered} G(a_1, a_2, \ldots, a_n, x) = \int_0^x \frac{dt}{t - a_1} G(a_2, \ldots, a_n, t) \\ \text{And } G\left(\vec{0}_n, x\right) = \frac{\log x}{n!} \end{gathered}}$$

- Ubiquitous in multiloop amplitudes and generally well-understood

*A. B. Goncharov, Math. Res. Lett. 5 (1998) 497; arXiv:1105.2076.

The Symbol of MPLs

Decompose an MPL into “letters” s_j

$$\mathcal{S}[G(a_1, a_2, \dots, a_n, x)] = \sum_i c_i s_{i_1} \otimes \cdots \otimes s_{i_n}$$

Integration kernel
at each iteration

- For MPLs, “$d\log$” letters
  - i.e. $s_j = d\log[x - a] = \frac{1}{x-a}$
- Non-“$d\log$” letters describe a more general class of functions

*A. B. Goncharov, Math. Res. Lett. 5 (1998) 497; arXiv:1105.2076.

Current Theoretical Uncertainty for Gluon-fusion Higgs

Correction* of the QCD perturbative expansion at each order (at 13 TeV): Charalampos

*[Anastasiou, Duhr, Falko, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger; arXiv:1602.00695]

$$\hat{\sigma} = \hat{\sigma}_{\mathrm{LO}}$$

$$+\hat{\sigma}_{\mathrm{NLO}}{}^{**}$$

$$+\hat{\sigma}_{\mathrm{NNLO}}{}^{***}$$

$$+\boldsymbol{\hat{\sigma}_{\mathrm{N3LO}}}{}^{*}$$

$$+\cdots$$

$$\boxed{O(100\%)}$$

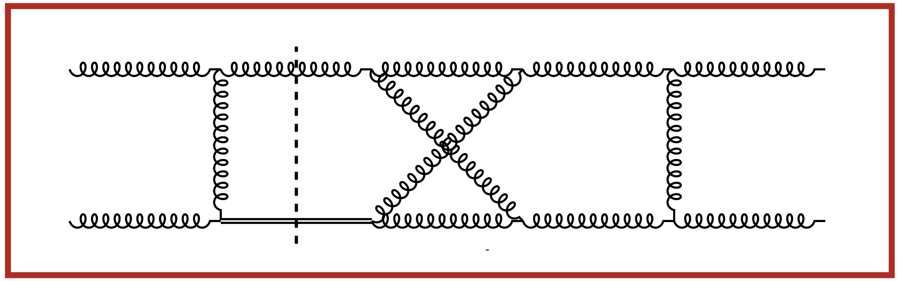
$$\boxed{O(25\%)}$$

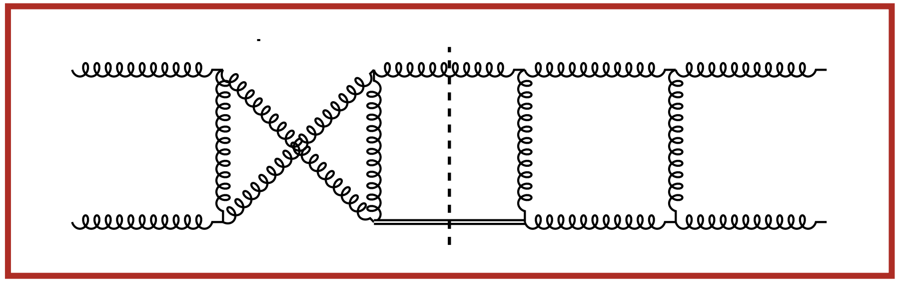
$$\boxed{O(5\%)}$$

**[Dawson; Nucl.Phys.B 359 (1991), 283-300]
***[Harlander, Kilgore; arXiv:0201206],
[Anastasiou, Melnikov; arXiv: 0207004]

The Gluon-fusion Higgs at $\mathrm{N^3LO}$*

Ingredients:





VVV Triple-Virtual Corrections

RV^2

RVV

Single-real Corrections

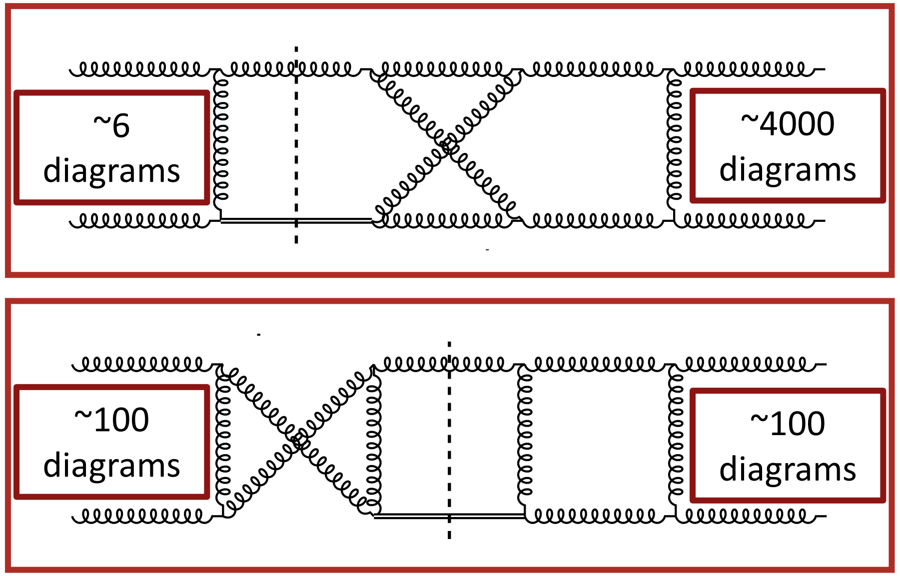
RRV Double-real Correction

RRR Triple-Real Correction

***[Mistlberger, JHEP 05 (2018) 028, arXiv: 1802.00833.]

The Gluon-fusion Higgs at $\mathrm{N}^3\mathrm{LO}^*$

Ingredients:



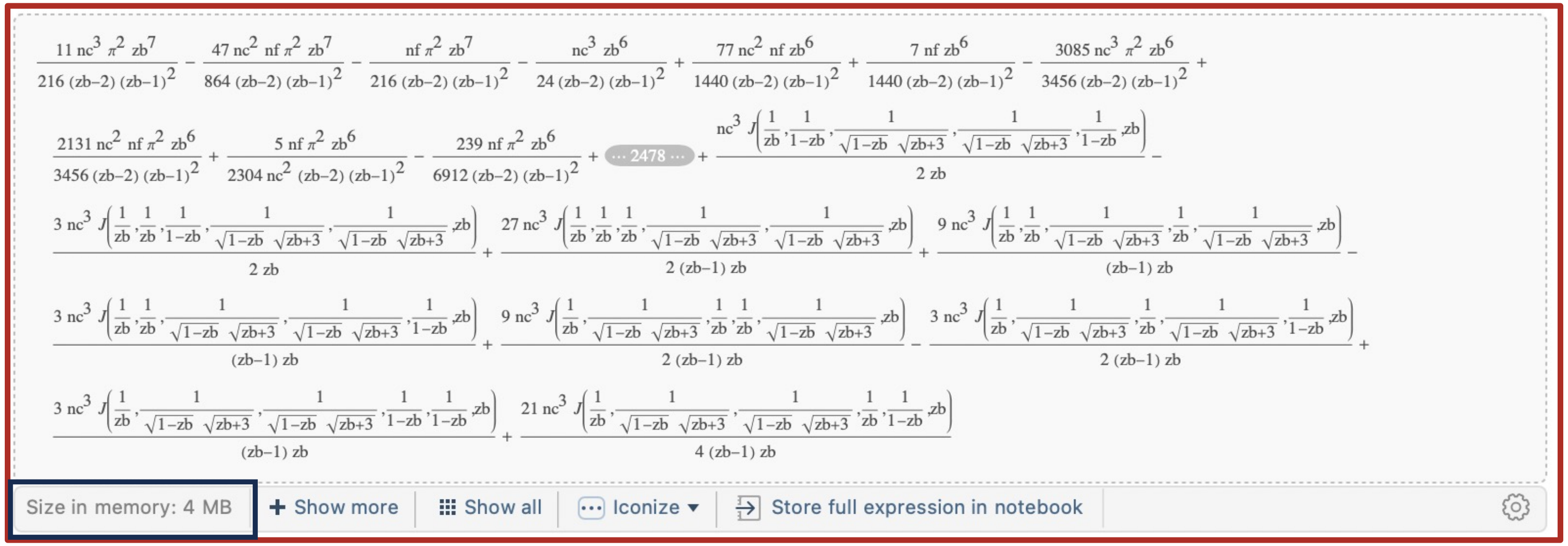
VVV	Triple-Virtual Corrections
RV^2 **RVV**	Single-real Corrections
RRV	Double-real Correction
RRR	Triple-Real Correction

*[Mistlberger, JHEP 05 (2018) 028, arXiv:1802.00833.]

The Gluon-fusion Higgs at $\mathrm{N}^3\mathrm{LO}$*

Result

$\hat{\sigma}_{\text{NNLO,reg}} \sim$



$$\frac{11\,\mathrm{nc}^3\,\pi^2\,\mathrm{zb}^7}{216\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}-\frac{47\,\mathrm{nc}^2\,\mathrm{nf}\,\pi^2\,\mathrm{zb}^7}{864\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}-\frac{\mathrm{nf}\,\pi^2\,\mathrm{zb}^7}{216\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}-\frac{\mathrm{nc}^3\,\mathrm{zb}^6}{24\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}+\frac{77\,\mathrm{nc}^2\,\mathrm{nf}\,\mathrm{zb}^6}{1440\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}+\frac{7\,\mathrm{nf}\,\mathrm{zb}^6}{1440\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}-\frac{3085\,\mathrm{nc}^3\,\pi^2\,\mathrm{zb}^6}{3456\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}+$$

$$\frac{2131\,\mathrm{nc}^2\,\mathrm{nf}\,\pi^2\,\mathrm{zb}^6}{3456\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}+\frac{5\,\mathrm{nf}\,\pi^2\,\mathrm{zb}^6}{2304\,\mathrm{nc}^2\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}-\frac{239\,\mathrm{nf}\,\pi^2\,\mathrm{zb}^6}{6912\,(\mathrm{zb}-2)\,(\mathrm{zb}-1)^2}+\cdots 2478 \cdots+\frac{\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{1-\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{1-\mathrm{zb}},\mathrm{zb}\right)}{2\,\mathrm{zb}}-$$

$$\frac{3\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\mathrm{zb}},\frac{1}{1-\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\mathrm{zb}\right)}{2\,\mathrm{zb}}+\frac{27\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\mathrm{zb}},\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\mathrm{zb}\right)}{2\,(\mathrm{zb}-1)\,\mathrm{zb}}+\frac{9\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\mathrm{zb}\right)}{(\mathrm{zb}-1)\,\mathrm{zb}}-$$

$$\frac{3\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{1-\mathrm{zb}},\mathrm{zb}\right)}{(\mathrm{zb}-1)\,\mathrm{zb}}+\frac{9\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\mathrm{zb}},\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\mathrm{zb}\right)}{2\,(\mathrm{zb}-1)\,\mathrm{zb}}-\frac{3\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{1-\mathrm{zb}},\mathrm{zb}\right)}{2\,(\mathrm{zb}-1)\,\mathrm{zb}}+$$

$$\frac{3\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{1-\mathrm{zb}},\frac{1}{1-\mathrm{zb}},\mathrm{zb}\right)}{(\mathrm{zb}-1)\,\mathrm{zb}}+\frac{21\,\mathrm{nc}^3\,\mathcal{J}\left(\frac{1}{\mathrm{zb}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\sqrt{1-\mathrm{zb}}\,\sqrt{\mathrm{zb}+3}},\frac{1}{\mathrm{zb}},\frac{1}{1-\mathrm{zb}},\mathrm{zb}\right)}{4\,(\mathrm{zb}-1)\,\mathrm{zb}}$$

- Now square root (from RRV) and elliptic letters (from RRR) enter!

*[Mistlberger, JHEP 05 (2018) 028, arXiv:1802.00833.]

Current Theoretical Uncertainty for Gluon-fusion Higgs

Uncertainty* due solely truncating the QCD perturbative expansion (at 13 TeV):

*[Anastasiou, Duhr, Falko, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger; arXiv:1602.00695]

$$\begin{aligned}\hat{\sigma} = \hat{\sigma}_{\text{LO}} & \\ & +\hat{\sigma}_{\text{NLO}} \\ & +\hat{\sigma}_{\text{NNLO}} \\ & +\hat{\sigma}_{\text{N3LO}} \\ & +\xcancel{\cdots}\end{aligned}$$

- State of the art sufficient for current experimental needs
- But **N4LO** will take time to compute – **start now!**

$$\boxed{\sim 2.5\%}$$

Current Theoretical Uncertainty for Gluon-fusion Higgs

Uncertainty due solely truncating the QCD perturbative expansion?

$$\begin{aligned}\hat{\sigma} = \hat{\sigma}_{\mathrm{LO}} \\ +\hat{\sigma}_{\mathrm{NLO}} \\ +\hat{\sigma}_{\mathrm{NNLO}} \\ +\hat{\sigma}_{\mathrm{N3LO}} \\ +\boldsymbol{\hat{\sigma}}_{\mathbf{N4LO}}\end{aligned}$$

- State of the art sufficient for current experimental needs
- But **N4LO** will take **years** to compute – **start now!**

Aim for $\sim 1\%$

Outline

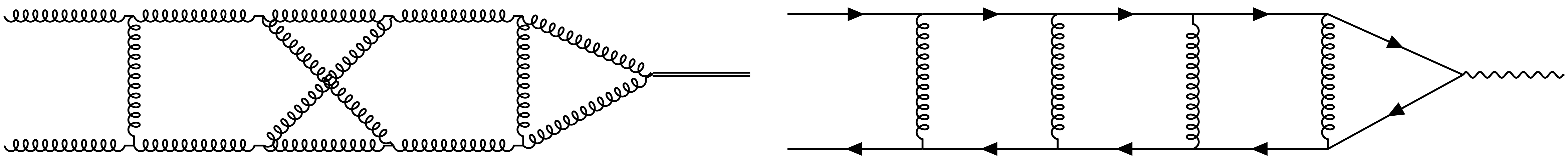
1. Higgs boson production at hadron colliders.
   - How and why do we count them?
   - How are they produced?
2. The journey to the precision frontier.
3. **The single-real ingredient to the inclusive production cross section.**

Fully virtual correction at N^{4}LO

Four loop amplitudes, “VVVV” contribution:

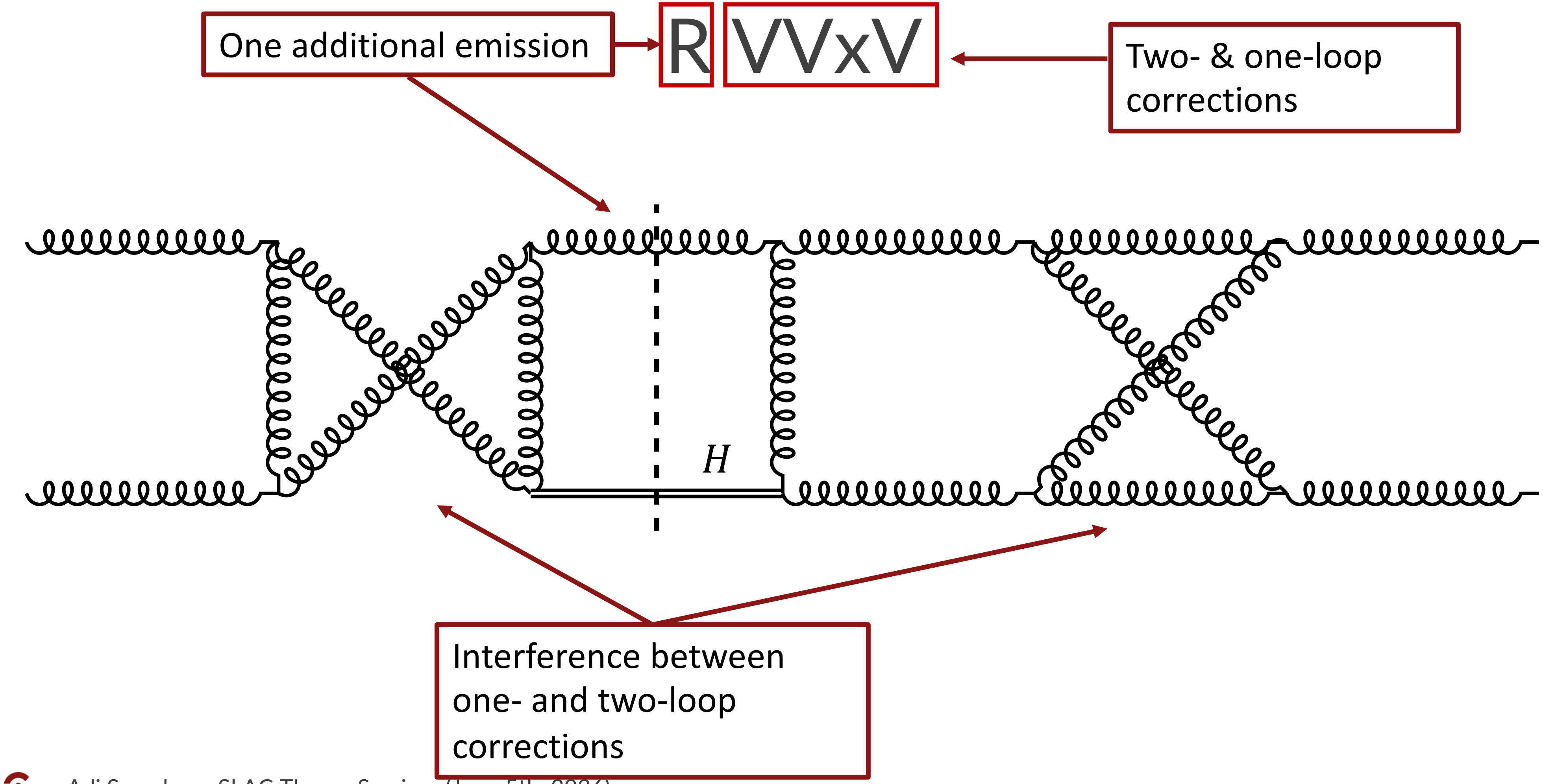
- Higgs and Drell-Yan first known approximately (2020) *
- Full computation (2022) **

*[Das, Moch, Vogt; arXiv: 2004.00563]
**[Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser; arXiv: 2202.04660]



What are the single-real ingredients? At N^{4}LO:

Now RVVxV: ”single-real, double-virtual cross single-virtual”.



One additional emission

R VVxV

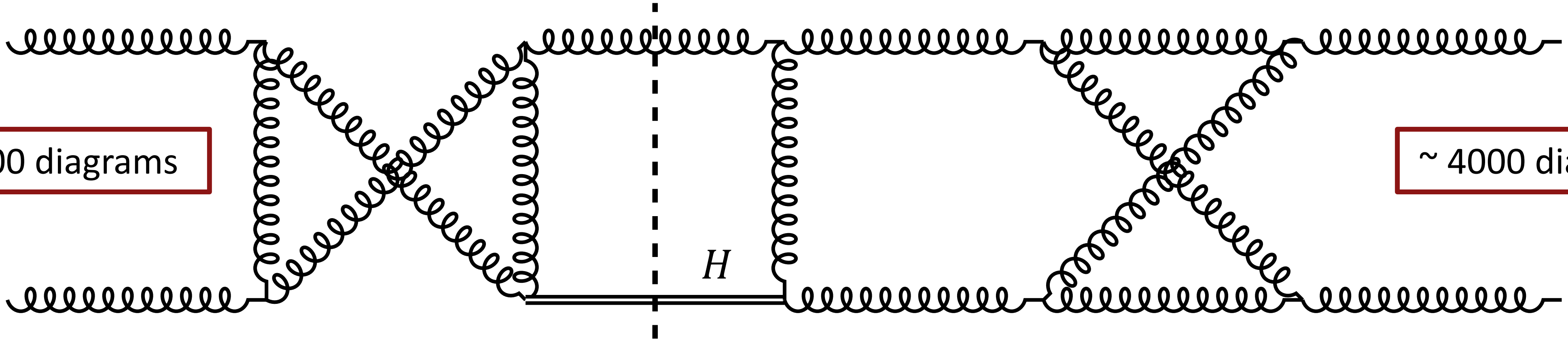
Two- & one-loop corrections

Interference between one- and two-loop corrections

What are the single-real ingredients? At N^{4}LO:

Now RVVxV: ”single-real, double-virtual cross single-virtual”.

RVVxV



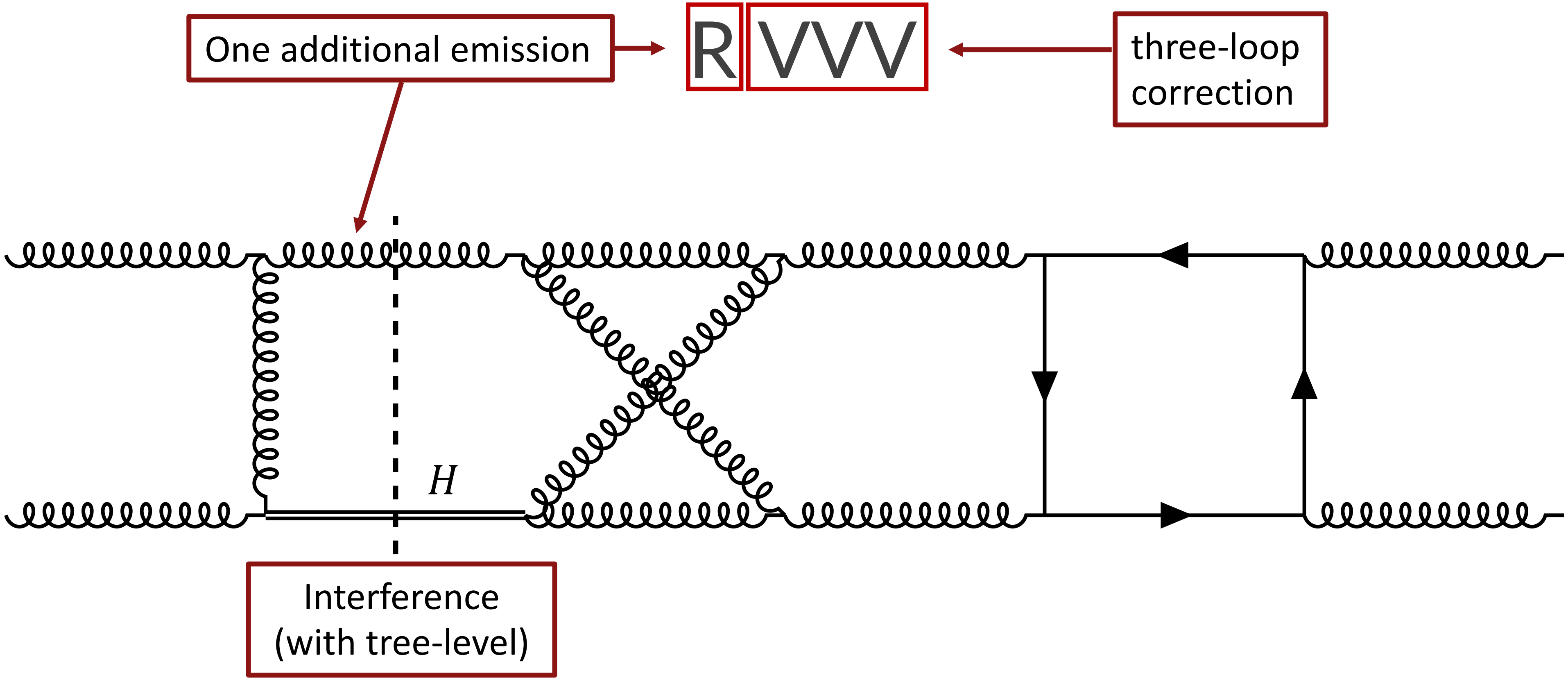
~ 100 diagrams

~ 4000 diagrams

Complexity grows fast!

What are the single-real ingredients? At N^{4}LO:

Now RVVV: ”single-real, triple-virtual”.



One additional emission

RVVV

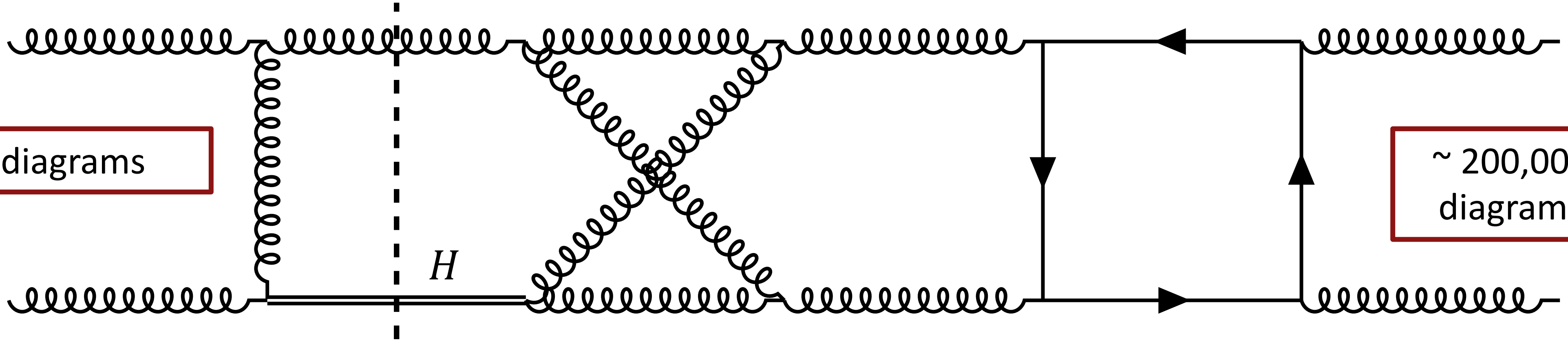
three-loop correction

Interference (with tree-level)

What are the single-real ingredients? At N^{4}LO:

Now RVVxV: ”single-real, double-virtual cross single-virtual”.

RVVV

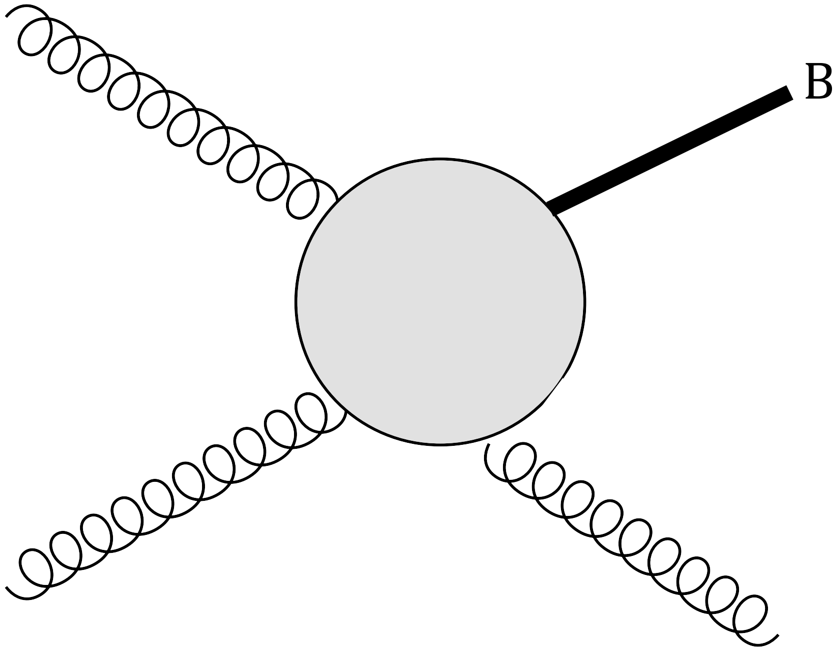


6 diagrams

~ 200,000 diagrams

The single-real ingredient as a probe

Only one additional emission is “easy”:

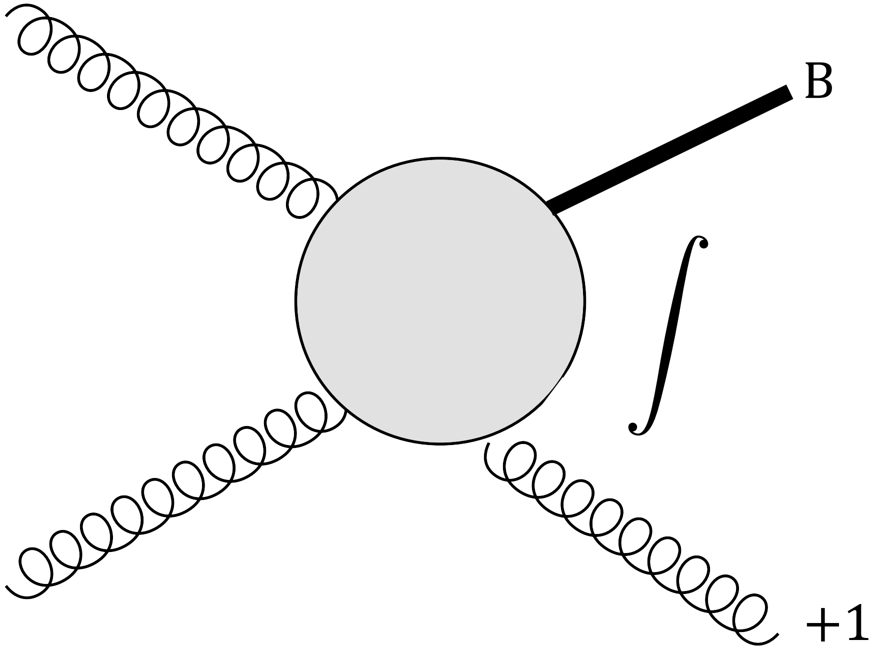


- Simplest (non-trivial) phase space
  - Can do **direct** phase space integration

Phase Space Integration

We only care about Higgs / DY production → Integrate over d.o.f. of final state

$$\hat{\sigma} \sim \boxed{\int d\,\Phi_{B+1}} A^{(1)} A^{*\,(2)}$$



Momentum conservation & rotational symmetry

⬇

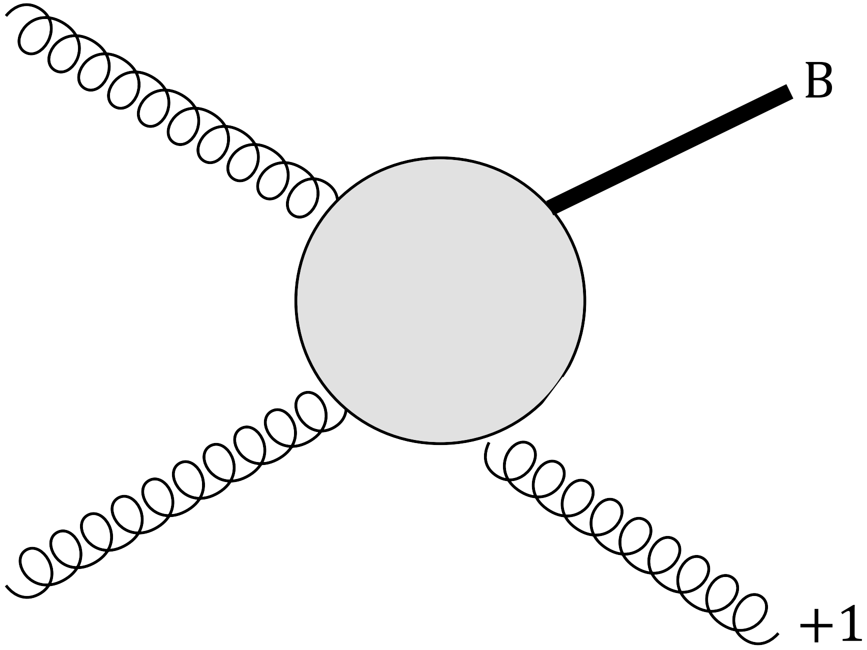
Only **one** actual d.o.f.:

Angle w.r.t. beamline:

$\boldsymbol{\lambda} \in [\mathbf{0}, \mathbf{1}]$

The single-real ingredient as a probe

Only one additional emission is “easy”:

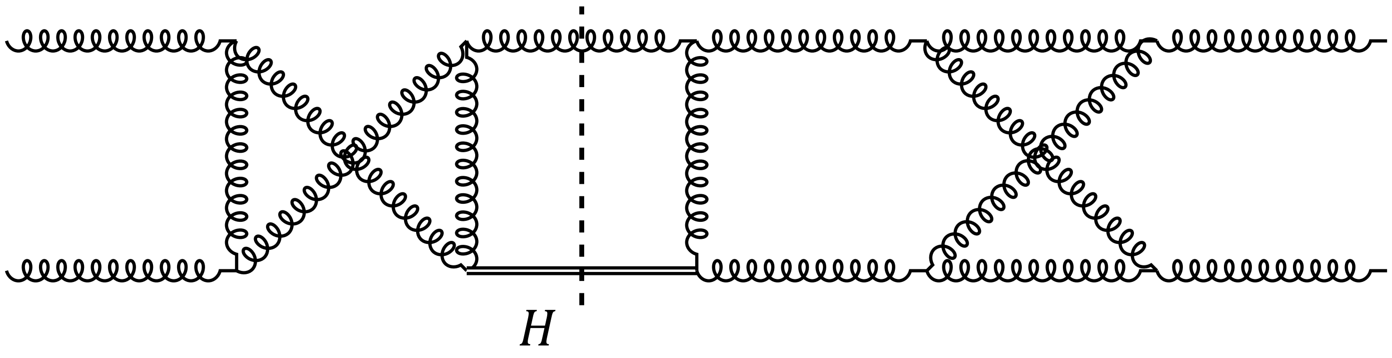


- Simplest (non-trivial) phase space
  - Can do **direct** phase space integration
  - Higher multiplicity involves the reverse unitarity approach (**loops + PS together**)
- Probe complexity at N^{4}LO:
  - Function space
  - Color
  - Computational tools

Production Channels

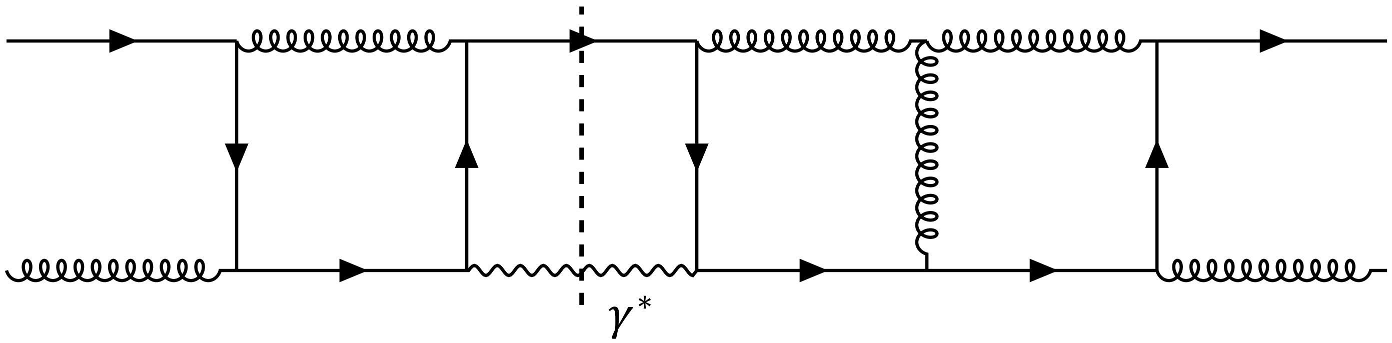
Higgs Production

- $g + g \to H + g$
- $q + \bar{q} \to H + g$
- $q + g \to H + q$



Drell-Yan Production

- $g + g \to \gamma^* + g$
    - Starts at 1-loop order
        - Only RVVxV
- $q + \bar{q} \to \gamma^* + g$
- $q + g \to \gamma^* + q$



RVVxV Contribution

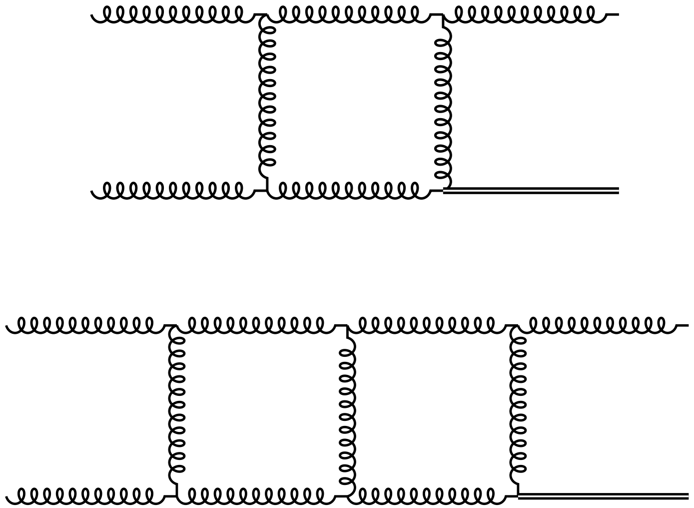
(Re)computing one- and two**-loop amplitudes

Generate all Feynman diagrams

Project onto gauge-invariant Lorentz basis

All particle live in $\mathrm{d} = 4 - 2\epsilon$ dimensions (conventional dimensional regularization)

Project onto basis of color tensors



Compute Loop integrals

- Use IBPs to obtain basis of “master integrals”
- Evaluate with method of differential equations
- In terms of generalized polylogarithms $G(\dots)$ up to transcendental weight eight

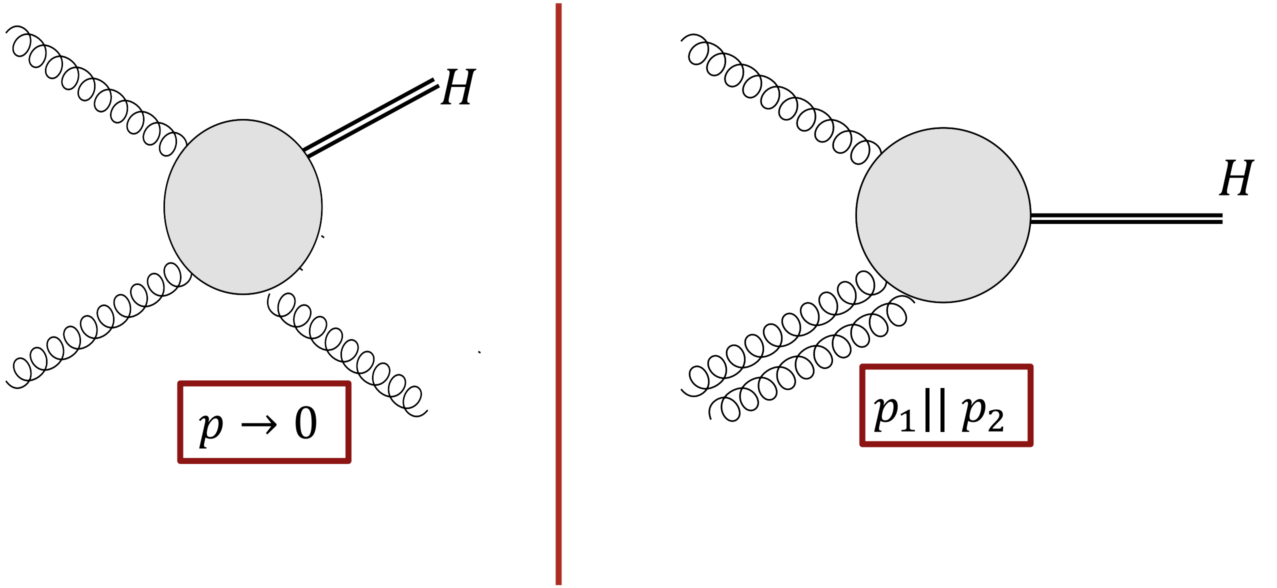
**[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2301.10849]

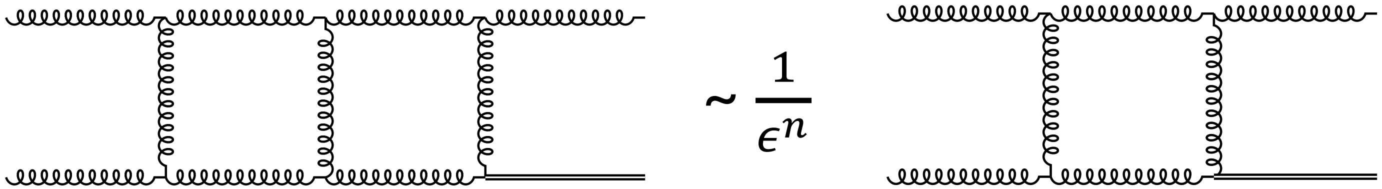
[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2306.10170]

Checks on Amplitudes

Amplitudes factorize in:

- Soft ($p \to 0$) limits
- Collinear ($p_1 || p_2$) limits
- Poles in ϵ ~ lower loop amplitudes

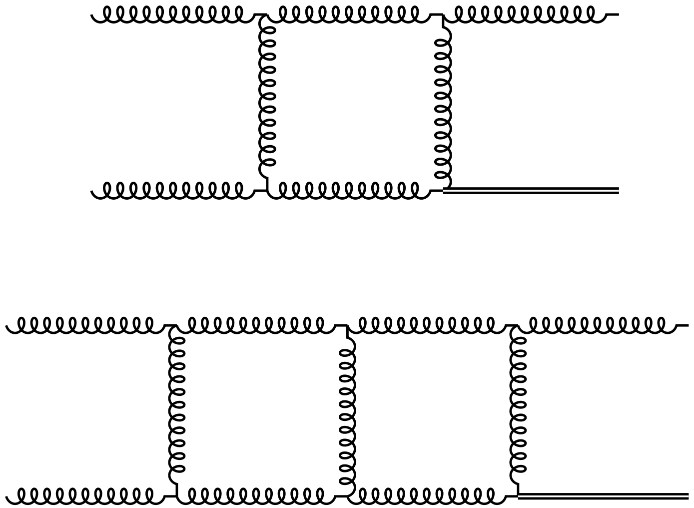




$$\sim \frac{1}{\epsilon^n}$$

(Re)computing one- and two**-loop amplitudes

Generate all Feynman diagrams



Project onto gauge-invariant Lorentz basis

All particle live in $d = 4 - 2\epsilon$ dimensions (conventional dimensional regularization)

Project onto basis of color tensors

Debut Mathematica color algebra package: CIFAR

Compute Loop integrals

- Use IBPs to obtain basis of “master integrals”
- Evaluate with method of differential equations
- In terms of generalized polylogarithms up to transcendental weight eight

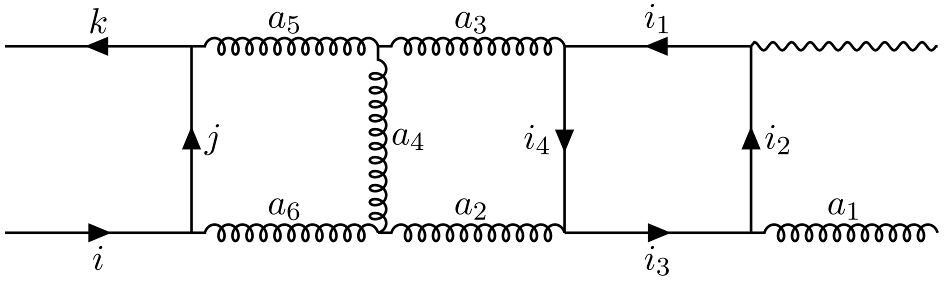
**[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2301.10849]

[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2306.10170]

Computing Color Factors in Gauge Theories

QCD gauge group → "color" factors in amplitude

- Gauge group is $\mathrm{SU}(n_c)$, with n_c number of colors
    - Factors reduce to polynomials in n_c
- Can instead consider **generalized gauge group**
    - Factors reduce to color group ("Casimir") invariants C



Proj. $t^{a_1}_{ik}$

$$\delta_{i_1 i_2} t^{a_1}_{i_2 i_3} t^{a_2}_{i_3 i_4} t^{a_3}_{i_4 i_1} t^{a_1}_{ik} t^{a_5}_{kj} t^{a_6}_{ji} f^{a_3 a_4 a_5} f^{a_2 a_6 a_4}$$

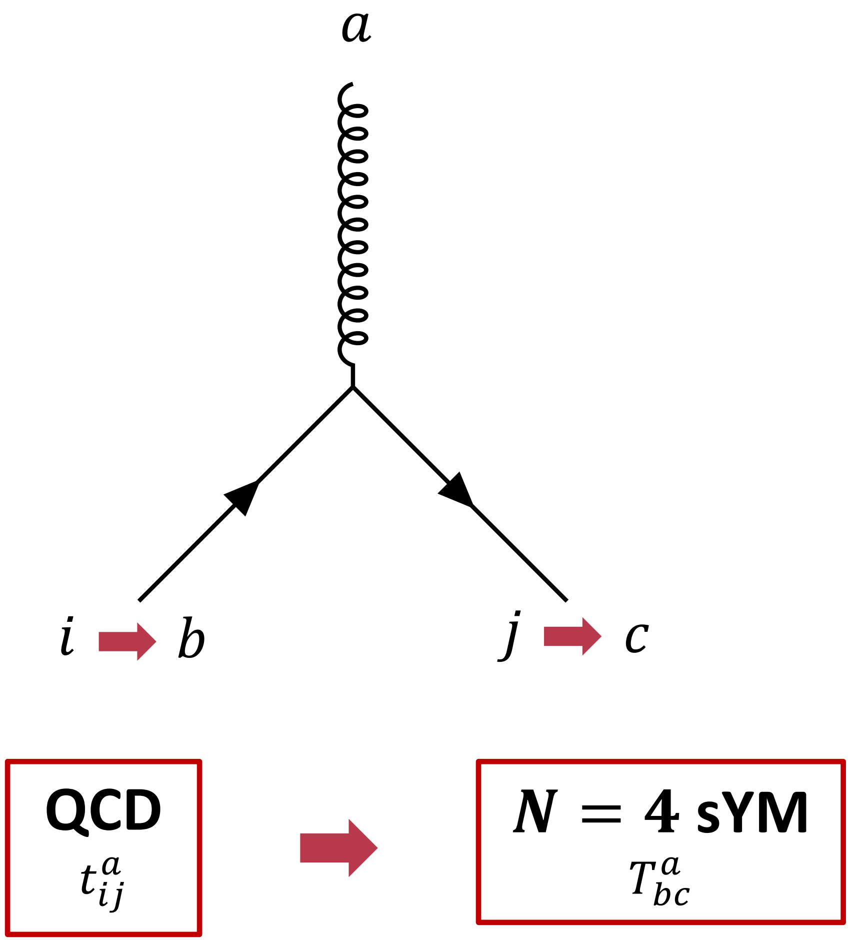
$$-\frac{1}{2} C_A C_3^{FF} - \frac{1}{8} T_F^2 D_A C_A^2$$

$$-\frac{1}{8} + \frac{3}{16} n_c^2 - \frac{1}{16} n_c^4$$

Computing Color Factors in Gauge Theories

QCD gauge group → “color” factors in amplitude

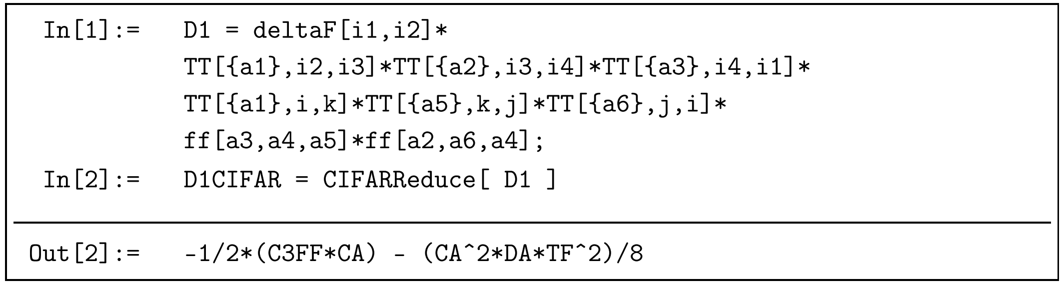
- Ubiquitous step in amplitude calculations
- Generalized gauge group → allows for comparison across theories
    - e.g. Set $C_F = C_A$ for $N = 4$ sYM (“Principle of maximal transcendentality”)
- Need an efficient and reliable computational pipeline



CIFAR: A Color Algebra Package

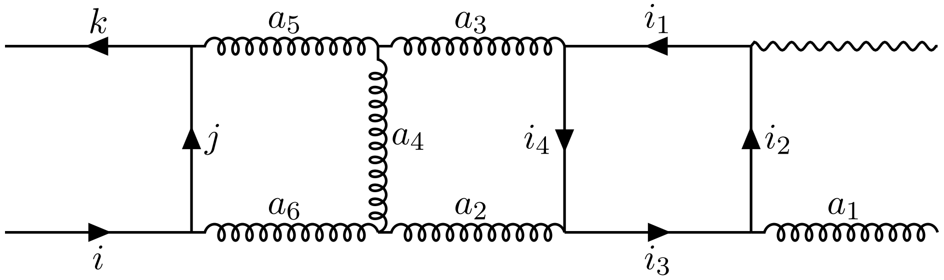
QCD gauge group → “color” factors in amplitude

- CIFAR can do this computation for us!



```
 In[1]:=    D1 = deltaF[i1,i2]*
            TT[{a1},i2,i3]*TT[{a2},i3,i4]*TT[{a3},i4,i1]*
            TT[{a1},i,k]*TT[{a5},k,j]*TT[{a6},j,i]*
            ff[a3,a4,a5]*ff[a2,a6,a4];
 In[2]:=    D1CIFAR = CIFARReduce[ D1 ]

Out[2]:=    -1/2*(C3FF*CA) - (CA^2*DA*TF^2)/8
```



Proj. $t^{a_1}_{ik}$

$$\delta_{i_1 i_2} t^{a_1}_{i_2 i_3} t^{a_2}_{i_3 i_4} t^{a_3}_{i_4 i_1} t^{a_1}_{ik} t^{a_5}_{kj} t^{a_6}_{ji} f^{a_3 a_4 a_5} f^{a_2 a_6 a_4}$$

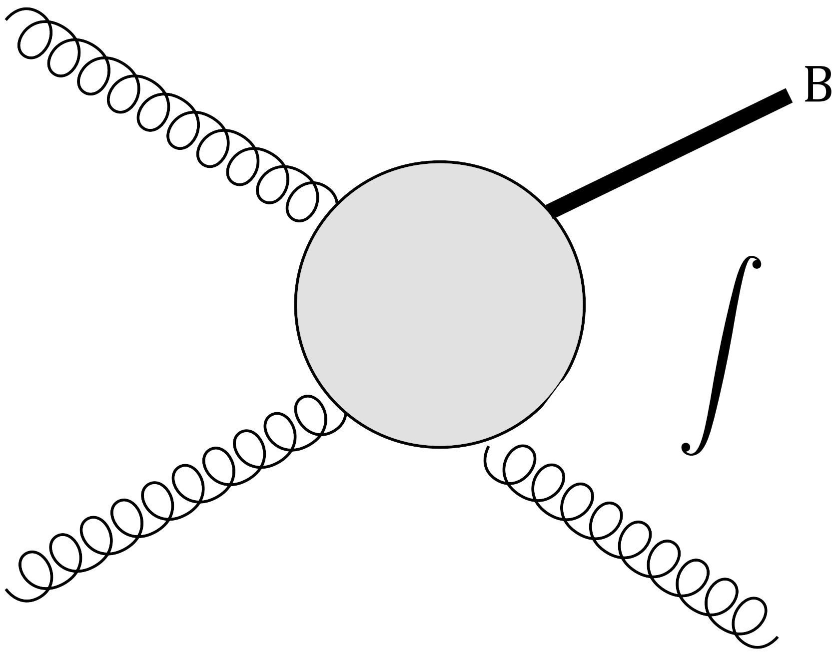
$$\boldsymbol{-\frac{1}{2} C_A C_3^{FF} - \frac{1}{8} T_F^2 D_A C_A^2}$$

$$-\frac{1}{8} + \frac{3}{16} n_c^2 - \frac{1}{16} n_c^4$$

- Quartic Casimirs: Ready for N^{4}LO
- Available at: github.com/adisurtya/CIFAR

Phase Space Integration

What do these integrals typically look like?

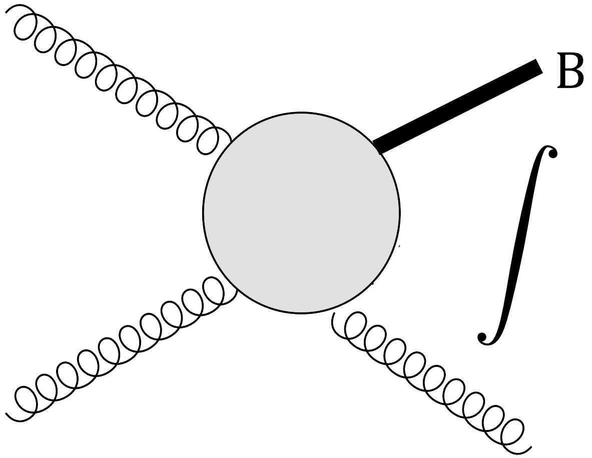


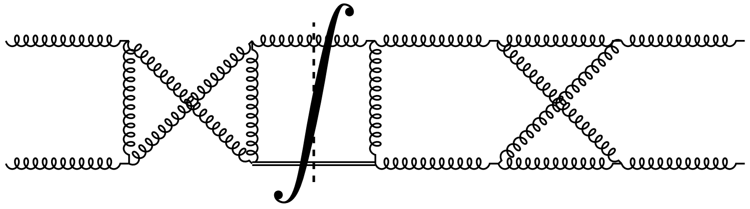
$$\hat{\sigma} \sim \int d\lambda \, \frac{\log\lambda}{\lambda - 1/\bar{z}} = \log\lambda \log\left(1 - \boxed{\bar{z}}\lambda\right) + \mathrm{Li}_2(\bar{z}\lambda)$$

Parametrizes energy of scattering

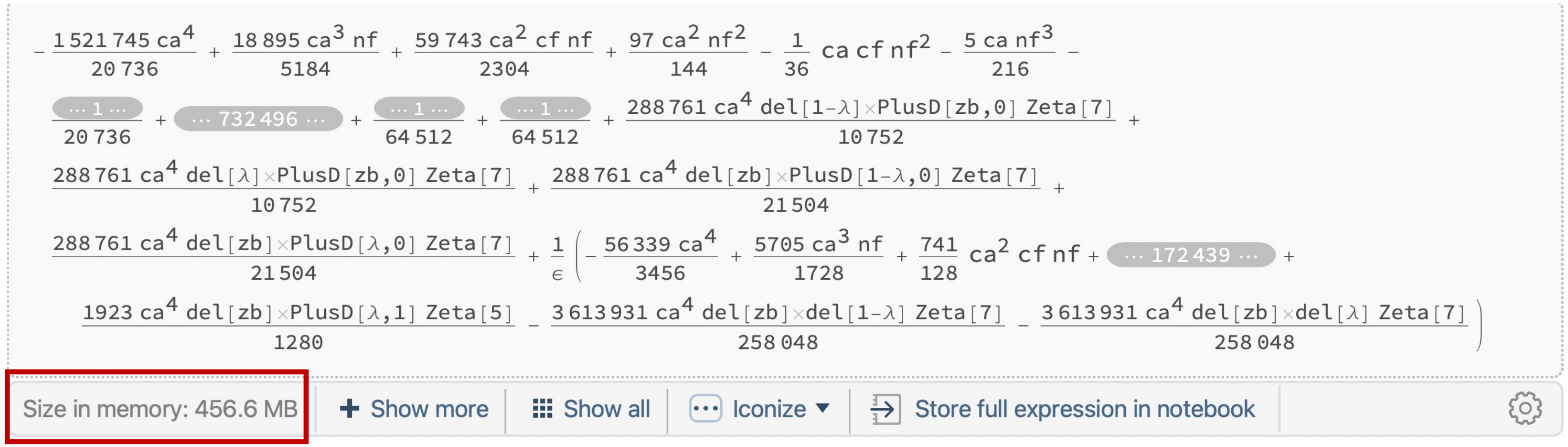
Phase Space Integration

What do these integrals typically look like **in practice**?





$gg \to Hg$ integrand:



$$-\frac{1\,521\,745\,\mathrm{ca}^4}{20\,736}+\frac{18\,895\,\mathrm{ca}^3\,\mathrm{nf}}{5184}+\frac{59\,743\,\mathrm{ca}^2\,\mathrm{cf}\,\mathrm{nf}}{2304}+\frac{97\,\mathrm{ca}^2\,\mathrm{nf}^2}{144}-\frac{1}{36}\,\mathrm{ca}\,\mathrm{cf}\,\mathrm{nf}^2-\frac{5\,\mathrm{ca}\,\mathrm{nf}^3}{216}-$$

$$\frac{\cdots 1 \cdots}{20\,736}+\cdots 732\,496 \cdots+\frac{\cdots 1 \cdots}{64\,512}+\frac{\cdots 1 \cdots}{64\,512}+\frac{288\,761\,\mathrm{ca}^4\,\mathrm{del}[1-\lambda]\times\mathrm{PlusD}[\mathrm{zb},0]\,\mathrm{Zeta}[7]}{10\,752}+$$

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$$\left.\frac{1923\,\mathrm{ca}^4\,\mathrm{del}[\mathrm{zb}]\times\mathrm{PlusD}[\lambda,1]\,\mathrm{Zeta}[5]}{1280}-\frac{3\,613\,931\,\mathrm{ca}^4\,\mathrm{del}[\mathrm{zb}]\times\mathrm{del}[1-\lambda]\,\mathrm{Zeta}[7]}{258\,048}-\frac{3\,613\,931\,\mathrm{ca}^4\,\mathrm{del}[\mathrm{zb}]\times\mathrm{del}[\lambda]\,\mathrm{Zeta}[7]}{258\,048}\right)$$

Size in memory: 456.6 MB

Huge and complicated! Contains various polylogs, distributions, renormalization factors…

Integration software based on PolyLogTools*

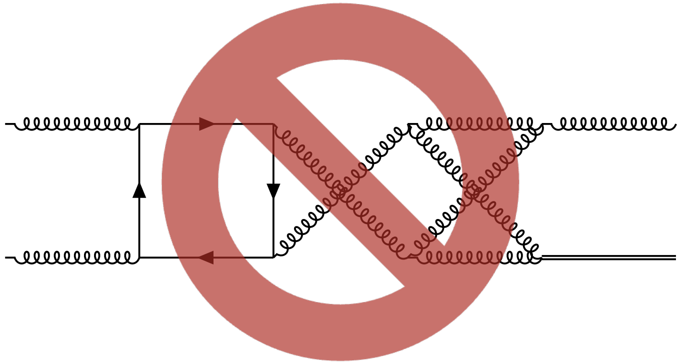
*[Claude Duhr, Falko Dulat; arXiv:1904.07279]

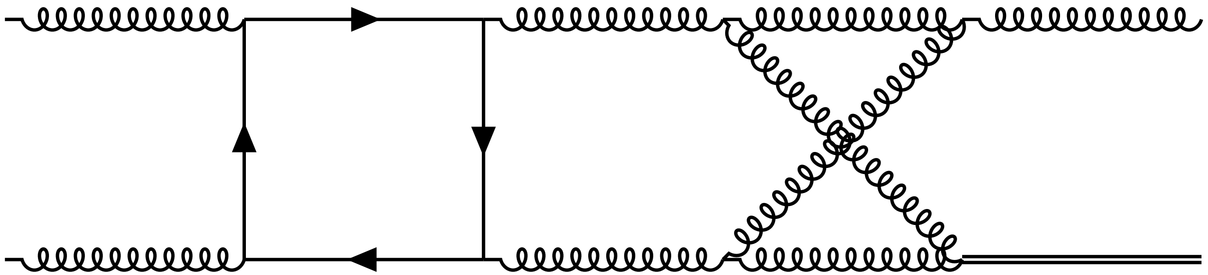
RVVV Contribution

Three-loop amplitudes

State-of-the-art: Leading color amplitudes

- Only leading power in ${n_C}^{\alpha}{n_f}^{\beta}$
- Drell-Yan known*
- Higgs recently computed**





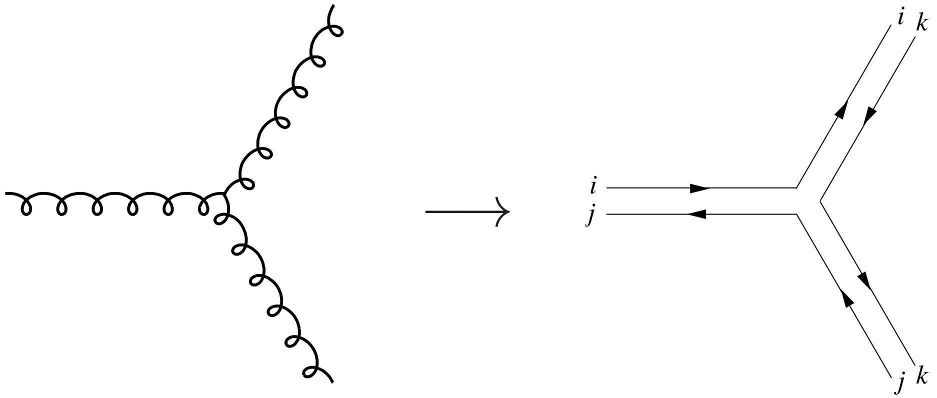
e.g. $\{n_C^6, n_C^5 n_f, n_C^4 n_f^2, n_C^3 n_f^3\}$ for $gg \to Hg$

*[Gehrmann, Jakubčík, Mella, Syrrakos, Tancredi; arXiv:2307.15405]
**[Chen, Guan, Mistlberger; arXiv: 2504.06490]

Generalized leading-color limit

Take $n_f, n_C \to \infty$

- Only planar color diagrams contribute
- Much easier to compute
  - E.g. Γ_{cusp} known to all orders in planar N=4 sYM* vs. N^{4}LO for non-planar **
- Subleading color suppressed by $\frac{1}{n_C^2} \sim \frac{1}{10}$.
  - For XS @ N^{4}LO, $\frac{1}{10}$ of 1%?



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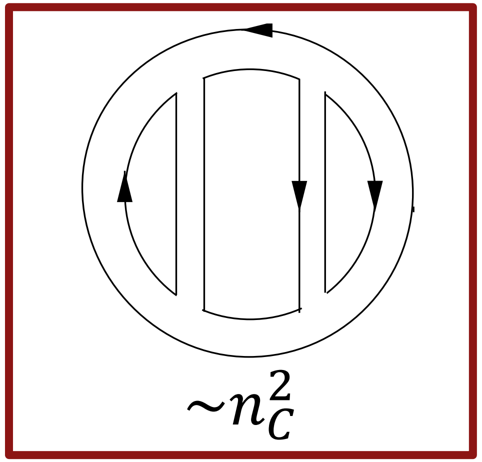

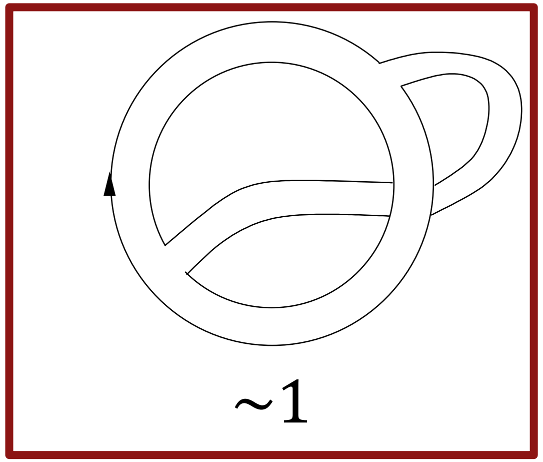

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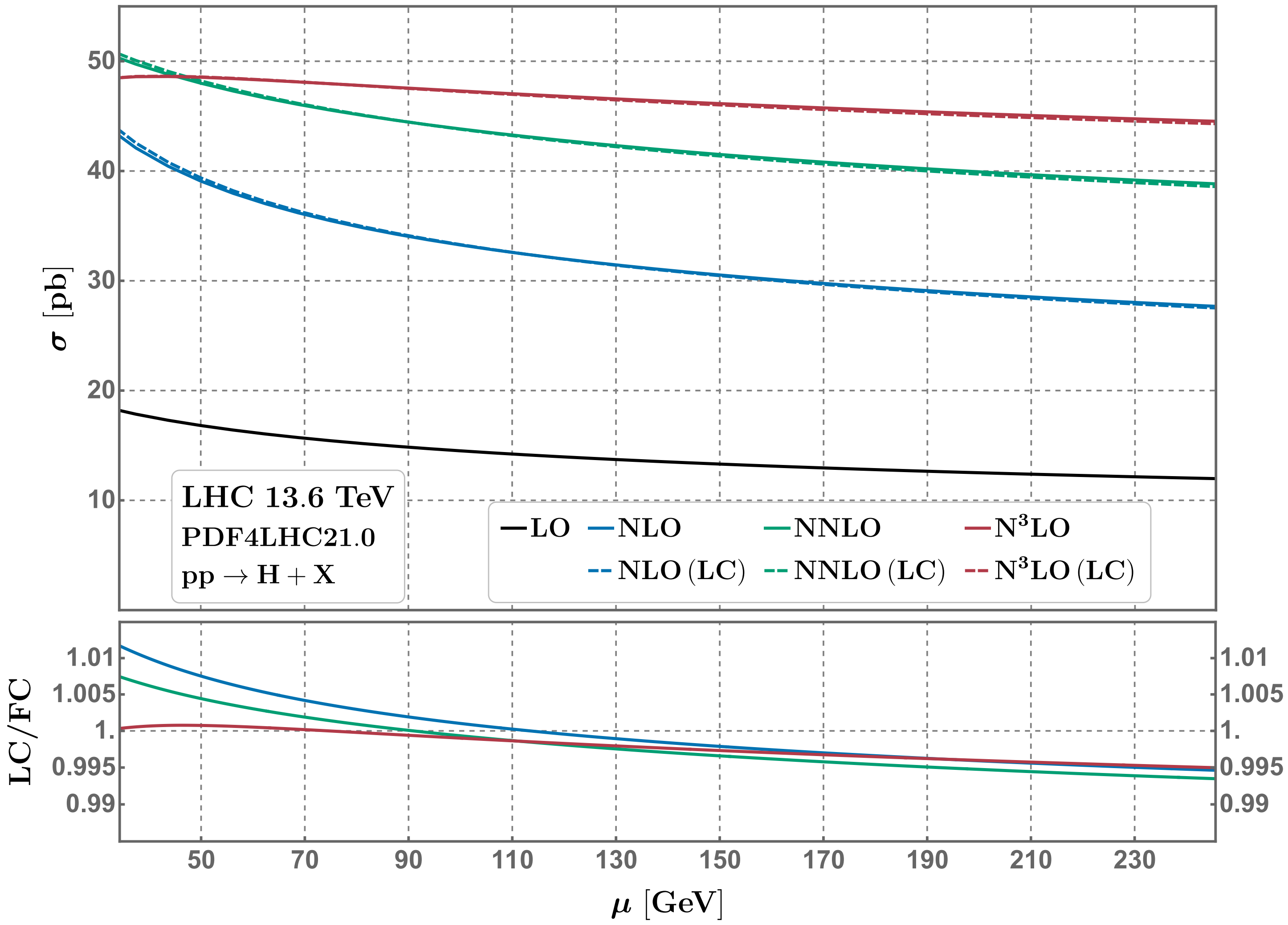
*[Beisert, Eden, Staudacher; arXiv:0610251]
**[Boels, Huber, Yang; arXiv:1705.03444]
***[davidtong.org/teaching/gauge-theory/]

Can we do **phenomenology** with only leading color?

Hadronic Cross Section up to N^{3}LO: Leading vs. Full Color



Leading color appears to capture the cross section at the <0.5% level!

Should hold for N^{4}LO.

RVVV Computation: Integration strategy for Higgs Channels

Problem: $\int_0^1 r(\lambda) G(a_1, \ldots, a_n, \lambda)\, d\lambda$

hard to evaluate when $a_i = \sqrt{\alpha + \beta\bar{z} + \gamma\bar{z}^2}\,\}$

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$\Downarrow$

Partial Integration Idea: Evaluate $\int_0^1 r(\lambda) M_j(\lambda)\, d\lambda$!

Canonical Master Integral

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⬇

Partial Integration Idea: Evaluate $\int_0^1 r(\lambda)M_j(\lambda)\, d\lambda$!

$$= R\ M\,\Big|_0^1 - \int_0^1 R\,\partial_\lambda M_i\, d\lambda$$

$R(\lambda) = \int r(\lambda)d\lambda$

RVVV Computation: Integration strategy for Higgs Channels

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$$= R\, M \Big|_0^1 - \int_0^1 R\, \partial_\lambda M_i\, d\lambda$$

Know from differential equations!

$$\partial_\lambda M_i = \epsilon A_{ij} M_j$$

RVVV Computation: Integration strategy for Higgs Channels

Problem: $\int_0^1 r(\lambda) G(a_1, \dots, a_n, \lambda)\, d\lambda$

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$$= R\, M \Big|_0^1 - \int_0^1 R\, \partial_\lambda M_i\, d\lambda$$

Next Iteration: $\epsilon \int_0^1 A_{ij} R(\lambda) M_j(\lambda)\, d\lambda$

Results

RVVxV: After Performing Phase Space Integration

Results

- Laurant expansion in dimensional regulator, ϵ
- With $\bar{z} = 1 - \frac{m_B^2}{s}$, contains:
  - Rational functions
  - Multiple Polylogs: $G(\dots, \bar{z})$
  - Distributions: $\delta(\bar{z})$, $\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+$
    - (soft singularities)

Checks

- Recomputed all lower-order single-real (R…) interference contributions
- Soft-singular parts for:
  - $g \; + \; g \; \rightarrow H + g$
  - $q + \bar{q} \rightarrow \gamma^* + g$
  - Applied single-emission soft current to $g \; + \; g \; \rightarrow H$ and $q + \bar{q} \rightarrow \gamma^*$ to find agreement

RVVV (LC): After Performing Phase Space Integration

Results

- With $\bar{z} = 1 - \frac{m_B^2}{s}$, contains:
  - Everything in RVVxV **and**
  - More general class of iterated integrals:
    - New (not $d\log$) letters:
      $\frac{1}{\bar{z}\sqrt{1-\bar{z}}}, \frac{1}{\sqrt{3+\bar{z}}\sqrt{1-\bar{z}}}$
    - Entered at RRV and RRR for N^3LO [***]

Checks

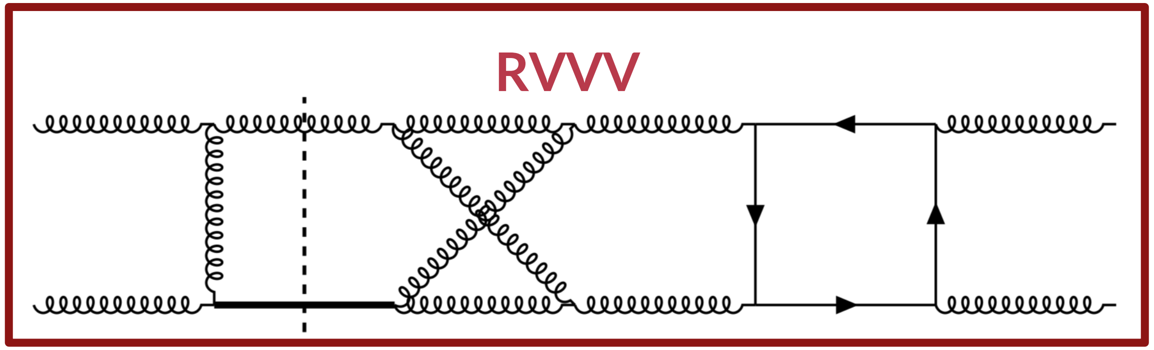
- Regulated IR singularities with known results:
  - Three-loop single-emission soft current[*]
  - Three-loop splitting amplitudes[**]
- Computed threshold expansion in $\bar{z}$ before and after phase-space integration

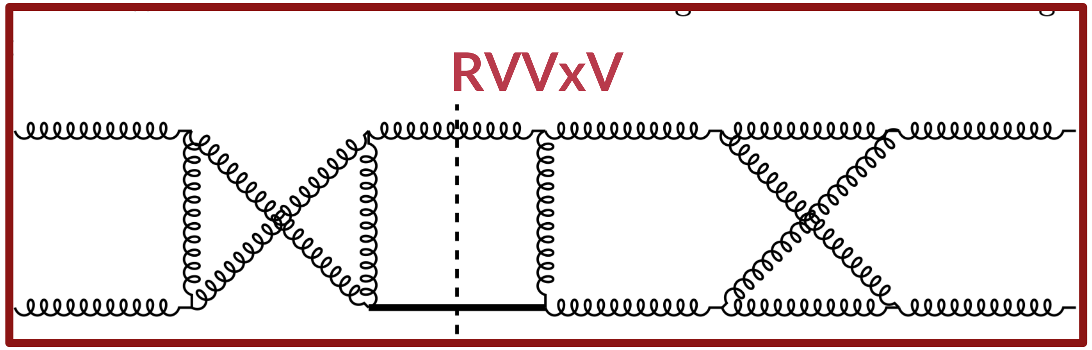
[*] [Herzog, Ma, Mistlberger, Suresh; arXiv:2309.07884]
[**] [Guan, Herzog, Ma, Mistlberger, Suresh; arXiv:2408.03019]
[***] [Mistlberger; arXiv:1802.00833]

Conclusion and Next Steps

Unlock Full Phenomenological Potential of LHC & Higgs Physics: Need N^{4}LO Precision

- Two key steps in computing the production cross section at N^{4}LO:





- Understanding complexity and building up technology (e.g. CIFAR) to tackle further N^{4}LO calculations
- How to compute other ingredients RRVV, RRV^2, …
  - Leading color? threshold expansion? …

Backup slides

Thank you!

Regulation of Soft Singularity

Expand soft singularity:

$$\left(\frac{1}{\bar{z}}\right)^{1+a\epsilon} = -\frac{1}{a\epsilon}\delta(\bar{z}) + \sum_{k=0}^{\infty}\frac{(-a\epsilon)^k}{k!}\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ ,$$

Regulation of Soft Singularity

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$$\left(\frac{1}{\bar{z}}\right)^{1+a\epsilon} = -\frac{\mathbf{1}}{\boldsymbol{a\epsilon}}\delta(\bar{z}) + \sum_{k=0}^{\infty} \frac{(-a\epsilon)^k}{k!} \left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ ,$$

Additional ϵ pole at soft limit $\bar{z} \to 0$

Regulation of Soft Singularity

Expand soft singularity:

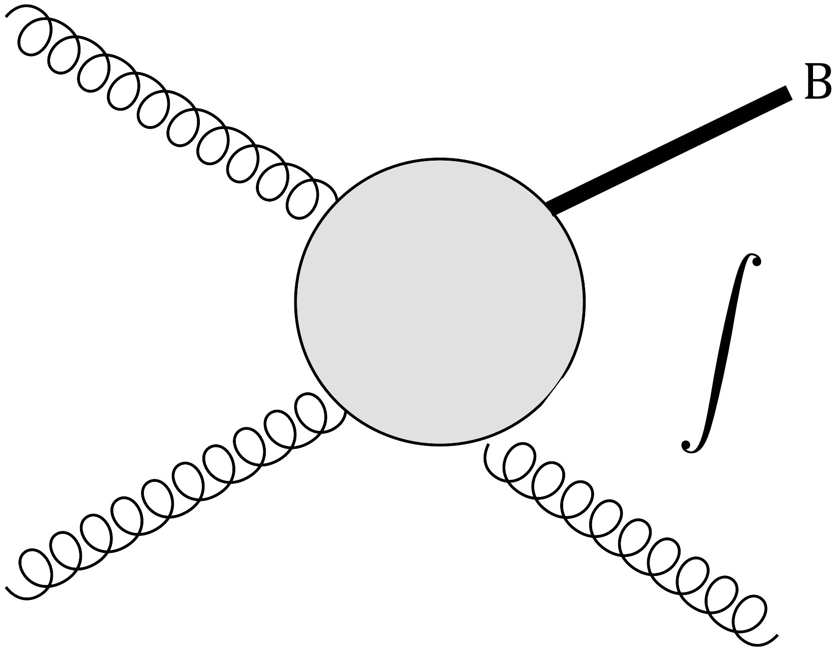
$$\left(\frac{1}{\bar{z}}\right)^{1+a\epsilon} = -\frac{1}{a\epsilon}\delta(\bar{z}) + \sum_{k=0}^{\infty}\frac{(-a\epsilon)^k}{k!}\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ ,$$

Plus distribution:

$$\int_0^1 d\bar{z}\left[\frac{\log^k \bar{z}}{\bar{z}}\right]_+ f(\bar{z}) \equiv \int_0^1 \frac{\log^k \bar{z}}{\bar{z}}\big(f(\bar{z}) - f(0)\big).$$

Phase Space Integration

What do these integrals typically look like?



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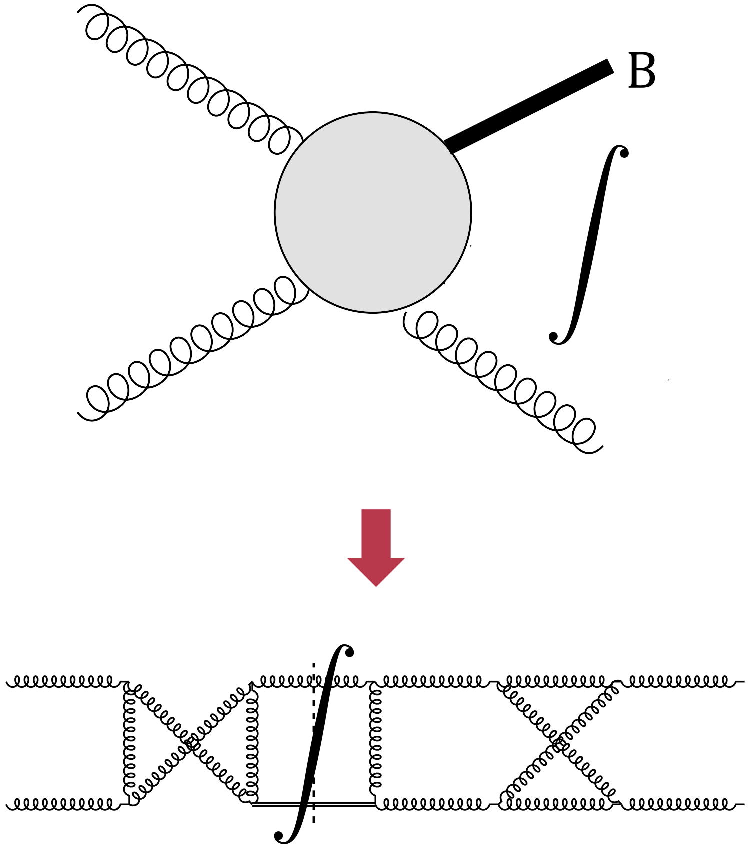
Parametrizes energy of scattering

Phase Space Integration

What do these integrals typically look like **in practice**?



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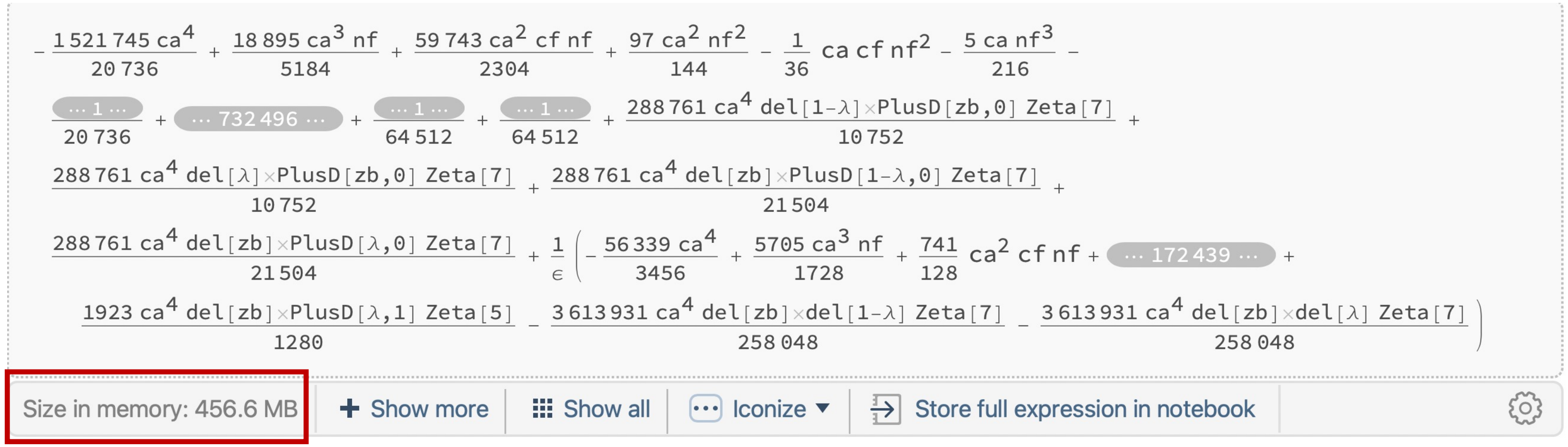

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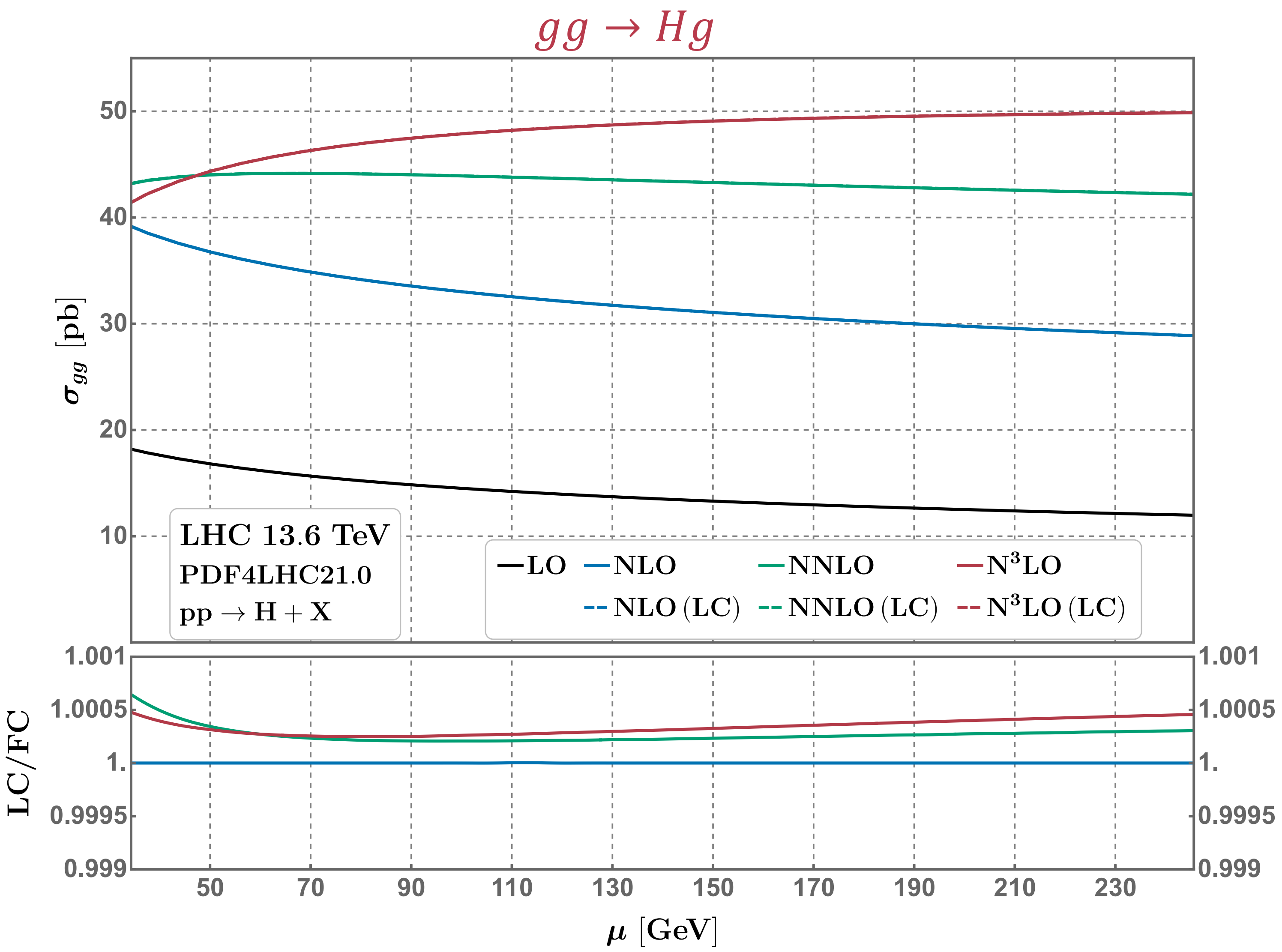
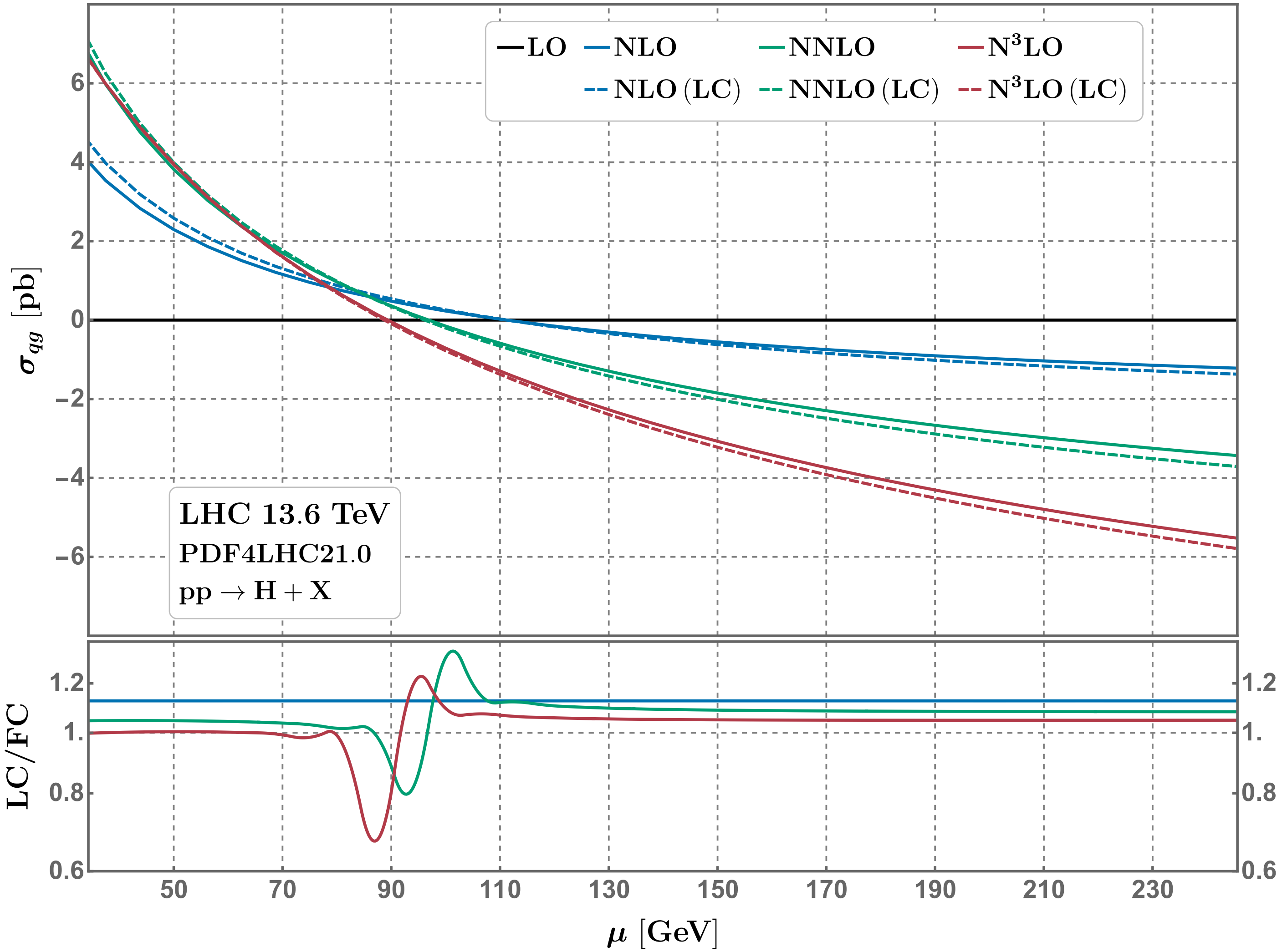
More Leading Color Phenomenology



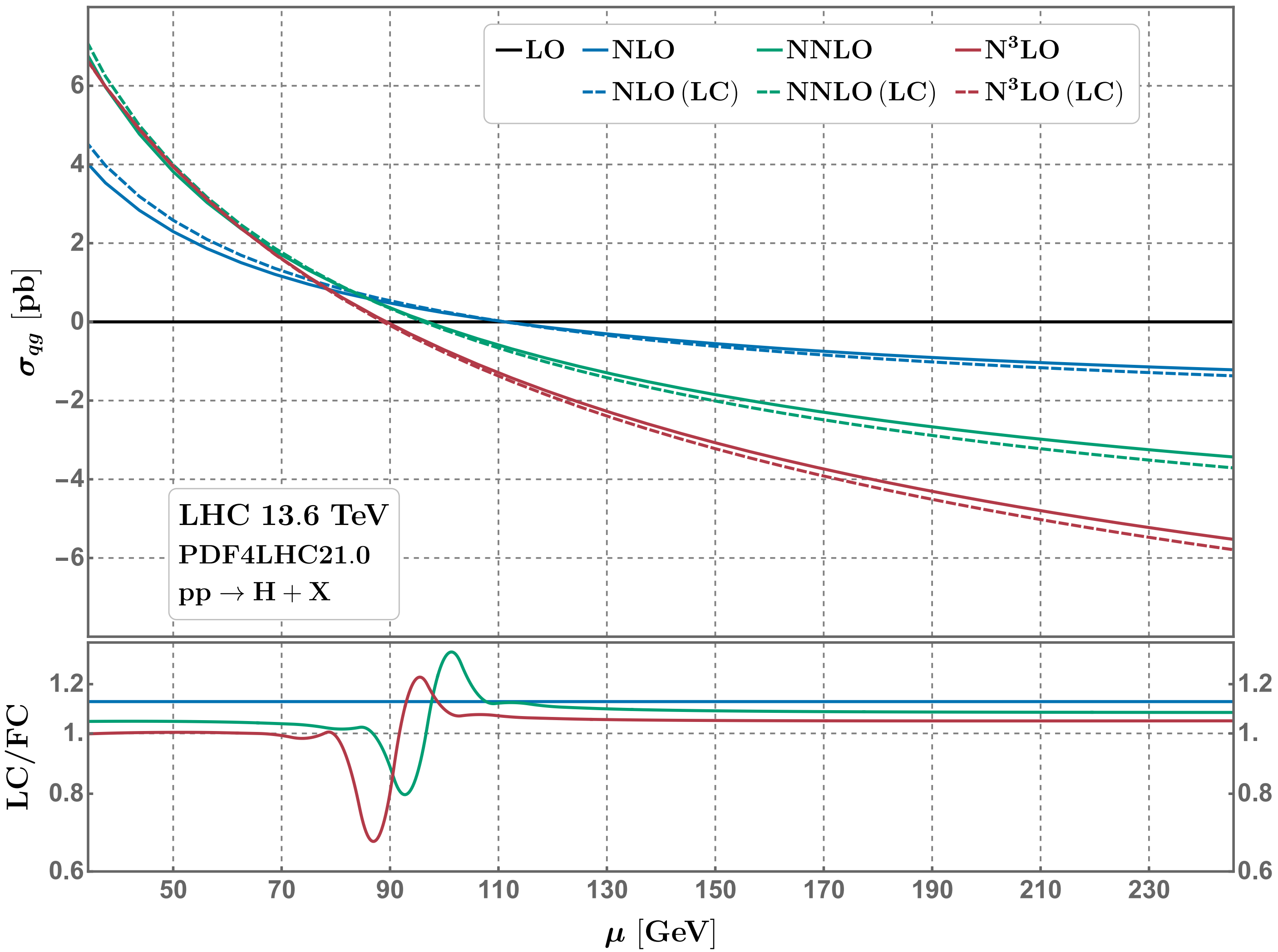
More Leading Color Phenomenology

$q\bar{q} \to Hg$



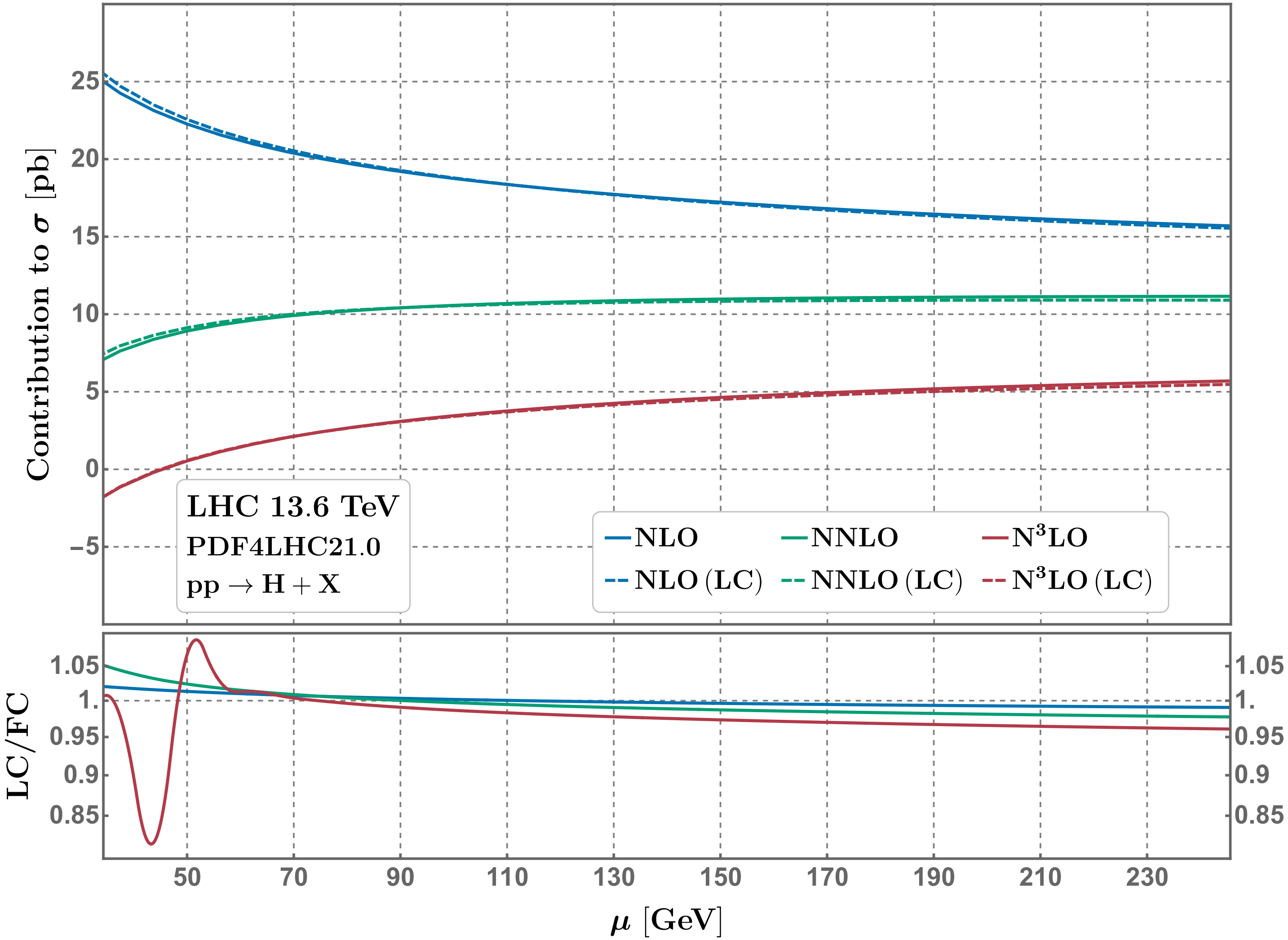
More Leading Color Phenomenology

$qg \to Hq$



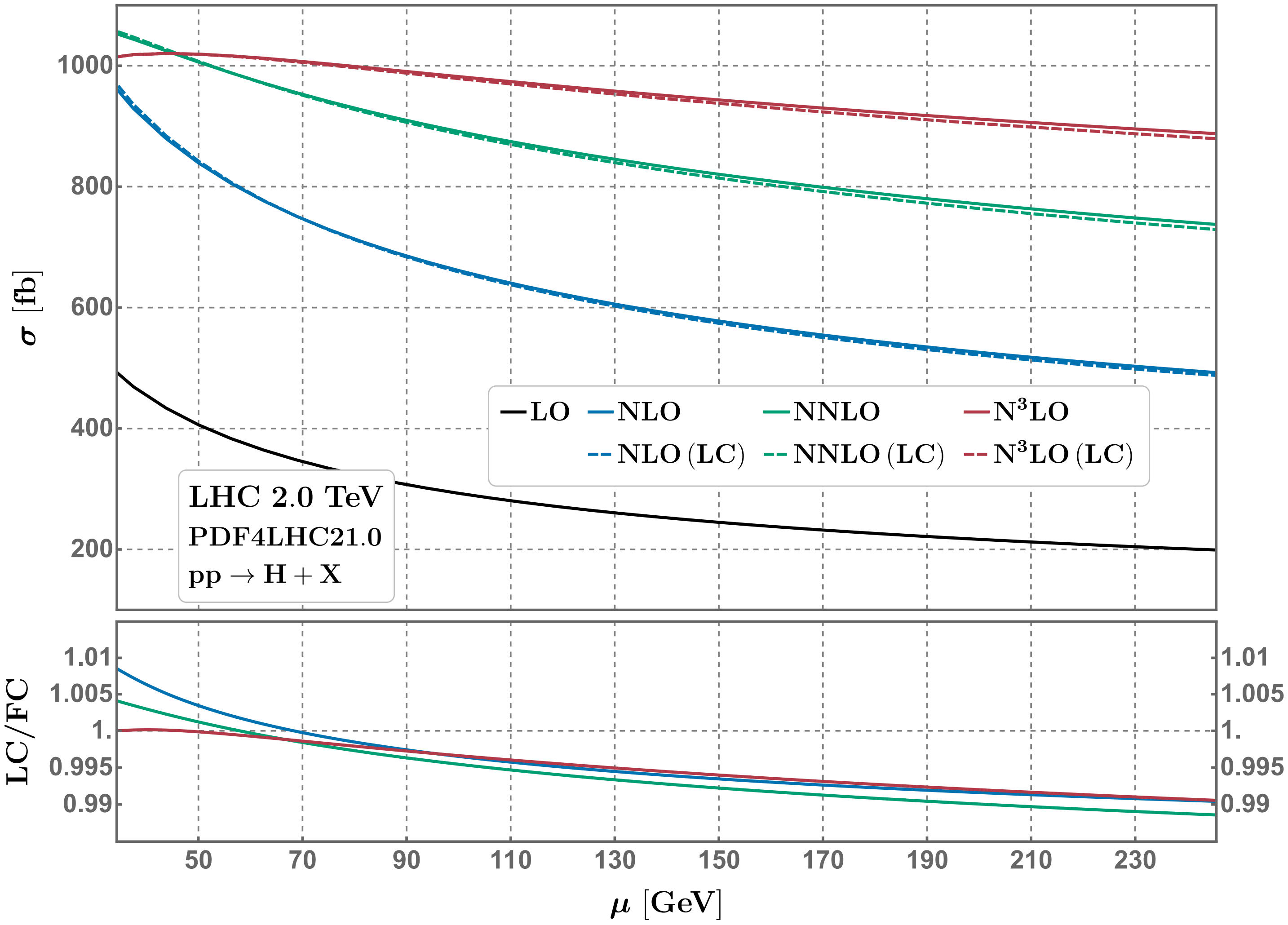
More Leading Color Phenomenology

Contribution to cross section at each order:



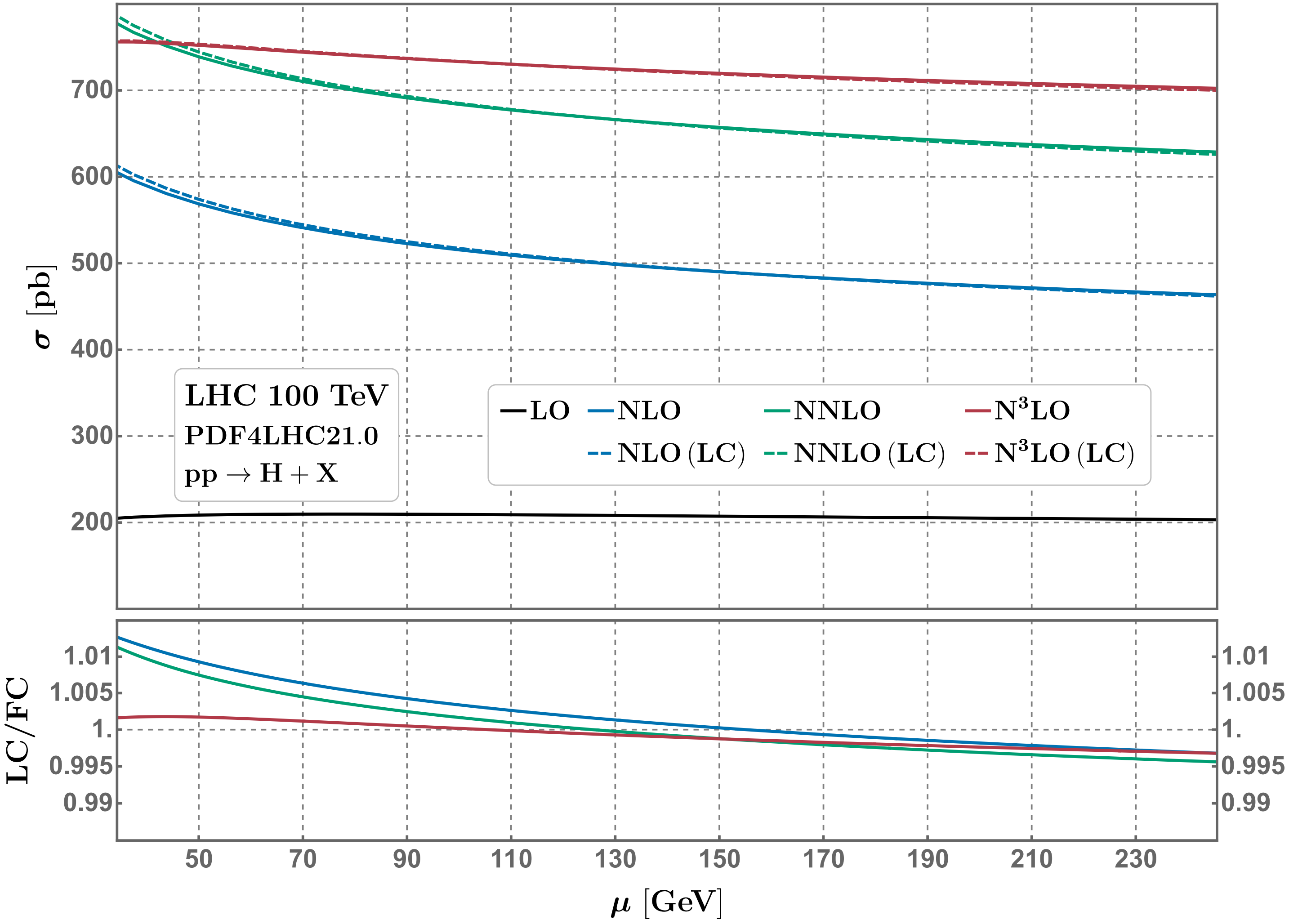
More Leading Color Phenomenology

Cross section at $\sqrt{s} = 2$ TeV



More Leading Color Phenomenology

Cross section at $\sqrt{s} = 100$ TeV



Integration strategy for RVVV Higgs Channels

Problem: $\int_0^1 r(\lambda) G(a_1, \dots, a_n, \lambda)\, d\lambda$

hard to evaluate when $a_i = \sqrt{\alpha + \beta\bar{z} + \gamma\bar{z}^2}\,\}$

Canonical Master Integral

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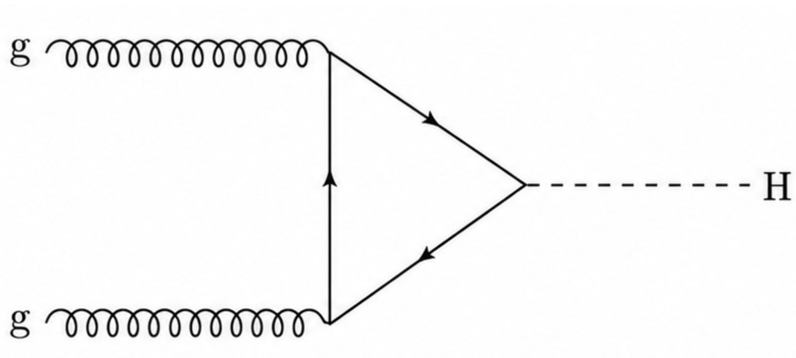
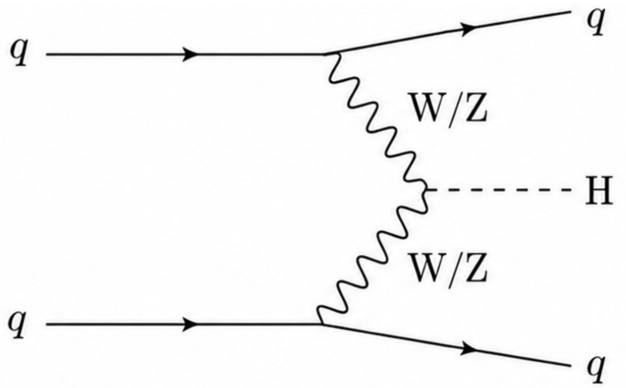
Next Iteration: $\epsilon \int_0^1 A_{ij}R(\lambda)M_j(\lambda)\, d\lambda$

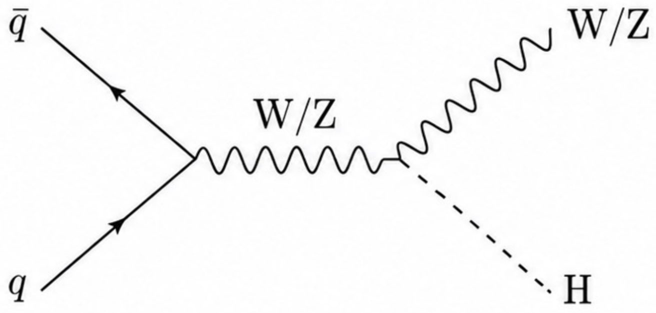
How are Higgs bosons produced at the LHC partonically?*



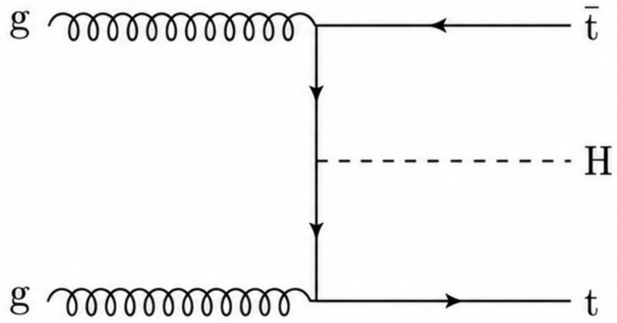
Gluon-fusion **88%**



Vector-boson fusion **4%**



W/Z Associated **7%**



$t\bar{t}$ associated **1%**

*[Higgs Cross Section Working Group; arXiv:1610.07922]

Drell-Yan: Translating t'Hooft to CDR