The On-Shell Path to Effective Field Theories: Anomalous Dimensions and Unitarity Bounds

SLAC Theory Seminar

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The Standard Model as an Effective Theory

- The Standard Model describes Nature's interactions with remarkable accuracy up to the TeV scale
- Yet it cannot be the final story: *neutrino masses*, *dark matter*, *baryogenesis*, *hierarchy problem*, ...
- **Effective Field Theory (EFT)** paradigm: the SM is the low-energy limit of a more fundamental theory emerging at some scale $\Lambda \gg v$

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Standard Model EFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d>4} \frac{1}{\Lambda^{d-4}} \sum_i c_i^{(d)}\, \mathcal{O}_i^{(d)}$$

- *Model-independent* parameterization of new physics through higher-dimensional, gauge-invariant operators $\mathcal{O}_i^{(d)}$ built from SM fields
- All the UV dynamics is encoded in the **Wilson coefficients** $c_i^{(d)}$

Two Fundamental Questions

1. How do Wilson coefficients *run*?

How does $c_i(\mu)$ evolve from the UV scale Λ down to the experimental scale $E \ll \Lambda$?

$\Downarrow$

Renormalization Group Equations

$$\mu \frac{\mathrm{d}c_i}{\mathrm{d}\mu} = \gamma_{i\leftarrow j}\, c_j + \cdots$$

$\rightsquigarrow$ *Anomalous Dimension Tensors*

2. When is the EFT *valid*?

For which values of c_i does the EFT admit a *consistent* UV completion?

$\Downarrow$

Theoretical Consistency Bounds

$$|c_i| \leq c_i^{\max}(\sqrt{s})$$

$\rightsquigarrow$ *Unitarity & Positivity* bounds

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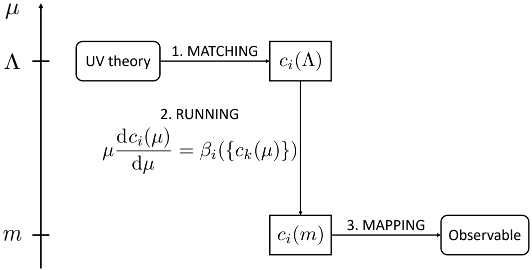
This talk: both questions admit an efficient unified treatment via on-shell amplitude methods

Why Running Matters

- Many of the most powerful probes of new physics are **low-energy observables** sensitive to operators generated at $\Lambda \gg v$

Examples

- **Electric Dipole Moments** (eEDM, nEDM)
- **Anomalous Magnetic Moments** ($(g-2)_\mu$)
- **Flavor Violation** ($\mu \to e\gamma$, $B \to K^{(*)}\ell\ell$, K-physics, ...)
- **Higgs & Electroweak Precision** (HL-LHC, FCC-ee, ...)



- Leading-log effects from running can dominate the phenomenology over a large separation of scales
- Reliable predictions *require* knowledge of the **anomalous dimension tensors** to the relevant loop order

Why Unitarity Bounds Matter

Key question

What is the energy scale at which an effective field theory breaks down?

- Unitarity violation $\rightsquigarrow$ New physics scale or strong dynamics
- Historical example: *no-lose Higgs theorem* $\quad m_H \lesssim 1\ \text{TeV}$

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Strength of Weak Interactions at Very High Energies and the Higgs Boson Mass

Benjamin W. Lee, C. Quigg,* and H. B. Thacker
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 28 February 1977)

It is shown that if the Higgs boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{1/2}$ partial-wave unitarity is not respected by the tree diagrams for two-body reactions of gauge bosons, and the weak interactions must become strong.

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Weak interactions at very high energies: The role of the Higgs-boson mass

Benjamin W. Lee,* C. Quigg,† and H. B. Thacker
Fermi National Accelerator Laboratory,‡ Batavia, Illinois 60510
(Received 20 April 1977)

We give an S-matrix-theoretic demonstration that if the Higgs-boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{1/2}$, parital-wave unitarity is not respected by the tree diagrams for two-body scattering of gauge bosons, and the weak interactions must become strong at high energies. We exhibit the relation of this bound to the structure of the Higgs-Goldstone Lagrangian, and speculate on the consequences of strongly coupled Higgs-Goldstone systems. Prospects for the observation of massive Higgs scalars are noted.

- Unitarity bounds are necessary for correct interpretation of experimental data (*e.g.*, tails of kinematical distributions, vector boson scattering, . . .)

Standard Methods Are Expensive

- Traditional Feynman-diagrammatic approach to EFT renormalization:
  - Combinatorial proliferation of diagrams with loops and external legs
  - Gauge-dependent expressions
  - Redundant operators related by equations of motion & field redefinitions
  - Non-trivial cancellations only at the very end

- Similar obstructions affect unitarity bounds:
  - Coupled-channel analyses with many helicity configurations
  - $2 \to N$ $(N \geq 3)$ processes not tractable with Wigner d-matrices
  - Spin-2 and higher-spin theories (gravity EFTs) impractical via Feynman rules

Can we bypass the complications by working directly with ***on-shell*** *&* ***physical*** *quantities?*

On-Shell Amplitude Methods

- Decades of work in collider physics have built a powerful alternative toolkit [Dixon '13]
  [Parke, Taylor '86] [Mangano, Parke '91] [Britto, Cachazo, Feng, Witten '05] [Peskin '11] [Elvang, Huang '13] …

Spinor-helicity formalism

Massless momentum $p^\mu \sigma_\mu^{\alpha\dot\alpha} = p^{\alpha\dot\alpha} = \lambda^\alpha \widetilde{\lambda}^{\dot\alpha}$

$$\langle ij\rangle = \lambda_i^\alpha \lambda_{j,\alpha} \quad [ij] = \widetilde{\lambda}_{i,\dot\alpha}\widetilde{\lambda}_j^{\dot\alpha} \quad \langle ij\rangle[ji] = 2p_i \cdot p_j$$

On-shell constructibility

- 3-point amplitudes fixed by Poincaré invariance & locality
- Higher-point tree amplitudes via *BCFW recursion*
- Loop amplitudes from *generalized unitarity cuts*

Why it matters for EFTs

- ✓ *Gauge invariance* manifest
- ✓ No *redundant operators*
- ✓ *Selection rules* (helicity, length, angular momentum) exposed
- ✓ Works natively for *arbitrary spin*
- ✓ *Recursive structure* reduces complexity loop-by-loop

Plan of the Talk

- ***Part 1* — Anomalous dimensions from unitarity cuts**
  - General operator mixing & leading mass effects [LB, Levati, Mastrolia, Paradisi '23]
  - Axion-Like Particle EFT [LB, Brunello, Levati, Mastrolia, Paradisi '24]
  - General Effective Gauge Theory [Aebischer, LB, Selimović '25a, '25b]
- ***Part 2* — Partial-wave unitarity bounds**
  - General $N \to M$ partial-wave decomposition [LB, Levati, Paradisi '25]
  - EFT of gravity & higher-spin theories [LB, Levati, Paradisi '25]
  - SMEFT & ALP phenomenology [LB, Levati, Paradisi '25] [LB, Paradisi, Sainaghi '26]

Part 1 — Anomalous Dimensions from Unitarity Cuts

On-Shell Methods for Renormalization

Core Features:

- **Unitarity Cuts:** Anomalous dimensions derived from discontinuities of scattering amplitudes
- **Phase-Space Integration:** Lorentz-invariant phase-space integrals replace full Feynman integrals
- **Advantages:**
  - Avoid complexities of standard loop calculations by focusing on physical, on-shell states
  - Gauge invariance is automatic
  - Explain zeros in anomalous dimensions $\rightsquigarrow$ Nonrenormalization Theorems based on

    1. HELICITY [Cheung, Shen '15]
    2. LENGTH [Bern, Parra-Martinez, Sawyer '20]
    3. ANGULAR MOMENTUM [Jiang, Shu, Xiao, Zheng '21]

Limitations & Generalizations

- Originally applied only to massless particles and operators with same dimensions
- Generalized to include **Leading Mass Effects** [LB, Levati, Mastrolia, Paradisi '23] via the *Higgs low-energy theorem* [Ellis, Gaillard, Nanopoulos '75] [Shifman, Vainshtein, Voloshin, Zakharov '79]:

$$\mathcal{L}_h^{\text{int}} = -\left(1 + \frac{h}{v}\right) \sum_f m_f \bar{f} f \implies \lim_{\{p_h\} \to 0} \mathcal{M}(A \to B + Nh) = \sum_f \left(\frac{m_f}{v} \frac{\partial}{\partial m_f}\right)^N \mathcal{M}(A \to B)$$

- Extended to handle the most **General Operator Mixing** [LB, Levati, Mastrolia, Paradisi '23]:

$$\beta_i := \mu \frac{\mathrm{d} c_i}{\mathrm{d} \mu} = \sum_{n>0} \frac{1}{n!} \gamma_{i \leftarrow j_1, \ldots, j_n} c_{j_1} \cdots c_{j_n} = \gamma_{i \leftarrow j} c_j + \frac{1}{2} \gamma_{i \leftarrow j,k} c_j c_k + \cdots ,$$

$$\gamma_{i \leftarrow j_1, \ldots, j_n} = \left. \frac{\partial^n \beta_i}{\partial c_{j_1} \cdots \partial c_{j_n}} \right|_*$$

S-Matrix & Dilatation Operator

- **Form Factor** associated with a local, gauge-invariant operator $\mathcal{O}_i$:

$$F_i(\vec{n}; q) = \frac{1}{\Lambda^{[\mathcal{O}_i]-4}} \langle \vec{n} | \mathcal{O}_i(q) | 0 \rangle$$

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- Exploiting the fundamental relations [Elias-Miró, Ingoldby, Riembau ’20]

  ◇ **Analyticity:** $F_i^*(\{s_{ij} - i\epsilon\}) = F_i(\{s_{ij} + i\epsilon\})$

  ◇ **Unitarity:** $\sum_{\vec{n}} \int d\Phi_{\vec{n}}\ |\vec{n}\rangle\langle\vec{n}| = \mathbb{1}\,, \quad d\Phi_{\vec{n}} = \prod_{i\in\vec{n}} \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}$

  ◇ **CPT Theorem:** $\langle\vec{n};\text{out}|\mathcal{O}_i(x)|0\rangle = \langle 0|\mathcal{O}_i^\dagger(-x)|\vec{n};\text{in}\rangle$

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  - ◇ **CPT Theorem:** $\langle\vec{n};\mathrm{out}|\mathcal{O}_i(x)|0\rangle = \langle 0|\mathcal{O}_i^\dagger(-x)|\vec{n};\mathrm{in}\rangle$

  it is possible to show that [Caron-Huot, Wilhelm '16]

$$e^{-i\pi D}F_i^*(\vec{n}) = (SF_i^*)(\vec{n}) \left(= \sum_{\vec{m}} \int d\Phi_{\vec{m}}\ \langle\vec{n}|S|\vec{m}\rangle\, F_i^*(\vec{m}) \right)$$

where $S = \mathbb{1} + i\mathcal{M}$ is the **S-Matrix** and $D = \sum_i p_i \cdot \partial/\partial p_i$ is the **Dilatation Operator**

Non-Perturbative Relations

Caron-Huot–Wilhelm equation

$$(e^{-i\pi D} - 1)F_i^* = i(\mathcal{M}F_i^*)$$

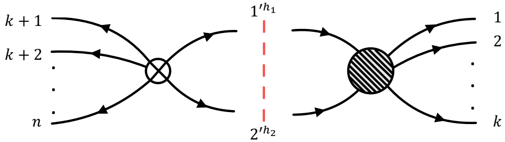
- In dimensional regularization and in **absence of masses**, $D \simeq -\mu\,\partial/\partial\mu$, which implies

Callan–Symanzik equation

$$DF_j = \left(\frac{\partial\beta_i}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}} + \delta_{ij}\beta_g\frac{\partial}{\partial g}\right)F_i$$

- Can be combined and expanded, *e.g.*, at one-loop

$$\left(\frac{\partial\beta_i^{(1)}}{\partial c_j} - \delta_{ij}\gamma_{i,\mathrm{IR}}^{(1)} + \delta_{ij}\beta_g^{(1)}\frac{\partial}{\partial g}\right)F_i^{(0)} = -\frac{1}{\pi}(\mathcal{M}F_j)^{(1)}$$



Master Formulae

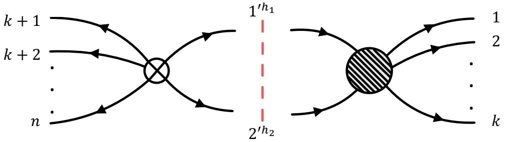
[LB, Levati, Mastrolia, Paradisi '23]

Linear operator mixing

$$\left(\gamma^{(1)}_{i\leftarrow j} - \delta_{ij}\gamma^{(1)}_{i,\mathrm{IR}}\right) F_i\big|^{(0)}_{*} = -\frac{1}{\pi}(\mathcal{M}F_j)\big|^{(1)}_{*}$$

$*$: Gaussian fixed point $\{c_i = 0\}$

$$(\mathcal{M}F_j)^{(1)}(\vec{n}) = \sum_k \sum_{h_1,h_2}$$



$$+ \text{ permutations}$$

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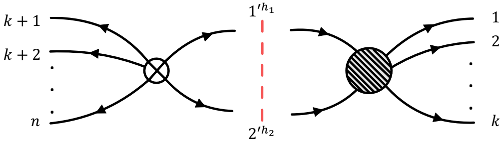


$+$ permutations

Non-linear operator mixing

$$\gamma^{(1)}_{i\leftarrow j,k} F_i|^{(0)}_* = -\frac{1}{\pi}\left.\frac{\partial}{\partial c_k}\right|_* (\mathcal{M}F_j)^{(1)}$$

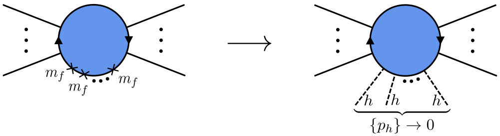
Leading Mass Effects

[LB, Levati, Mastrolia, Paradisi '23]

The amplitude requires ***N* fermion mass insertions** not to vanish

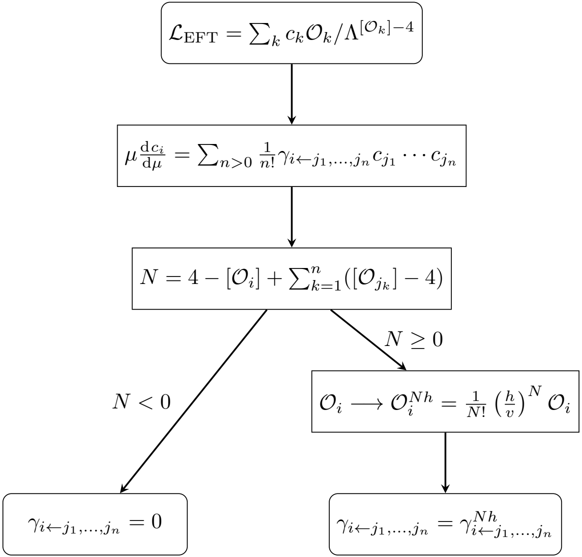
$\Downarrow$

Consider an equivalent amplitude entailing ***N* extra massless Higgs fields**



$$\mathcal{L}_h^{\text{int}} = -\left(1 + \frac{h}{v}\right) \sum_f m_f \bar{f} f$$

N: superficial degree of divergence



Stokes Integration

[Mastrolia '09] [LB, Brunello, Levati, Mastrolia, Paradisi '24]

- Efficient way to perform double-cut phase-space integrals
- IR singularities automatically discarded
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- **Steps:**
  1. Parameterize virtual spinors as

$$\begin{pmatrix} \lambda_x \\ \lambda_y \end{pmatrix} = \frac{1}{\sqrt{1+z\bar{z}}} \begin{pmatrix} 1 & \bar{z} \\ -z & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

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    2. Integrate in $\bar{z}$, keeping only *rational* contributions

$$G(z,\bar{z}) = \text{Rational}\left[\int d\bar{z}\, \frac{g(z,\bar{z})}{(1+z\bar{z})^2}\right]$$

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  3. Apply **Residue Theorem** by summing over the z-poles $\mathcal{P}_G$ of G:

$$\int d\Phi_2\, g = -\frac{1}{8\pi} \sum_{z_0 \in \mathcal{P}_G} \text{Res}_{(z,\bar{z})=(z_0,z_0^*)} G(z,\bar{z})$$

Application: Axion-Like Particle EFT

[LB, Brunello, Levati, Mastrolia, Paradisi '24]

- Axion-Like Particles (ALPs) ϕ: *pseudo-Nambu-Goldstone bosons* of a spontaneously broken global symmetry
- *Generalize* the QCD axion: $m_\phi f_\phi \sim m_\pi f_\pi$ correlation is relaxed, m_ϕ and $f_\phi \sim \Lambda$ become *independent parameters*
- Open up a much wider phenomenological landscape, probed across many scales: cosmology, astrophysics, beam-dumps, colliders, rare processes

CP-violating ALP EFT below the electroweak scale

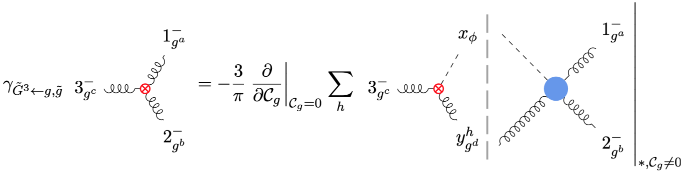
$$\mathcal{L}_\phi = \frac{\widetilde{\mathcal{C}}_\gamma}{\Lambda}\phi F\widetilde{F} + \frac{\widetilde{\mathcal{C}}_g}{\Lambda}\phi G\widetilde{G} + \mathcal{Y}_P^{ij}\phi\,\bar{f}_i i\gamma_5 f_j + \frac{\mathcal{C}_\gamma}{\Lambda}\phi FF + \frac{\mathcal{C}_g}{\Lambda}\phi GG + \mathcal{Y}_S^{ij}\phi\,\bar{f}_i f_j$$

- Simultaneous presence of both sectors $\Longrightarrow$ rich phenomenology: EDMs, $(g-2)$
  [Di Luzio, Gröber, Paradisi '20] [Di Luzio *et al.* '21]
- **Goal:** one-loop renormalization of the full CP-violating ALP EFT via on-shell methods

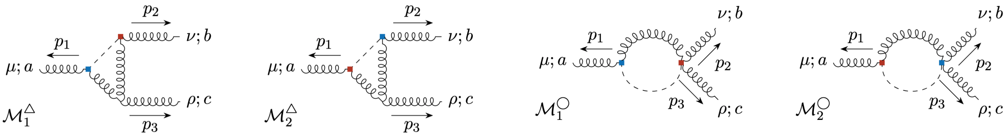
Axion-Like Particle RGEs

[LB, Brunello, Levati, Mastrolia, Paradisi ’24]

- Computed the full anomalous dimension matrix of the ALP sector
- Computed ALP-induced running of SM operators (*e.g.*, dipoles, Weinberg operator), both **above** and **below** the EW scale



- Used and compared different integration methods (angular *vs.* Stokes integration)
- Compared on-shell and standard techniques



+8 diagrams

Application: General Gauge EFT

[Aebischer, LB, Selimović '25a, '25b]

- BSM scenarios require *dedicated* RGE calculations
  (axions, ALPs, Z', flavor non-universal gauge models, GUTs, dark sectors, ...)
- **Strategy:** compute one-loop RGEs *once*, in maximal generality, specialize later
- EFT extension of the classical program for general *renormalizable* gauge theories
  [Machacek, Vaughn '83, '84, '85] [Luo, Wang, Xiao '02] [Poole, Thomsen '19] [Schienbein *et al.* '18]

Setup

– Gauge group $G = \prod_{\alpha=1}^{N_G} G_\alpha$
– Arbitrary real scalars ϕ_a, LH Weyl fermions ψ_i, gauge bosons $A_\mu^{A_\alpha}$
– Physical operator basis up to dim-6, built on-shell via **amplitude-operator correspondence**

The payoff

Any specific EFT reduces to a *group-theoretical lookup*:

[Gauge group] + **[Particle content]** $\underset{\text{group algebra}}{\rightsquigarrow}$ **[RGEs]**

Dimension-5 Operators

[Aebischer, LB, Selimović '25a, '25b]

Name	Operator	Symmetry	Form factor
$\mathcal{O}_{\phi^5}$	$\phi_a\phi_b\phi_c\phi_d\phi_e$	$[C_{\phi^5}]_{abcde} = [C_{\phi^5}]_{(abcde)}$	$F_{\phi^5}(1_a, 2_b, 3_c, 4_d, 5_e) = 5!\left[C_{\phi^5}\right]_{abcde}$
$\mathcal{O}_{\phi F^2}$	$\phi_a F_{\mu\nu}^{A_\alpha} F^{B_\beta\,\mu\nu}$	$\left[C_{\phi F^2}\right]_a^{A_\alpha B_\beta} = \left[C_{\phi F^2}\right]_a^{(A_\alpha B_\beta)}$	$F_{\phi F^2}(1_a, 2^-_{A_\alpha}, 3^-_{B_\beta}) = -\mathcal{S}_{\alpha\beta}\left[C_{\phi F^2}\right]_a^{A_\alpha B_\beta}\langle 23\rangle^2$
$\mathcal{O}_{\phi\widetilde{F}^2}$	$\phi_a F_{\mu\nu}^{A_\alpha} \widetilde{F}^{B_\beta\,\mu\nu}$	$\left[C_{\phi\widetilde{F}^2}\right]_a^{A_\alpha B_\beta} = \left[C_{\phi\widetilde{F}^2}\right]_a^{(A_\alpha B_\beta)}$	$F_{\phi\widetilde{F}^2}(1_a, 2^-_{A_\alpha}, 3^-_{B_\beta}) = -i\mathcal{S}_{\alpha\beta}\left[C_{\phi\widetilde{F}^2}\right]_a^{A_\alpha B_\beta}\langle 23\rangle^2$
$\mathcal{O}_{\psi^2\phi^2}$	$\psi_i\psi_j\phi_a\phi_b$	$\left[C_{\psi^2\phi^2}\right]_{ijab} = \left[C_{\psi^2\phi^2}\right]_{(ij)ab} = \left[C_{\psi^2\phi^2}\right]_{ij(ab)}$	$F_{\psi^2\phi^2}(1_i^-, 2_j^-, 3_a, 4_b) = 4\left[C_{\psi^2\phi^2}\right]_{ijab}\langle 12\rangle$
$\mathcal{O}_{\psi^2 F}$	$\psi_i\sigma^{\mu\nu}\psi_j F_{\mu\nu}^{A_\alpha}$	$\left[C_{\psi^2 F}\right]_{ij}^{A_\alpha} = \left[C_{\psi^2 F}\right]_{[ij]}^{A_\alpha}$	$F_{\psi^2 F}(1_i^-, 2_j^-, 3^-_{A_\alpha}) = 2\sqrt{2}\left[C_{\psi^2 F}\right]_{ij}^{A_\alpha}\langle 13\rangle\langle 23\rangle$

Dimension-6 Operators

[Aebischer, LB, Selimović '25a, '25b]

Name	Operator	Symmetry
$\mathcal{O}_{\phi^6}$	$\phi_a\phi_b\phi_c\phi_d\phi_e\phi_f$	$[C_{\phi^6}]_{abcdef} = [C_{\phi^6}]_{(abcdef)}$
$\mathcal{O}_{D^2\phi^4}$	$(D_\mu\phi)_a(D^\mu\phi)_b\phi_c\phi_d$	$[C_{D^2\phi^2}]_{abcd} = [C_{D^2\phi^2}]_{(ab)cd} = [C_{D^2\phi^2}]_{ab(cd)}$
$\mathcal{O}_{\phi^2F^2}$	$\phi_a\phi_b F^{A_\alpha}_{\mu\nu}F^{B_\beta\,\mu\nu}$	$\left[C_{\phi^2F^2}\right]^{A_\alpha B_\beta}_{ab} = \left[C_{\phi^2F^2}\right]^{(A_\alpha B_\beta)}_{ab} = \left[C_{\phi^2F^2}\right]^{A_\alpha B_\beta}_{(ab)}$
$\mathcal{O}_{\phi^2\widetilde{F}^2}$	$\phi_a\phi_b F^{A_\alpha}_{\mu\nu}\widetilde{F}^{B_\beta\,\mu\nu}$	$\left[C_{\phi^2\widetilde{F}^2}\right]^{A_\alpha B_\beta}_{ab} = \left[C_{\phi^2\widetilde{F}^2}\right]^{(A_\alpha B_\beta)}_{ab} = \left[C_{\phi^2\widetilde{F}^2}\right]^{A_\alpha B_\beta}_{(ab)}$
$\mathcal{O}_{F^3}$	$F^{A_\alpha\,\nu}_{\mu}F^{B_\alpha\,\rho}_{\nu}F^{C_\alpha\,\mu}_{\rho}$	$[C_{F^3}]^{A_\alpha B_\alpha C_\alpha} = [C_{F^3}]^{[A_\alpha B_\alpha C_\alpha]}$
$\mathcal{O}_{\widetilde{F}^3}$	$F^{A_\alpha\,\nu}_{\mu}F^{B_\alpha\,\rho}_{\nu}\widetilde{F}^{C_\alpha\,\mu}_{\rho}$	$\left[C_{\widetilde{F}^3}\right]^{A_\alpha B_\alpha C_\alpha} = \left[C_{\widetilde{F}^3}\right]^{[A_\alpha B_\alpha C_\alpha]}$
$\mathcal{O}_{\psi^2\phi^3}$	$\psi_i\psi_j\phi_a\phi_b\phi_c$	$\left[C_{\psi^2\phi^3}\right]_{ijabc} = \left[C_{\psi^2\phi^3}\right]_{ij(abc)} = \left[C_{\psi^2\phi^3}\right]_{(ij)abc}$
$\mathcal{O}_{\psi^2\phi F}$	$\psi_i\sigma^{\mu\nu}\psi_j\phi_a F^{A_\alpha}_{\mu\nu}$	$\left[C_{\psi^2\phi F}\right]^{A_\alpha}_{ija} = \left[C_{\psi^2\phi F}\right]^{A_\alpha}_{[ij]a}$
$\mathcal{O}_{\psi^4}$	$(\psi_i\psi_j)(\psi_k\psi_l)$	$\left[C_{\psi^4}\right]_{ijkl} = \left[C_{\psi^4}\right]_{(ij)kl} = \left[C_{\psi^4}\right]_{ij(kl)} = \left[C_{\psi^4}\right]_{klij}$
$\mathcal{O}_{\overline{\psi}^2\psi^2}$	$(\psi_i^\dagger\overline{\sigma}^\mu\psi_j)(\psi_k^\dagger\overline{\sigma}_\mu\psi_l)$	$\left[C_{\overline{\psi}^2\psi^2}\right]_{ijkl} = \left[C_{\overline{\psi}^2\psi^2}\right]_{kjil} = \left[C_{\overline{\psi}^2\psi^2}\right]_{ilkj} = \left[C_{\overline{\psi}^2\psi^2}\right]^*_{jilk}$
$\mathcal{O}_{D\overline{\psi}\psi\phi^2}$	$i(\psi_i^\dagger\overline{\sigma}^\mu\psi_j)\,[(D_\mu\phi)_a\phi_b - \phi_a(D_\mu\phi)_b]$	$\left[C_{D\overline{\psi}\psi\phi^2}\right]_{ijab} = \left[C_{D\overline{\psi}\psi\phi^2}\right]_{ij[ab]} = \left[C_{D\overline{\psi}\psi\phi^2}\right]^*_{jiba}$

Dimension-6 Operators

[Aebischer, LB, Selimović '25a, '25b]

Name	Form factor
$\mathcal{O}_{\phi^6}$	$F_{\phi^5}(1_a, 2_b, 3_c, 4_d, 5_e, 6_f) = 6!\left[C_{\phi^6}\right]_{abcdef}$
$\mathcal{O}_{D^2\phi^4}$	$F_{D^2\phi^4}(1_a, 2_b, 3_c, 4_d) = -2\left(\left[\widehat{C}_{D^2\phi^4}\right]_{abcd} s_{12} + \left[\widehat{C}_{D^2\phi^4}\right]_{acbd} s_{13}\right)$
$\mathcal{O}_{\phi^2 F^2}$	$F_{\phi^2 F^2}(1_a, 2_b, 3^-_{A_\alpha}, 4^-_{B_\beta}) = -2\mathcal{S}_{\alpha\beta}\left[C_{\phi^2 F^2}\right]^{A_\alpha B_\beta}_{ab} \langle 34\rangle^2$
$\mathcal{O}_{\phi^2 \widetilde{F}^2}$	$F_{\phi^2 \widetilde{F}^2}(1_a, 2_b, 3^-_{A_\alpha}, 4^-_{B_\beta}) = -2i\mathcal{S}_{\alpha\beta}\left[C_{\phi^2 \widetilde{F}^2}\right]^{A_\alpha B_\beta}_{ab} \langle 34\rangle^2$
$\mathcal{O}_{F^3}$	$F_{F^3}(1^-_{A_\alpha}, 2^-_{B_\alpha}, 3^-_{C_\alpha}) = -3i\sqrt{2}\left[C_{F^3}\right]^{A_\alpha B_\alpha C_\alpha} \langle 12\rangle\langle 23\rangle\langle 31\rangle$
$\mathcal{O}_{\widetilde{F}^3}$	$F_{\widetilde{F}^3}(1^-_{A_\alpha}, 2^-_{B_\alpha}, 3^-_{C_\alpha}) = 3\sqrt{2}\left[C_{\widetilde{F}^3}\right]^{A_\alpha B_\alpha C_\alpha} \langle 12\rangle\langle 23\rangle\langle 31\rangle$
$\mathcal{O}_{\psi^2\phi^3}$	$F_{\psi^2\phi^3}(1^-_i, 2^-_j, 3_a, 4_b, 5_c) = 12\left[C_{\psi^2\phi^3}\right]_{ijabc} \langle 12\rangle$
$\mathcal{O}_{\psi^2\phi F}$	$F_{\psi^2\phi F}(1^-_i, 2^-_j, 3_a, 4^-_{A_\alpha}) = 2\sqrt{2}\left[C_{\psi^2\phi F}\right]^{A_\alpha}_{ija} \langle 14\rangle\langle 24\rangle$
$\mathcal{O}_{\psi^4}$	$F_{\psi^4}(1^-_i, 2^-_j, 3^-_k, 4^-_l) = 8\left(\left[C_{\psi^4}\right]_{ijkl} - \left[C_{\psi^4}\right]_{iljk}\right)\langle 12\rangle\langle 34\rangle + 8\left(\left[C_{\psi^4}\right]_{ikjl} - \left[C_{\psi^4}\right]_{iljk}\right)\langle 13\rangle\langle 42\rangle$
$\mathcal{O}_{\overline{\psi}^2\psi^2}$	$F_{\overline{\psi}^2\psi^2}(1^+_i, 2^-_j, 3^+_k, 4^-_l) = 8\left[C_{\overline{\psi}^2\psi^2}\right]_{ijkl} \langle 24\rangle[31]$
$\mathcal{O}_{D\overline{\psi}\psi\phi^2}$	$F_{D\overline{\psi}\psi\phi^2}(1^+_i, 2^-_j, 3_a, 4_b) = -4\left[C_{D\overline{\psi}\psi\phi^2}\right]_{ijab} \langle 23\rangle[31]$

General Gauge EFT Results

[Aebischer, LB, Selimović '25a, '25b]

- Full one-loop anomalous dimensions for *all* dim-5 and dim-6 operators
  $\rightsquigarrow$ **184 anomalous dimensions** in total
- Included running of **renormalizable couplings** induced by dim-5 and dim-6 operators
- Every RGE expressed as [coupling combinations] × [group-theoretical invariants]

Usage

Project the general EFT onto your specific model $\Longrightarrow$
contract the group-theoretical invariants $\Longrightarrow$ RGEs

- *Cross-checks:* SMEFT, LEFT, ALP EFT, scalar $O(n)$ EFT
- Probe *light* new states via their **RG mixing into well-constrained SMEFT operators**, largely independently of mass or lifetime [Galda, Neubert, Renner '21]

Part 2 — Partial-Wave Unitarity Bounds

Standard Approach to Unitarity Bounds

- **Standard approach**: $2 \to 2$ partial-wave decomposition with **Wigner d-matrices**

$$\mathcal{A}_{h_1,h_2\to h_3,h_4}(s,\theta,\varphi) = 8\pi \sum_J (2J+1)\; a^J_{h_1,h_2\to h_3,h_4}(s)\; d^J_{h_1-h_2,h_3-h_4}(\theta)\; e^{i(h_1-h_2-h_3+h_4)\varphi}$$

h_i: helicities J: total angular momentum [Jacob, Wick '59]

- Two critical **gaps**:

  1. $N \to M$ amplitudes (*e.g.*, $2 \to 3$), relevant for high-energy future colliders, do not admit such decomposition

  2. Spin-2 or higher-spin theories: Feynman rules impractical

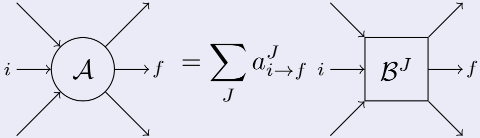
- **New formalism needed** to generalize partial-wave unitarity bounds [LB, Levati, Paradisi '25]

Amplitudes & Partial-Wave Decomposition

Linear algebra problem

Project an amplitude $|\mathcal{A}_{i\to f}\rangle$ onto a *kinematic basis* $|\mathcal{B}^J_{i\to f}\rangle$ with definite angular momentum J

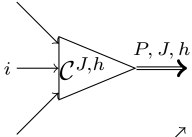
$$|\mathcal{A}_{i\to f}\rangle = \sum_J a^J_{i\to f}\, |\mathcal{B}^J_{i\to f}\rangle$$



- $|\mathcal{A}_{i\to f}\rangle\,,\ |\mathcal{B}^J_{i\to f}\rangle \in V_{i\to f}$ vector space
- $a^J_{i\to f}$: partial-wave coefficients that encode the *dynamics*

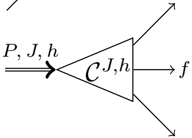
Poincaré Clebsch-Gordan Coefficients

$$\langle P, J, h | i \rangle = \mathcal{C}^{J,h}_{i\to *}\, \delta^{(4)}\left(P - \sum_{k\in i} p_k\right) =$$



$\in V_{i\to *}$

$$\langle f | P, J, h \rangle = \mathcal{C}^{J,h}_{*\to f}\, \delta^{(4)}\left(P - \sum_{k\in f} p_k\right) =$$

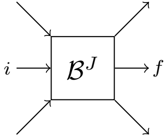


$\in V_{*\to f}$

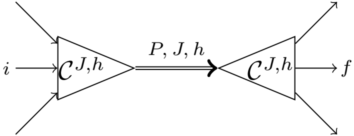
$|P, J, h\rangle$: Poincaré irreducible multiparticle state

[Jiang, Shu, Xiao, Zheng '20]

$$\left|\mathcal{B}^J_{i\to f}\right\rangle = \sum_h \left|\mathcal{C}^{J,h}_{i\to *}\right\rangle \otimes \left|\mathcal{C}^{J,h}_{*\to f}\right\rangle$$



$= \sum_h$

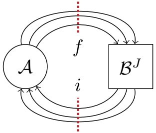


Perturbative Unitarity Bounds

[LB, Levati, Paradisi '25]

- *Inner product* via Lorentz-invariant phase-space integrals

$$a^J_{i\to f} = \frac{1}{2J+1}\langle \mathcal{B}^J_{i\to f}|\mathcal{A}_{i\to f}\rangle = \frac{1}{2J+1}\int \mathrm{d}\Phi_i\,\mathrm{d}\Phi_f\,\mathcal{A}_{i\to f}\left(\mathcal{B}^J_{i\to f}\right)^* = \frac{1}{2J+1}$$



$$\langle \mathcal{B}^J_{i\to f}|\mathcal{B}^{J'}_{i\to f}\rangle = (2J+1)\delta^{JJ'} \quad\Longleftrightarrow\quad \int \mathrm{d}\Phi_X\ |\mathcal{B}^J_{i\to X}\rangle \otimes |\mathcal{B}^{J'}_{X\to f}\rangle = |\mathcal{B}^J_{i\to f}\rangle\,\delta^{JJ'}$$

- *Generalized optical theorem*:

$$|\mathcal{A}_{i\to f}\rangle - |\mathcal{A}^*_{f\to i}\rangle = i\sum_X \int \mathrm{d}\Phi_X\ |\mathcal{A}_{i\to X}\rangle \otimes |\mathcal{A}^*_{f\to X}\rangle$$

$$a^J_{i\to f} - \left(a^J_{f\to i}\right)^* = i\sum_X a^J_{i\to X}\left(a^J_{f\to X}\right)^*$$

- *Partial-wave unitarity bounds*:

$$\left|\mathrm{Re}\,a^J_{i\to i}\right| \leq 1 \qquad 0 \leq \mathrm{Im}\,a^J_{i\to i} \leq 2 \qquad \left|a^J_{i\to f}\right| \leq 1 \quad (f\neq i)$$

Determination of $|\mathcal{B}^J_{i\to f}\rangle$: a 3-Step Algorithm

1. Find a basis for $V_{i\to f}$ of kinematic *monomials* in spinor-helicity variables $\{\lambda_k, \widetilde{\lambda}_k\}_{k=1}^n$
   [Shadmi, Weiss '18] [De Angelis '22] [Li, Ren, Xiao, Yu, Zheng '22]
2. Find the eigenvectors with definite $J_\mathcal{I}$ of the *Pauli-Lubanski operator squared*

$$\mathbb{W}^2_\mathcal{I} = \frac{1}{8}\mathbb{P}^2_\mathcal{I}\left(\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\mathbb{M}_{\mathcal{I},\alpha\beta}\mathbb{M}_{\mathcal{I},\gamma\delta} + \epsilon^{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\widetilde{\mathbb{M}}_{\mathcal{I},\dot\alpha\dot\beta}\widetilde{\mathbb{M}}_{\mathcal{I},\dot\gamma\dot\delta}\right) + \frac{1}{4}\mathbb{P}^{\alpha\dot\alpha}_\mathcal{I}\mathbb{P}^{\beta\dot\beta}_\mathcal{I}\mathbb{M}_{\mathcal{I},\alpha\beta}\widetilde{\mathbb{M}}_{\mathcal{I},\dot\alpha\dot\beta}$$

   where $\mathcal{I} = i$ or f and [Witten '03]

$$\mathbb{P}^{\alpha\dot\alpha}_\mathcal{I} = \sum_{i\in\mathcal{I}}\lambda^\alpha_i\widetilde{\lambda}^{\dot\alpha}_i$$

$$\mathbb{M}^{\alpha\beta}_\mathcal{I} = \sum_{i\in\mathcal{I}}\left(\lambda^\alpha_i\frac{\partial}{\partial\lambda_{i,\beta}} + \lambda^\beta_i\frac{\partial}{\partial\lambda_{i,\alpha}}\right) \qquad \widetilde{\mathbb{M}}^{\dot\alpha\dot\beta}_\mathcal{I} = \sum_{i\in\mathcal{I}}\left(\widetilde{\lambda}^{\dot\alpha}_i\frac{\partial}{\partial\widetilde{\lambda}_{i,\dot\beta}} + \widetilde{\lambda}^{\dot\beta}_i\frac{\partial}{\partial\widetilde{\lambda}_{i,\dot\alpha}}\right)$$

   The eigenvalues are $-P^2_\mathcal{I}J_\mathcal{I}(J_\mathcal{I}+1)$
3. Normalize them such that their norm is $\sqrt{2J_\mathcal{I}+1}$

Photon & Gravity EFT

[LB, Levati, Paradisi '25]

Anomalous quartic couplings (P- and CP-violating)

For *generic* spin-S ($S \in \mathbb{N}$) massless particles

$$\frac{\mathcal{L}^{(S)}}{\sqrt{-g}} = c_1^{(S)} (\mathcal{Q}^{(S)})^2 + c_2^{(S)} (\widetilde{\mathcal{Q}}^{(S)})^2 + c_3^{(S)} \mathcal{Q}^{(S)} \widetilde{\mathcal{Q}}^{(S)}$$

$$\mathcal{Q}^{(1)} = F_{\mu\nu} F^{\mu\nu} \qquad \widetilde{\mathcal{Q}}^{(1)} = F_{\mu\nu} \widetilde{F}^{\mu\nu} \qquad \mathcal{Q}^{(2)} = M_{\mathrm{P}}^2 \, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \qquad \widetilde{\mathcal{Q}}^{(2)} = M_{\mathrm{P}}^2 \, R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma}$$

Photon & Gravity EFT

[LB, Levati, Paradisi '25]

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Full $2 \to 2$ helicity amplitude:

$|\mathcal{A}_{i\to f}\rangle =$

	$(3^{+S},4^{+S})$	$(3^{+S},4^{-S})$	$(3^{-S},4^{-S})$
$(1^{+S},2^{+S})$	$8\, c_+^{(S)} \langle 12\rangle^{2S} [34]^{2S}$	0	$8\, c_-^{(S)} (\langle 12\rangle^{2S}\langle 34\rangle^{2S} + \langle 13\rangle^{2S}\langle 24\rangle^{2S} + \langle 14\rangle^{2S}\langle 23\rangle^{2S})$
$(1^{+S},2^{-S})$	0	$8\, c_+^{(S)} \langle 14\rangle^{2S} [23]^{2S}$	0
$(1^{-S},2^{-S})$	$8\, (c_-^{(S)})^* ([12]^{2S}[34]^{2S} + [13]^{2S}[24]^{2S} + [14]^{2S}[23]^{2S})$	0	$8\, c_+^{(S)} \langle 34\rangle^{2S} [12]^{2S}$

with $c_+^{(S)} = c_1^{(S)} + c_2^{(S)} \in \mathbb{R}$ and $c_-^{(S)} = c_1^{(S)} - c_2^{(S)} + i\, c_3^{(S)} \in \mathbb{C}$

Photon & Gravity EFT

[LB, Levati, Paradisi '25]

1. Normalized eigenvectors of $\mathbb{W}_{12}^2$ corresponding to $J_{12} = 0$ (with $s = 2p_1 \cdot p_2$):

$$\left|\mathcal{B}^0_{i\to f}\right\rangle = \left(\begin{array}{c||ccc} & (3^{+S},4^{+S}) & (3^{+S},4^{-S}) & (3^{-S},4^{-S}) \\ \hline\hline (1^{+S},2^{+S}) & \frac{16\pi}{s^{2S}}\langle 12\rangle^{2S}[34]^{2S} & 0 & \frac{16\pi}{s^{2S}}\langle 12\rangle^{2S}\langle 34\rangle^{2S} \\ (1^{+S},2^{-S}) & 0 & 0 & 0 \\ (1^{-S},2^{-S}) & \frac{16\pi}{s^{2S}}[12]^{2S}[34]^{2S} & 0 & \frac{16\pi}{s^{2S}}\langle 34\rangle^{2S}[12]^{2S} \end{array}\right)$$

2. Partial-wave coefficients:

$$a^0_{i\to f} = \langle \mathcal{B}^0_{i\to f}|\mathcal{A}_{i\to f}\rangle = \left(\begin{array}{c||ccc} & (3^{+S},4^{+S}) & (3^{+S},4^{-S}) & (3^{-S},4^{-S}) \\ \hline\hline (1^{+S},2^{+S}) & \frac{s^{2S}}{2\pi}c_+^{(S)} & 0 & \frac{s^{2S}}{2\pi}\frac{2S+3}{2S+1}c_-^{(S)} \\ (1^{+S},2^{-S}) & 0 & 0 & 0 \\ (1^{-S},2^{-S}) & \frac{s^{2S}}{2\pi}\frac{2S+3}{2S+1}\left(c_-^{(S)}\right)^* & 0 & \frac{s^{2S}}{2\pi}c_+^{(S)} \end{array}\right)$$

3. Non-zero eigenvalues:

$$\frac{s^{2S}}{2\pi}\left(c_+^{(S)} \pm \frac{2S+3}{2S+1}\left|c_-^{(S)}\right|\right) \quad \Longrightarrow \quad \frac{s^{2S}}{2\pi}\left|c_+^{(S)} \pm \frac{2S+3}{2S+1}\left|c_-^{(S)}\right|\right| \leq 1$$

Positivity Bounds

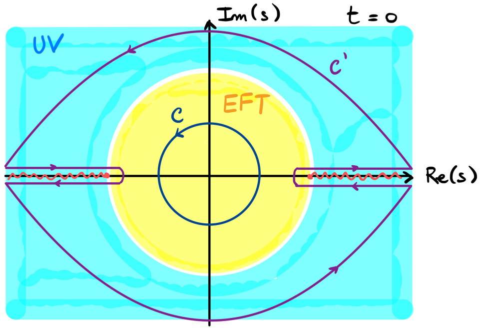
[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

- Certain signs of Wilson coefficients violate **fundamental principles**:
  - Unitarity, Causality, and Analyticity

- Forward and crossing symmetric $2 \to 2$ amplitude

  $\mathcal{A}(s) = \lim_{t\to 0} \mathcal{A}(s,t) = \sum_{n\geq 0} c_n s^n$

  - Froissart bound:
    $|\mathcal{A}(s)| = o(|s|^n)$ as $|s| \to \infty$ $(n \geq 2)$
  - Schwartz reflection principle:
    $\mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon) = 2i\,\mathrm{Im}\,\mathcal{A}(s)$
  - Optical theorem: $\mathrm{Im}\,\mathcal{A}(s) = s\,\sigma(s)$

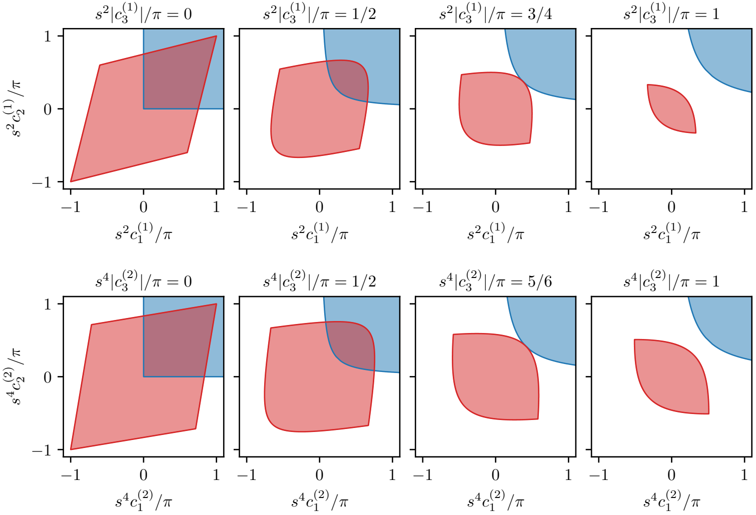


imply $$c_n = \frac{1+(-1)^n}{\pi}\int_{\Lambda^2}^{\infty} \mathrm{d}s\, \frac{\sigma(s)}{s^n} \geq 0$$

- In our example: $|c_-^{(S)}| \leq c_+^{(S)}$

Synergy of Unitarity & Positivity Bounds

[LB, Levati, Paradisi '25]



Shaded regions: allowed

Partial-wave unitarity ($\mathscr{U}$)

$$|c_+^{(S)}| + \frac{2S+3}{2S+1}|c_-^{(S)}| \leq \frac{2\pi}{s^{2S}}$$

Positivity ($\mathscr{P}$)

$$c_1^{(S)} \geq 0 \qquad c_2^{(S)} \geq 0$$

$$(c_3^{(S)})^2 \leq 4\, c_1^{(S)} c_2^{(S)}$$

$$\frac{\mathrm{Vol}(\mathscr{U} \cap \mathscr{P})}{\mathrm{Vol}(\mathscr{U})} = \frac{1}{32}\left(\frac{2S+3}{S+1}\right)^2$$

$2 \to 2$ *vs.* $2 \to 3$: SMEFT Examples

[LB, Levati, Paradisi '25]

Dimension-6 example

$$\mathcal{L}^{(6)} \supset C_{eH}^{pr} \left(H^\dagger H\right) \bar{\ell}_p e_r H + C_{dH}^{pr} \left(H^\dagger H\right) \bar{q}_p d_r H + C_{uH}^{pr} \left(H^\dagger H\right) \bar{q}_p u_r \widetilde{H} + \text{H.c.}$$

$$\sqrt{\text{Tr}\left[3C_{uH}C_{uH}^\dagger + 3C_{dH}C_{dH}^\dagger + C_{eH}C_{eH}^\dagger\right]} \leq \frac{32\pi^2}{\sqrt{3s}}$$

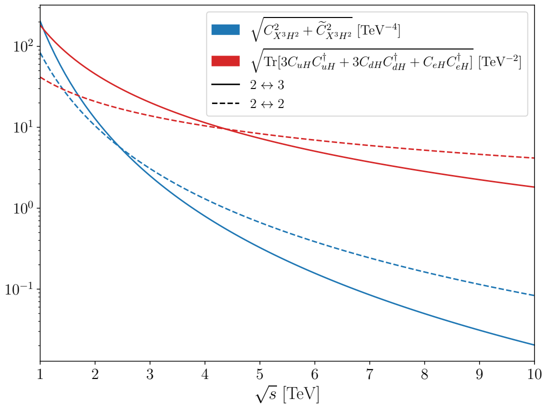
$$\sqrt{\text{Tr}\left[3C_{uH}C_{uH}^\dagger + 3C_{dH}C_{dH}^\dagger + C_{eH}C_{eH}^\dagger\right]} \leq \frac{4\sqrt{2}\pi}{\sqrt{3sv^2}}$$

Dimension-8 example

$$\mathcal{L}^{(8)} \supset C_{X^3H^2} \left(H^\dagger H\right) f^{ABC} X_\mu^{A\nu} X_\nu^{B\rho} X_\rho^{C\mu} + \widetilde{C}_{X^3H^2} \left(H^\dagger H\right) f^{ABC} X_\mu^{A\nu} X_\nu^{B\rho} \widetilde{X}_\rho^{C\mu}$$

$$\sqrt{C_{X^3H^2}^2 + \widetilde{C}_{X^3H^2}^2} \leq \frac{32\sqrt{10}\pi^2}{s^2} \frac{1}{\sqrt{C_2(G)d(G)}}$$

$$\sqrt{C_{X^3H^2}^2 + \widetilde{C}_{X^3H^2}^2} \leq \frac{8\sqrt{2}\pi}{vs^{3/2}} \frac{1}{\sqrt{C_2(G)}}$$



With gauge group $G = SU(3)$, $d(G) = 8$ and $C_2(G) = 3$

SMEFT Example

[LB, Paradisi, Sainaghi '26]

Computed unitarity bounds for *all* dim-6 SMEFT Wilson coefficients of Warsaw basis

Semi-leptonic operators:

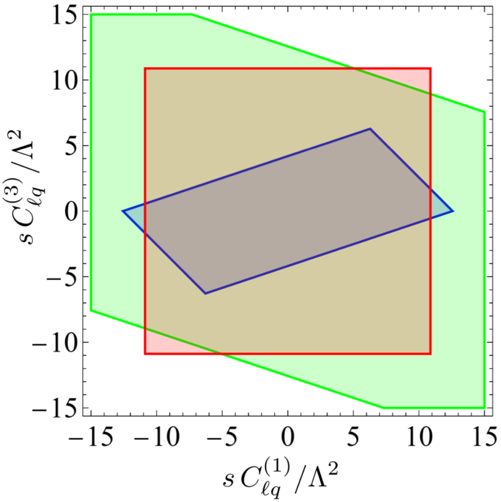
$$O_{\ell q}^{(1)} = (\bar{\ell}_3 \gamma^\mu \ell_3)(\bar{q}_3 \gamma_\mu q_3)$$
$$O_{\ell q}^{(3)} = (\bar{\ell}_3 \gamma^\mu \sigma^I \ell_3)(\bar{q}_3 \gamma_\mu \sigma^I q_3)$$

Full **coupled-channel analysis**:

$$\ell_p^i q_r^{j\alpha} \to \ell_s^k q_t^{\ell\beta} \qquad J = 0$$

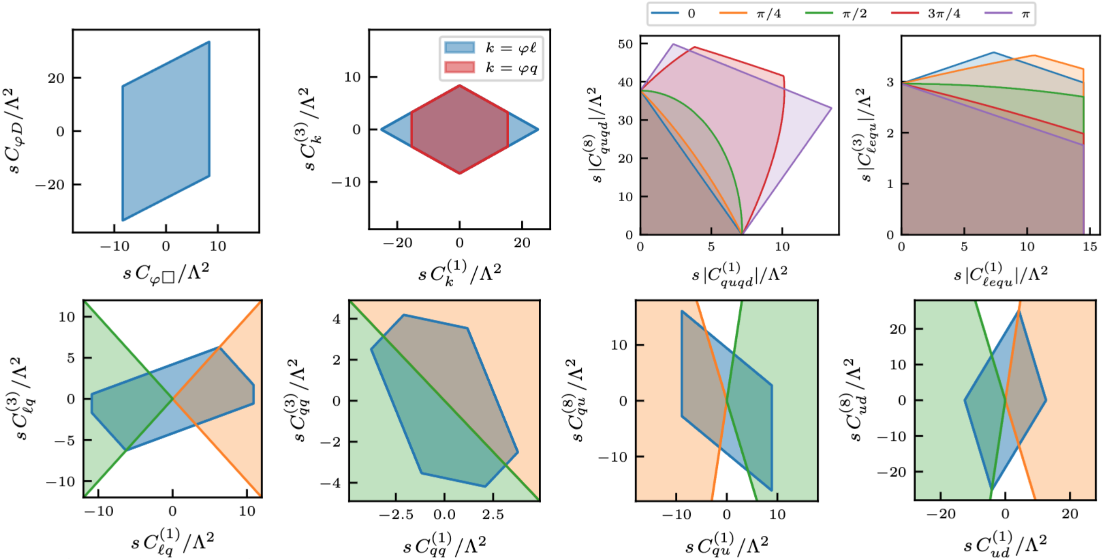
$$\ell_p^i \bar{q}_r^{j\alpha} \to \ell_s^k \bar{q}_t^{\ell\beta} \qquad J = 1$$

$$\ell_p^i \bar{\ell}_r^j \to q_s^{k\alpha} \bar{q}_t^{\ell\beta} \qquad J = 1$$



SMEFT Correlations

[LB, Paradisi, Sainaghi '26]



SMEFT Marginalized Bounds

[LB, Paradisi, Sainaghi ’26]

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$		$(\bar{L}L)(\bar{L}L)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
C_G	$4\pi/(9g_s)$	C_φ	$32\pi^3/3$	$C_{e\varphi}$	$32\pi^2/\sqrt{3}$	$C_{\ell\ell}$	2π	$C_{\ell edq}$	$8\pi/\sqrt{3}$
$C_{\widetilde{G}}$	$4\pi/(9g_s)$	$C_{\varphi\Box}$	$8\pi/3$	$C_{u\varphi}$	$32\pi^2/3$	$C_{qq}^{(1)}$	$6\pi/5$	$C_{quqd}^{(1)}$	$16\pi(1+\sqrt{2})/9$
C_W	$2\pi/(3g)$	$C_{\varphi D}$	$32\pi/3$	$C_{d\varphi}$	$32\pi^2/3$	$C_{qq}^{(3)}$	$4\pi/3$	$C_{quqd}^{(8)}$	$4\pi(2+7\sqrt{2})/3$
$C_{\widetilde{W}}$	$2\pi/(3g)$					$C_{\ell q}^{(1)}$	$2\pi\sqrt{3}$	$C_{\ell equ}^{(1)}$	$8\pi/\sqrt{3}$
						$C_{\ell q}^{(3)}$	2π	$C_{\ell equ}^{(3)}$	$(2+\sqrt{2})\pi/3$

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$C_{\varphi G}$	π	C_{uG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(1)}$	8π	C_{ee}	2π	$C_{\ell e}$	4π
$C_{\varphi\widetilde{G}}$	π	C_{dG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(3)}$	$8\pi/3$	C_{uu}	$3\pi/2$	$C_{\ell u}$	4π
$C_{\varphi W}$	$2\pi\sqrt{2/3}$	C_{eW}	$4\pi\sqrt{2/3}$	$C_{\varphi e}$	8π	C_{dd}	$3\pi/2$	$C_{\ell d}$	4π
$C_{\varphi\widetilde{W}}$	$2\pi\sqrt{2/3}$	C_{uW}	$4\pi\sqrt{2/3}$	$C_{\varphi q}^{(1)}$	$2\pi\sqrt{6}$	C_{eu}	4π	C_{qe}	4π
$C_{\varphi B}$	$2\pi\sqrt{2}$	C_{uB}	4π	$C_{\varphi q}^{(3)}$	$8\pi/3$	C_{ed}	4π	$C_{qu}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{B}}$	$2\pi\sqrt{2}$	C_{eB}	4π	$C_{\varphi u}$	$4\pi\sqrt{3}$	$C_{ud}^{(1)}$	4π	$C_{qu}^{(8)}$	$3\pi(1+1/\sqrt{2})$
$C_{\varphi WB}$	4π	C_{dW}	$4\pi\sqrt{2/3}$	$C_{\varphi d}$	$4\pi\sqrt{3}$	$C_{ud}^{(8)}$	8π	$C_{qd}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{W}B}$	4π	C_{dB}	4π	$C_{\varphi ud}$	8π			$C_{qd}^{(8)}$	$3\pi(1+1/\sqrt{2})$

SMEFT Marginalized Bounds

[LB, Paradisi, Sainaghi '26]

Verified

$\boldsymbol{X^3}$		$\boldsymbol{\varphi^6}$ and $\boldsymbol{\varphi^4 D^2}$		$\boldsymbol{\psi^2\varphi^3}$		$\boldsymbol{(\bar{L}L)(\bar{L}L)}$		$\boldsymbol{(\bar{L}R)(\bar{R}L)}$ and $\boldsymbol{(\bar{L}R)(\bar{L}R)}$	
C_G	$4\pi/(9g_s)$	C_φ	$32\pi^3/3$	$C_{e\varphi}$	$32\pi^2/\sqrt{3}$	$C_{\ell\ell}$	2π	$C_{\ell edq}$	$8\pi/\sqrt{3}$
$C_{\widetilde{G}}$	$4\pi/(9g_s)$	$C_{\varphi\Box}$	$8\pi/3$	$C_{u\varphi}$	$32\pi^2/3$	$C_{qq}^{(1)}$	$6\pi/5$	$C_{quqd}^{(1)}$	$16\pi(1+\sqrt{2})/9$
C_W	$2\pi/(3g)$	$C_{\varphi D}$	$32\pi/3$	$C_{d\varphi}$	$32\pi^2/3$	$C_{qq}^{(3)}$	$4\pi/3$	$C_{quqd}^{(8)}$	$4\pi(2+7\sqrt{2})/3$
$C_{\widetilde{W}}$	$2\pi/(3g)$					$C_{\ell q}^{(1)}$	$2\pi\sqrt{3}$	$C_{\ell equ}^{(1)}$	$8\pi/\sqrt{3}$
						$C_{\ell q}^{(3)}$	2π	$C_{\ell equ}^{(3)}$	$(2+\sqrt{2})\pi/3$

$\boldsymbol{X^2\varphi^2}$		$\boldsymbol{\psi^2 X\varphi}$		$\boldsymbol{\psi^2\varphi^2 D}$		$\boldsymbol{(\bar{R}R)(\bar{R}R)}$		$\boldsymbol{(\bar{L}L)(\bar{R}R)}$	
$C_{\varphi G}$	π	C_{uG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(1)}$	8π	C_{ee}	2π	$C_{\ell e}$	4π
$C_{\varphi\widetilde{G}}$	π	C_{dG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(3)}$	$8\pi/3$	C_{uu}	$3\pi/2$	$C_{\ell u}$	4π
$C_{\varphi W}$	$2\pi\sqrt{2/3}$	C_{eW}	$4\pi\sqrt{2/3}$	$C_{\varphi e}$	8π	C_{dd}	$3\pi/2$	$C_{\ell d}$	4π
$C_{\varphi\widetilde{W}}$	$2\pi\sqrt{2/3}$	C_{uW}	$4\pi\sqrt{2/3}$	$C_{\varphi q}^{(1)}$	$2\pi\sqrt{6}$	C_{eu}	4π	C_{qe}	4π
$C_{\varphi B}$	$2\pi\sqrt{2}$	C_{uB}	4π	$C_{\varphi q}^{(3)}$	$8\pi/3$	C_{ed}	4π	$C_{qu}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{B}}$	$2\pi\sqrt{2}$	C_{eB}	4π	$C_{\varphi u}$	$4\pi\sqrt{3}$	$C_{ud}^{(1)}$	4π	$C_{qu}^{(8)}$	$3\pi(1+1/\sqrt{2})$
$C_{\varphi WB}$	4π	C_{dW}	$4\pi\sqrt{2/3}$	$C_{\varphi d}$	$4\pi\sqrt{3}$	$C_{ud}^{(8)}$	8π	$C_{qd}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{W}B}$	4π	C_{dB}	4π	$C_{\varphi ud}$	8π			$C_{qd}^{(8)}$	$3\pi(1+1/\sqrt{2})$

SMEFT Marginalized Bounds

[LB, Paradisi, Sainaghi ’26]

Verified **Improved**

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$		$(\bar{L}L)(\bar{L}L)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
C_G	$4\pi/(9g_s)$	C_φ	$32\pi^3/3$	$C_{e\varphi}$	$32\pi^2/\sqrt{3}$	$C_{\ell\ell}$	2π	$C_{\ell edq}$	$8\pi/\sqrt{3}$
$C_{\widetilde{G}}$	$4\pi/(9g_s)$	$C_{\varphi\Box}$	$8\pi/3$	$C_{u\varphi}$	$32\pi^2/3$	$C_{qq}^{(1)}$	$6\pi/5$	$C_{quqd}^{(1)}$	$16\pi(1+\sqrt{2})/9$
C_W	$2\pi/(3g)$	$C_{\varphi D}$	$32\pi/3$	$C_{d\varphi}$	$32\pi^2/3$	$C_{qq}^{(3)}$	$4\pi/3$	$C_{quqd}^{(8)}$	$4\pi(2+7\sqrt{2})/3$
$C_{\widetilde{W}}$	$2\pi/(3g)$					$C_{\ell q}^{(1)}$	$2\pi\sqrt{3}$	$C_{\ell equ}^{(1)}$	$8\pi/\sqrt{3}$
						$C_{\ell q}^{(3)}$	2π	$C_{\ell equ}^{(3)}$	$(2+\sqrt{2})\pi/3$

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$C_{\varphi G}$	π	C_{uG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(1)}$	8π	C_{ee}	2π	$C_{\ell e}$	4π
$C_{\varphi\widetilde{G}}$	π	C_{dG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(3)}$	$8\pi/3$	C_{uu}	$3\pi/2$	$C_{\ell u}$	4π
$C_{\varphi W}$	$2\pi\sqrt{2/3}$	C_{eW}	$4\pi\sqrt{2/3}$	$C_{\varphi e}$	8π	C_{dd}	$3\pi/2$	$C_{\ell d}$	4π
$C_{\varphi\widetilde{W}}$	$2\pi\sqrt{2/3}$	C_{uW}	$4\pi\sqrt{2/3}$	$C_{\varphi q}^{(1)}$	$2\pi\sqrt{6}$	C_{eu}	4π	C_{qe}	4π
$C_{\varphi B}$	$2\pi\sqrt{2}$	C_{uB}	4π	$C_{\varphi q}^{(3)}$	$8\pi/3$	C_{ed}	4π	$C_{qu}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{B}}$	$2\pi\sqrt{2}$	C_{eB}	4π	$C_{\varphi u}$	$4\pi\sqrt{3}$	$C_{ud}^{(1)}$	4π	$C_{qu}^{(8)}$	$3\pi(1+1/\sqrt{2})$
$C_{\varphi WB}$	4π	C_{dW}	$4\pi\sqrt{2/3}$	$C_{\varphi d}$	$4\pi\sqrt{3}$	$C_{ud}^{(8)}$	8π	$C_{qd}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{W}B}$	4π	C_{dB}	4π	$C_{\varphi ud}$	8π			$C_{qd}^{(8)}$	$3\pi(1+1/\sqrt{2})$

SMEFT Marginalized Bounds

[LB, Paradisi, Sainaghi '26]

Verified **Improved** **New**

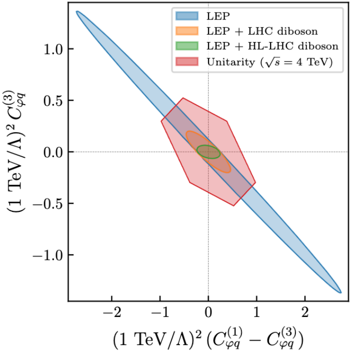
X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$		$(\bar{L}L)(\bar{L}L)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
C_G	$4\pi/(9g_s)$	C_φ	$32\pi^3/3$	$C_{e\varphi}$	$32\pi^2/\sqrt{3}$	$C_{\ell\ell}$	2π	$C_{\ell edq}$	$8\pi/\sqrt{3}$
$C_{\widetilde{G}}$	$4\pi/(9g_s)$	$C_{\varphi\Box}$	$8\pi/3$	$C_{u\varphi}$	$32\pi^2/3$	$C_{qq}^{(1)}$	$6\pi/5$	$C_{quqd}^{(1)}$	$16\pi(1+\sqrt{2})/9$
C_W	$2\pi/(3g)$	$C_{\varphi D}$	$32\pi/3$	$C_{d\varphi}$	$32\pi^2/3$	$C_{qq}^{(3)}$	$4\pi/3$	$C_{quqd}^{(8)}$	$4\pi(2+7\sqrt{2})/3$
$C_{\widetilde{W}}$	$2\pi/(3g)$					$C_{\ell q}^{(1)}$	$2\pi\sqrt{3}$	$C_{\ell equ}^{(1)}$	$8\pi/\sqrt{3}$
						$C_{\ell q}^{(3)}$	2π	$C_{\ell equ}^{(3)}$	$(2+\sqrt{2})\pi/3$

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$C_{\varphi G}$	π	C_{uG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(1)}$	8π	C_{ee}	2π	$C_{\ell e}$	4π
$C_{\varphi\widetilde{G}}$	π	C_{dG}	$2\pi\sqrt{3}$	$C_{\varphi\ell}^{(3)}$	$8\pi/3$	C_{uu}	$3\pi/2$	$C_{\ell u}$	4π
$C_{\varphi W}$	$2\pi\sqrt{2/3}$	C_{eW}	$4\pi\sqrt{2/3}$	$C_{\varphi e}$	8π	C_{dd}	$3\pi/2$	$C_{\ell d}$	4π
$C_{\varphi\widetilde{W}}$	$2\pi\sqrt{2/3}$	C_{uW}	$4\pi\sqrt{2/3}$	$C_{\varphi q}^{(1)}$	$2\pi\sqrt{6}$	C_{eu}	4π	C_{qe}	4π
$C_{\varphi B}$	$2\pi\sqrt{2}$	C_{uB}	4π	$C_{\varphi q}^{(3)}$	$8\pi/3$	C_{ed}	4π	$C_{qu}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{B}}$	$2\pi\sqrt{2}$	C_{eB}	4π	$C_{\varphi u}$	$4\pi\sqrt{3}$	$C_{ud}^{(1)}$	4π	$C_{qu}^{(8)}$	$3\pi(1+1/\sqrt{2})$
$C_{\varphi WB}$	4π	C_{dW}	$4\pi\sqrt{2/3}$	$C_{\varphi d}$	$4\pi\sqrt{3}$	$C_{ud}^{(8)}$	8π	$C_{qd}^{(1)}$	$2\pi\sqrt{2}$
$C_{\varphi\widetilde{W}B}$	4π	C_{dB}	4π	$C_{\varphi ud}$	8π			$C_{qd}^{(8)}$	$3\pi(1+1/\sqrt{2})$

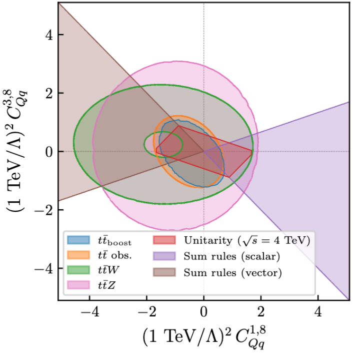
SMEFT Phenomenology

[LB, Paradisi, Sainaghi '26]

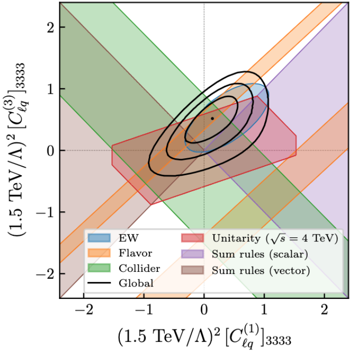
[Celada et al. '24]



[Brivio et al. '19]



[Allwicher, Cornella, Isidori, Stefanek '23]



1. Diboson production: $O_{\varphi q}^{(1)} = \sum_{p=1,2} i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_p)$ *vs.* $O_{\varphi q}^{(3)} = \sum_{p=1,2} i(\varphi^\dagger \overleftrightarrow{D}^I_\mu \varphi)(\bar{q}_p \sigma^I \gamma^\mu q_p)$
2. Top physics: $O_{Qq}^{1,8} = \sum_{p=1,2} (\bar{q}_3 \gamma_\mu T^A q_3)(\bar{q}_p \gamma^\mu T^A q_p)$ *vs.* $O_{Qq}^{3,8} = \sum_{p=1,2} (\bar{q}_3 \gamma_\mu T^A \sigma^I q_3)(\bar{q}_p \gamma^\mu T^A \sigma^I q_p)$
3. $U(2)^5$ flavor symmetric scenario: $[O_{\ell q}^{(1)}]_{3333} = (\bar{\ell}_3 \gamma^\mu \ell_3)(\bar{q}_3 \gamma_\mu q_3)$ *vs.* $[O_{\ell q}^{(3)}]_{3333} = (\bar{\ell}_3 \gamma^\mu \sigma^I \ell_3)(\bar{q}_3 \gamma_\mu \sigma^I q_3)$

Axion-Like Particle EFT

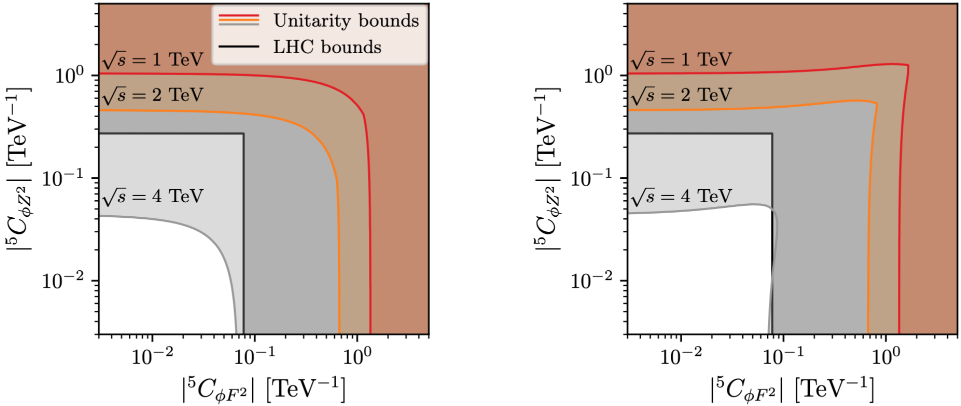
[LB, Levati, Paradisi ’25]

Non-resonant ALP searches at LHC

[Gavela, No, Sanz, Trocóniz ’19]

$$\mathcal{L}_\phi^{(5)} \supset {}^5C_{\phi F^2}\, \phi\, F_{\mu\nu} \widetilde{F}^{\mu\nu} + {}^5C_{\phi Z^2}\, \phi\, Z_{\mu\nu} \widetilde{Z}^{\mu\nu}$$

${}^5C_{\phi F^2}\, {}^5C_{\phi Z^2} < 0$ ${}^5C_{\phi F^2}\, {}^5C_{\phi Z^2} > 0$



Weak-Violating Axion-Like Particle

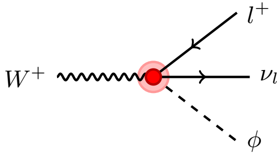
[LB, Levati, Paradisi '25]

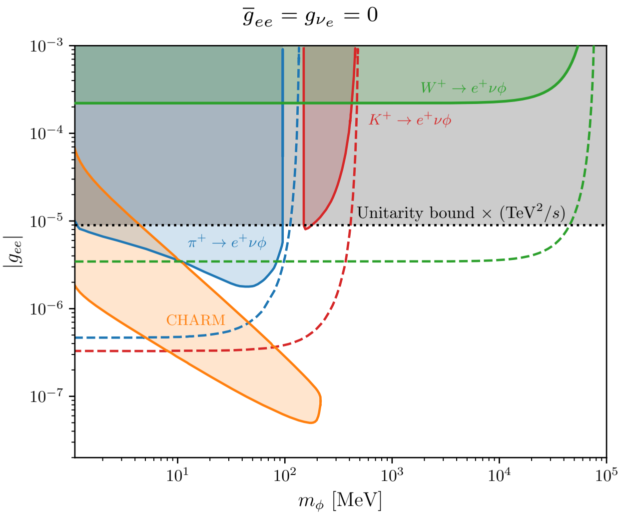
[Altmannshofer, Dror, Gori '22]

$$\mathcal{L} = \frac{\partial_\mu \phi}{2m_l}\left(\overline{g}_{ll}\,\bar{l}\gamma^\mu l + g_{ll}\,\bar{l}\gamma^\mu\gamma_5 l + g_{\nu_l}\,\overline{\nu}_l\gamma^\mu P_L\nu_l\right)$$

$$\Downarrow$$

$$\mathcal{L} \supset \frac{-ig}{2\sqrt{2}m_l}(g_{ll} - \overline{g}_{ll} + g_{\nu_l})(\bar{l}\gamma^\mu P_L\nu_l)W^-_\mu\,\phi + \text{H.c.}$$





Conclusions & Outlook

- On-shell methods unify the study of EFT *running* and *validity*: two faces of the same amplitude-based toolkit

Part 1: Running

- Leading mass effects & general operator mixing framework
- Full ALP EFT renormalization
- One-loop RGEs of the general gauge EFT: 184 anomalous dimensions, ready for any model

Part 2: Validity

- Vectorial formalism for *arbitrary* $N \to M$ partial-wave bounds
- Photon/gravity/higher-spin EFTs
- Full dim-6 SMEFT unitarity bounds
- Full ALP EFT unitarity bounds up to dim-8

Outlook

Future experiments demand *theory-consistent* EFT interpretations

Flavor structure amplifies theoretical bounds · *Theory priors* sharpen statistical inference

Goal: connect first principles with *collider*, *flavor*, and *DM* phenomenology

Thank you for your attention!

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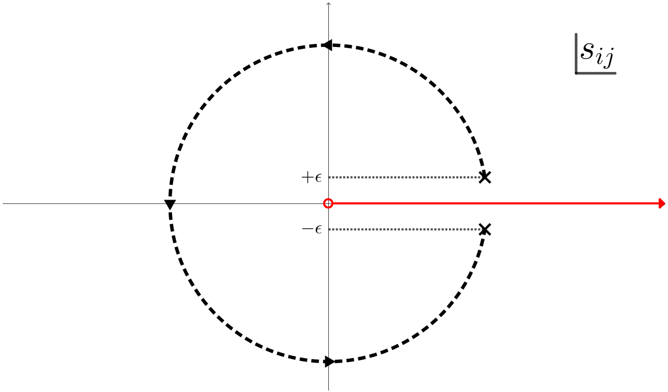
Dilatation Operator & Complex Rotations



The **Dilatation Operator**

$$D = \sum_i p_i \cdot \frac{\partial}{\partial p_i}$$

generates the **Complex Rotations**:

$$p_i \to e^{i\alpha} p_i \implies F_{\mathcal{O}} \to e^{i\alpha D} F_{\mathcal{O}} \,.$$

For $\boldsymbol{\alpha = \pi}$ their infinitesimal imaginary part ϵ changes sign:

$$F_{\mathcal{O}}(\{s_{ij} - i\epsilon\}) = e^{i\pi D} F_{\mathcal{O}}(\{s_{ij} + i\epsilon\}) \,.$$

IR Anomalous Dimensions

- In theories with **massless fields**, IR singularities originate from configurations where loop momenta become **soft** or **collinear**
- The **IR anomalous dimension** only depends on the external state $\langle \vec{n}|$ [Becher, Neubert '09]

$$\gamma_{\rm IR}^{(1)}(\{s_{ij}\};\mu) = \frac{g^2}{4\pi^2}\sum_{i<j} T_i^a T_j^a \log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma_{i,\rm coll.}^{(1)}$$

- Since the **stress-energy tensor $T_{\mu\nu}$** is **UV protected**, $\gamma_{\rm IR}$ can be computed as

$$\gamma_{\rm IR}^{(1)} F_{T_{\mu\nu}}^{(0)}(\vec{n}) = \frac{1}{\pi}(\mathcal{M}F_{T_{\mu\nu}})^{(1)}(\vec{n})$$

Physical Operator Classification

- Physical operators are identified through the **Amplitude-Operator Correspondence** [Li, Ren, Xiao, Yu, Zheng '22]
- Little group and dimensional analysis enough to construct the physical operators:

$$[\mathcal{M}_n] = 4 - n$$

- Contact amplitudes (from local operators $\mathcal{O}_i$) are polynomials in inner products:

$$\mathcal{M} = c_i \times \mathcal{K}_i(\{\langle ij\rangle, [ij]\}) \qquad [\mathcal{K}_i] = [\mathcal{O}_i] - \ell(\mathcal{O}_i) \geq 0$$

- Redundancies:

$$\sum_{i=1}^{n} p_i = 0$$

$$\langle ij\rangle\langle k\ell\rangle + \langle ki\rangle\langle j\ell\rangle + \langle jk\rangle\langle i\ell\rangle = 0$$

$$[ij][k\ell] + [ki][j\ell] + [jk][i\ell] = 0$$

Dimension-5 Operators

Classification of independent kinematics: $[\mathcal{K}_i] = 5 - \ell_i$

$$\ell_i = 5 \quad \Longrightarrow \quad \mathcal{K}_i = \left\{ 1 \right. \quad \Longrightarrow \quad \mathcal{O}_i = \left\{ \phi_1\, \phi_2\, \phi_3\, \phi_4\, \phi_5 \,, \right.$$

$$\ell_i = 4 \quad \Longrightarrow \quad \mathcal{K}_i = \left\{ \langle 12 \rangle \right. \quad \Longrightarrow \quad \mathcal{O}_i = \left\{ \psi_{1L}\, \psi_{2L}\, \phi_3\, \phi_4 \,, \right.$$

$$\ell_i = 3 \quad \Longrightarrow \quad \mathcal{K}_i = \begin{cases} \langle 12 \rangle^2 \\ \langle 12 \rangle \langle 13 \rangle \end{cases} \quad \Longrightarrow \quad \mathcal{O}_i = \begin{cases} F_{1L}\, F_{2L}\, \phi_3 \,, \\ F_{1L}\, \psi_{2L}\, \psi_{3L} \,. \end{cases}$$

Group-dressed Lagrangian:

$$\mathcal{L}^{(5)}_{\text{bos}} = \sum_{\alpha=1}^{N_G} \sum_{\beta=1}^{\alpha} \left[\left[C_{\phi F^2} \right]_a^{A_\alpha B_\beta} \phi_a F^{A_\alpha}_{\mu\nu} F^{B_\beta\, \mu\nu} + \left[C_{\phi \widetilde{F}^2} \right]_a^{A_\alpha B_\beta} \phi_a F^{A_\alpha}_{\mu\nu} \widetilde{F}^{B_\beta\, \mu\nu} \right] + \left[C_{\phi^5} \right]_{abcde} \phi_a \phi_b \phi_c \phi_d \phi_e$$

$$\mathcal{L}^{(5)}_{\text{fer}} = \left[C_{\psi^2 \phi^2} \right]_{ijab} \psi_i \psi_j \phi_a \phi_b + \sum_{\alpha=1}^{N_G} \left[C_{\psi^2 F} \right]_{ij}^{A_\alpha} \psi_i \sigma^{\mu\nu} \psi_j F^{A_\alpha}_{\mu\nu} + \text{H.c.}$$

Dimension-6 Operators

Classification of independent kinematics: $[\mathcal{K}_i] = 6 - \ell_i$

$$\ell_i = 6 \quad \Longrightarrow \quad \mathcal{K}_i = \Big\{ 1 \quad \Longrightarrow \quad \mathcal{O}_i = \Big\{ \phi_1\, \phi_2\, \phi_3\, \phi_4\, \phi_5\, \phi_6\,,$$

$$\ell_i = 5 \quad \Longrightarrow \quad \mathcal{K}_i = \Big\{ \langle 12 \rangle \quad \Longrightarrow \quad \mathcal{O}_i = \Big\{ \psi_{1L}\, \psi_{2L}\, \phi_3\, \phi_4\, \phi_5\,,$$

$$\ell_i = 4 \quad \Longrightarrow \quad \mathcal{K}_i = \begin{cases} \langle 12 \rangle^2 \\ \langle 12 \rangle \langle 13 \rangle \\ \langle 12 \rangle \langle 34 \rangle \\ \langle 12 \rangle [34] \\ \langle 12 \rangle [23] \\ \langle 12 \rangle [12] \end{cases} \quad \Longrightarrow \quad \mathcal{O}_i = \begin{cases} F_{1L}\, F_{2L}\, \phi_3\, \phi_4\,, \\ F_{1L}\, \psi_{2L}\, \psi_{3L}\, \phi_4\,, \\ \psi_{1L}\, \psi_{2L}\, \psi_{3L}\, \psi_{4L}\,, \\ \psi_{1L}\, \psi_{2L}\, \psi_{3R}\, \psi_{4R}\,, \\ \psi_{1L}\, D\phi_2\, \psi_{3R}\, \phi_4\,, \\ D\phi_1\, D\phi_2\, \phi_3\, \phi_4\,. \end{cases}$$

$$\ell_i = 3 \quad \Longrightarrow \quad \mathcal{K}_i = \Big\{ \langle 12 \rangle \langle 23 \rangle \langle 13 \rangle \quad \Longrightarrow \quad \mathcal{O}_i = \Big\{ F_{1L}\, F_{2L}\, F_{3L}\,.$$