Imaging Boosted Boson Jets with Energy Correlators

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with Kyle Lee (Argonne), Xiaoyuan Zhang (MIT)
arXiv: 2601.20933 + ongoing work

Theory Seminar, SLAC

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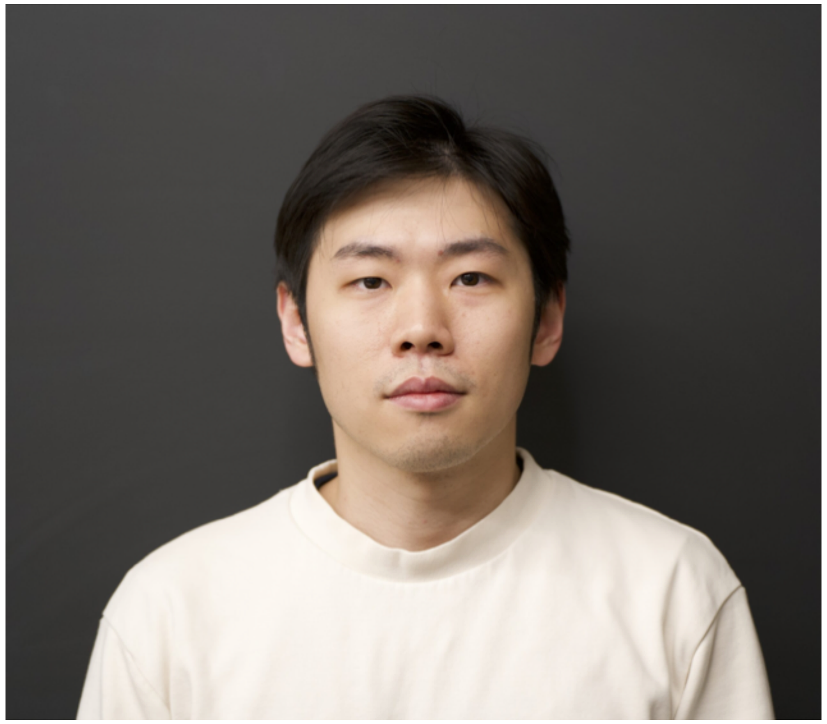
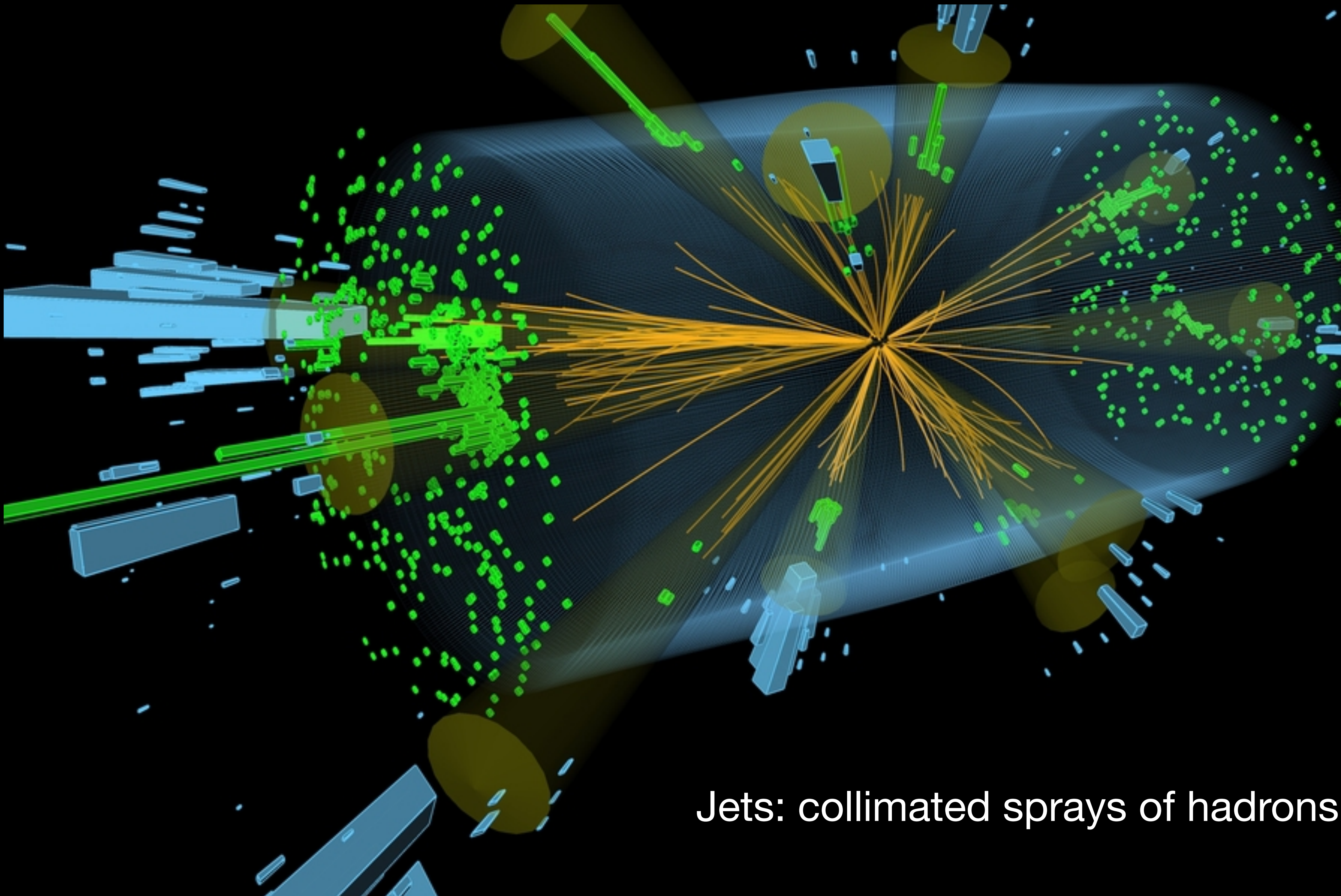
SLAC NATIONAL ACCELERATOR LABORATORY



Kyle Lee (Argonne)

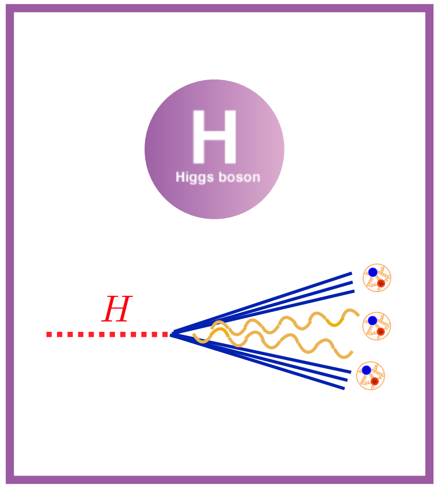
Xiaoyuan Zhang (MIT)



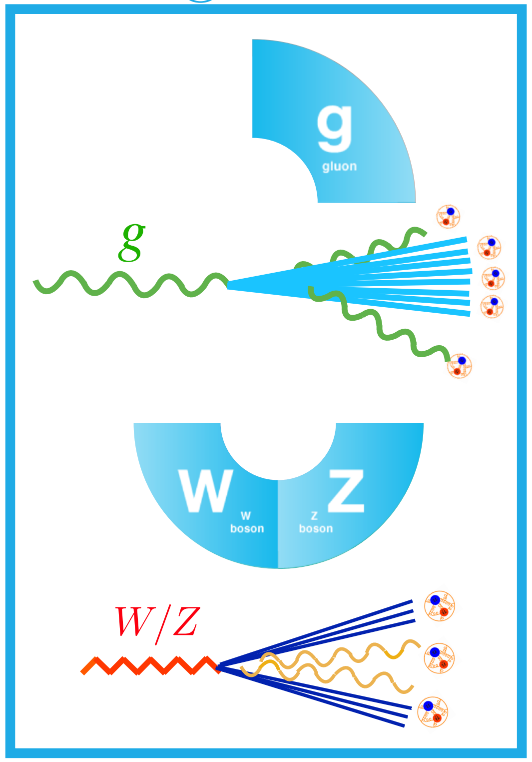
Jets: collimated sprays of hadrons

Introduction

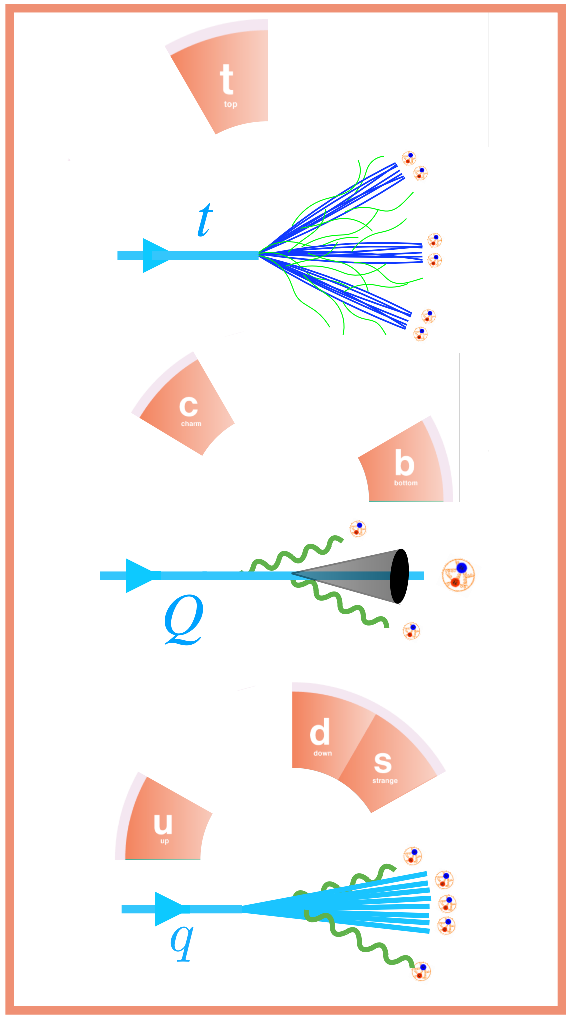
Higgs

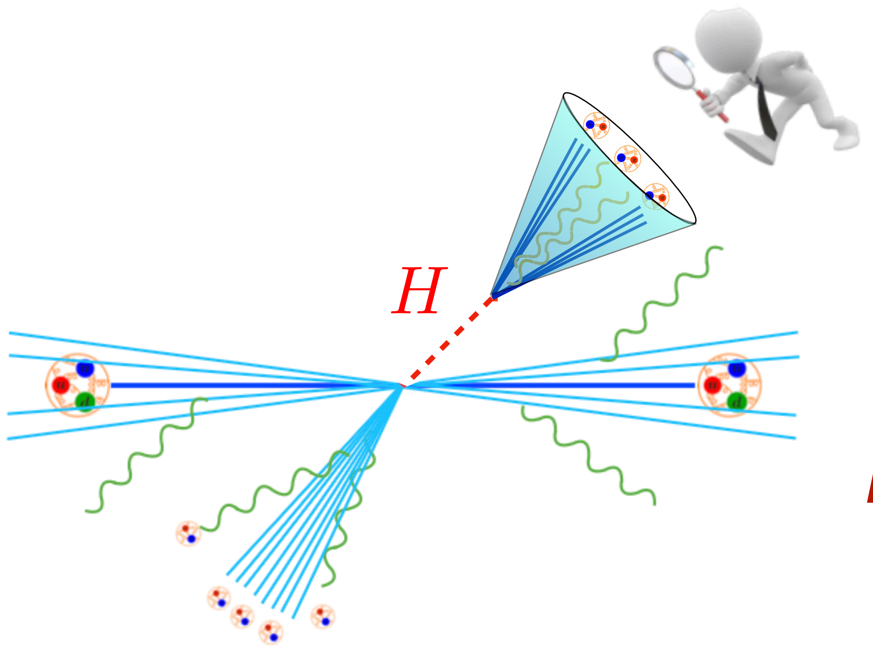


Gauge Bosons



Quarks





How to distinguish different particle-initiated jets?

Look into jet substructure!

Introduction

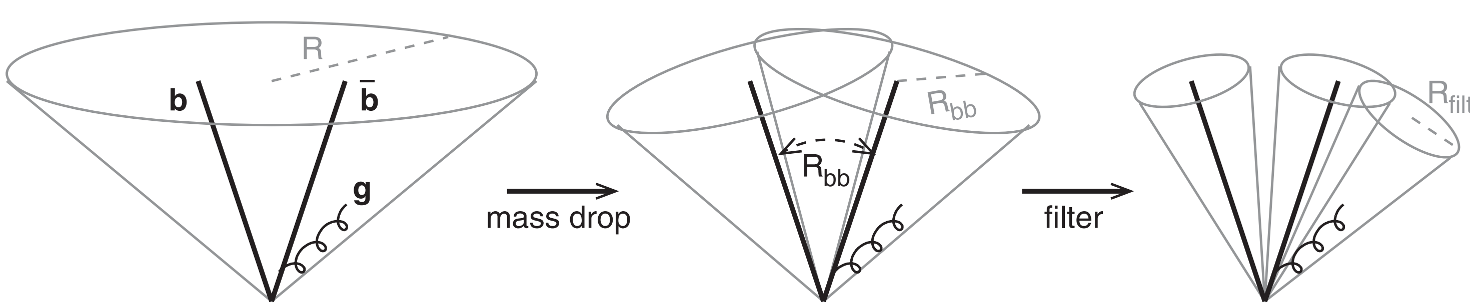
Jet Substructure as a New Higgs-Search Channel at the Large Hadron Collider

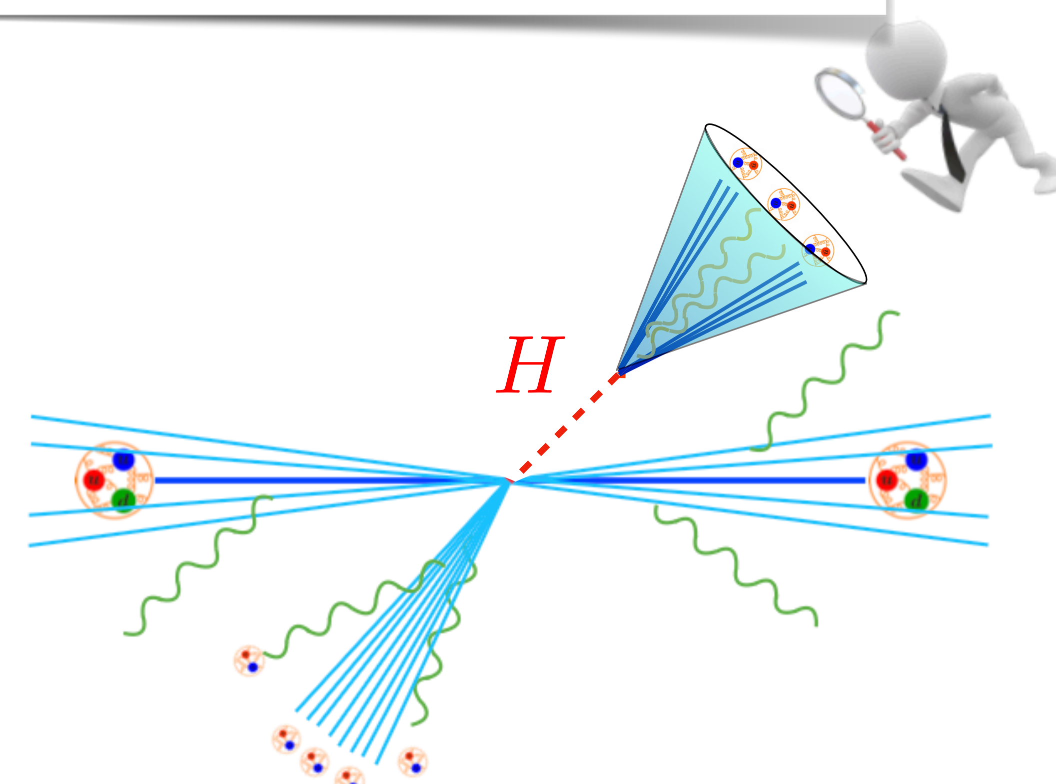
Jonathan M. Butterworth and Adam R. Davison
Department of Physics & Astronomy, University College London, United Kingdom

Mathieu Rubin and Gavin P. Salam
LPTHE; UPMC Univ. Paris 6; Univ. Denis Diderot; CNRS UMR 7589; Paris, France
(Received 2 March 2008; published 18 June 2008)

It is widely considered that, for Higgs boson searches at the CERN Large Hadron Colider, WH and ZH production where the Higgs boson decays to $b\bar{b}$ are poor search channels due to large backgrounds. We show that at high transverse momenta, employing state-of-the-art jet reconstruction and decomposition techniques, these processes can be recovered as promising search channels for the standard model Higgs boson around 120 GeV in mass.

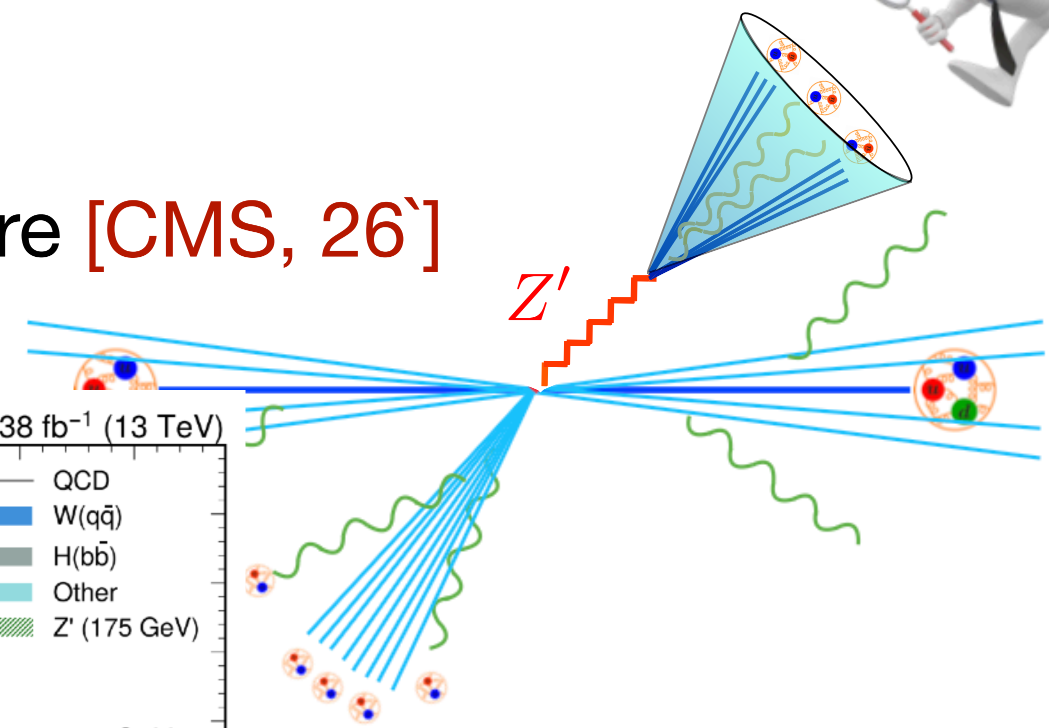
DOI: 10.1103/PhysRevLett.100.242001 PACS numbers: 13.87.Ce, 13.87.Fh



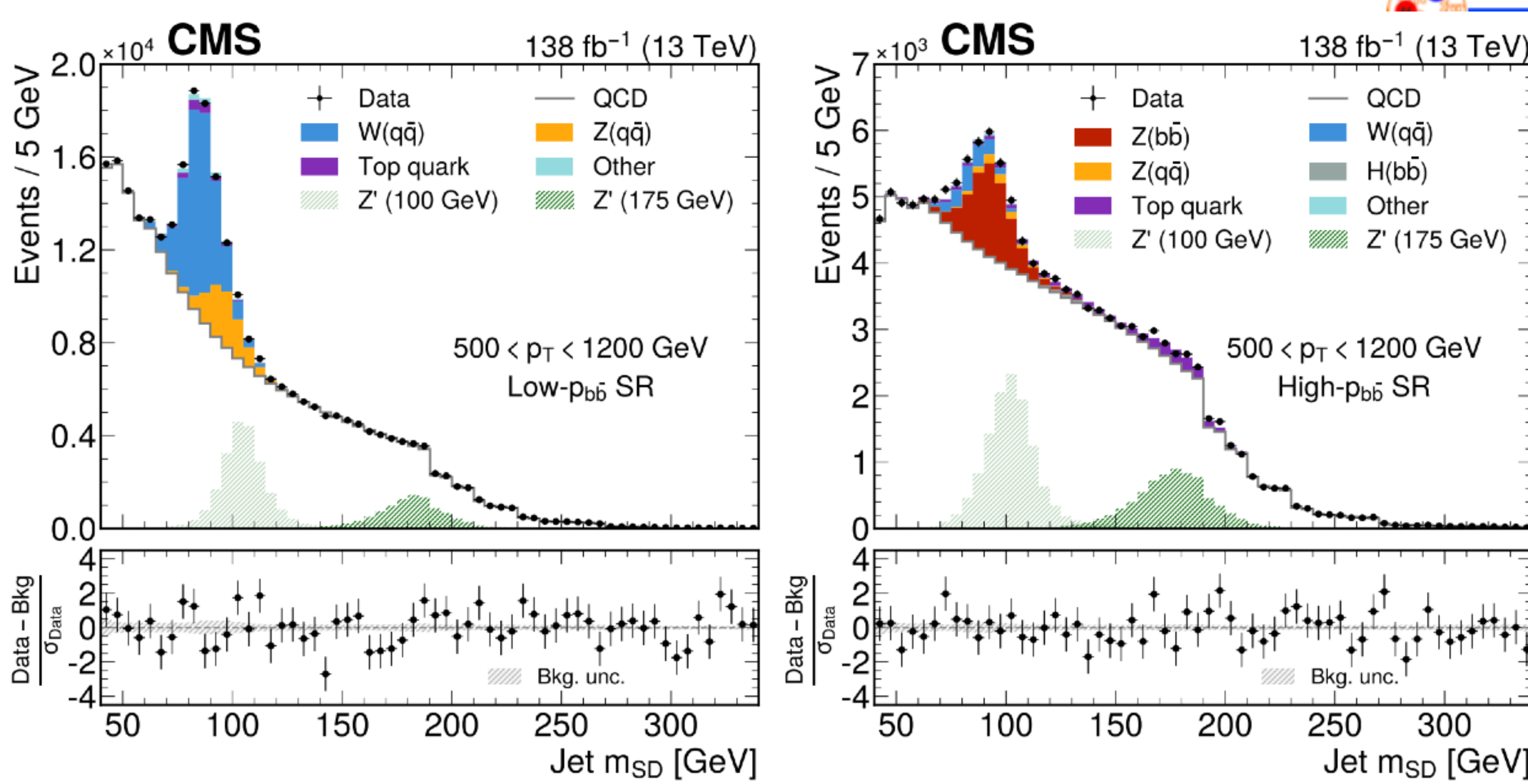


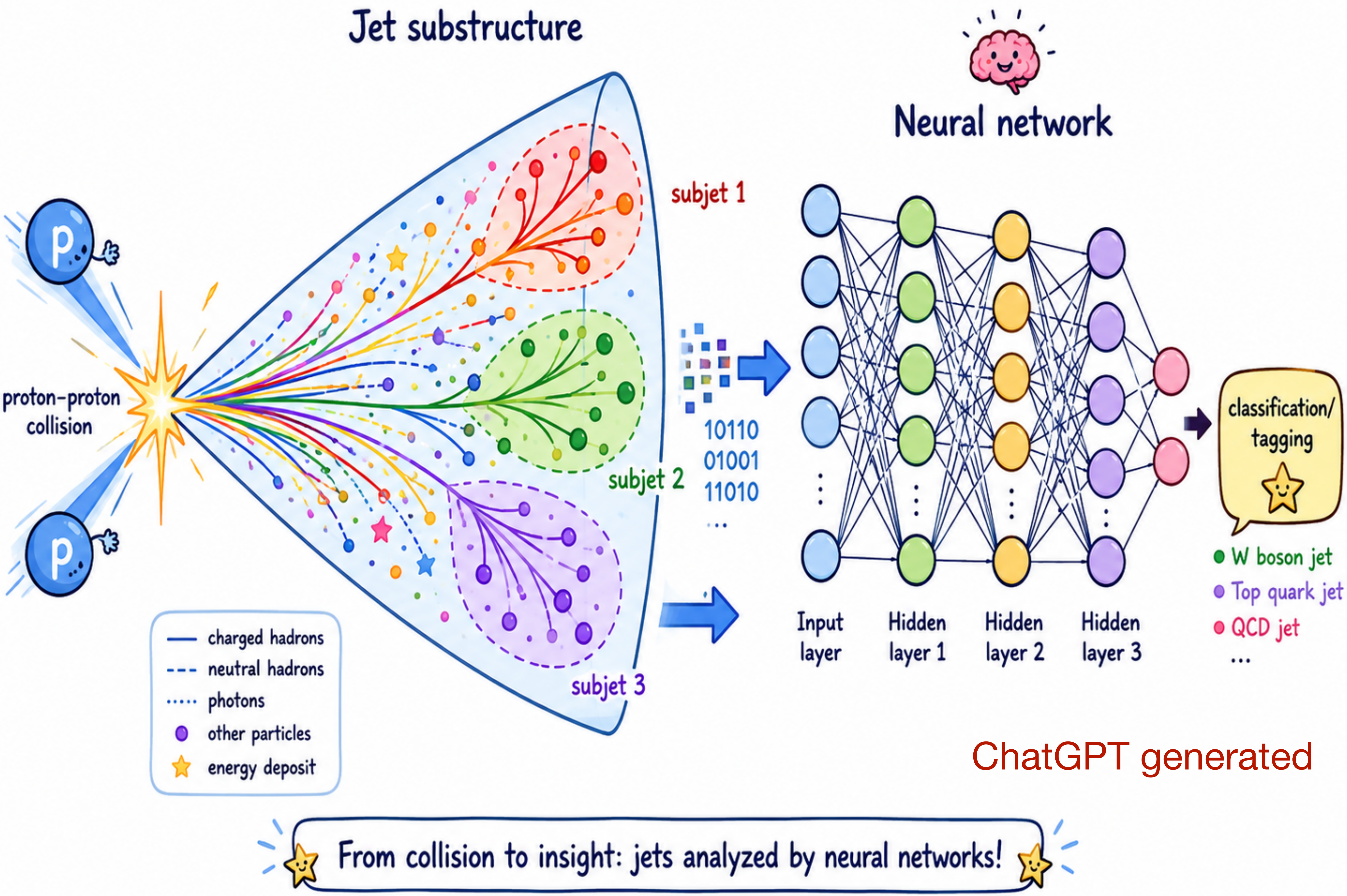
Jet Substructure

SM measurement, Search for BSM, anomaly detection



- Limits for light Z' through jet substructure [CMS, 26`]





ChatGPT generated

Can we do precision physics inside jets?

How to compute jet substructure through first principle QCD?

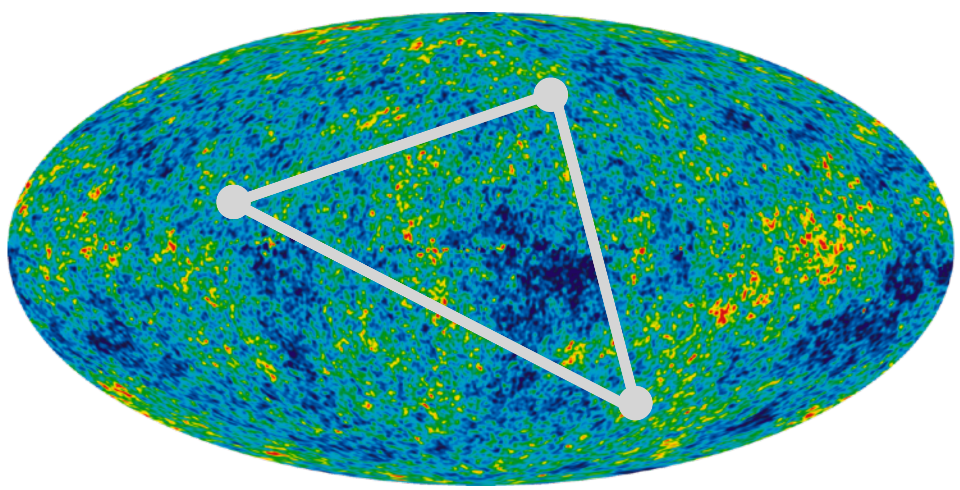
Can we do precision physics inside jets?

How to compute jet substructure through first principle QCD?

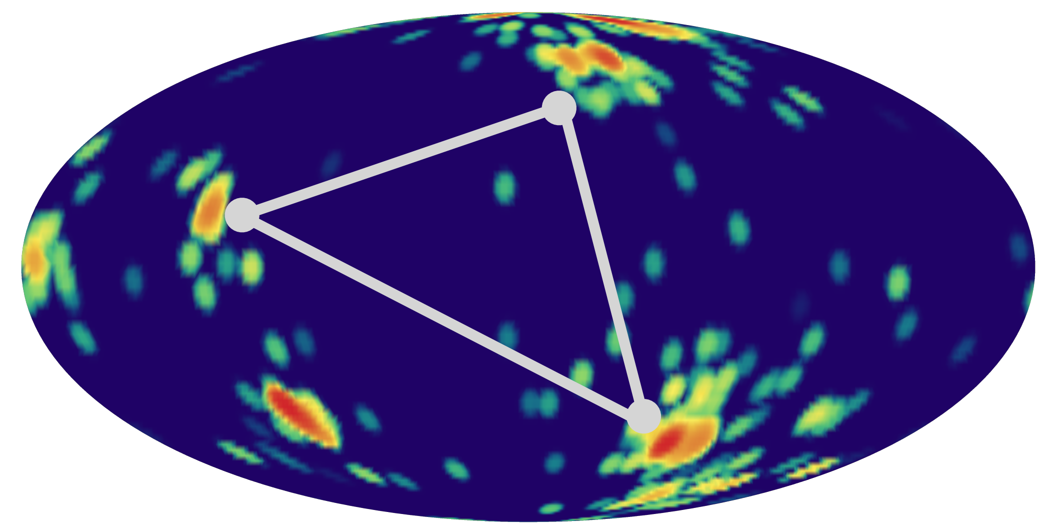
Use energy correlators!

CMB v.s. Collider

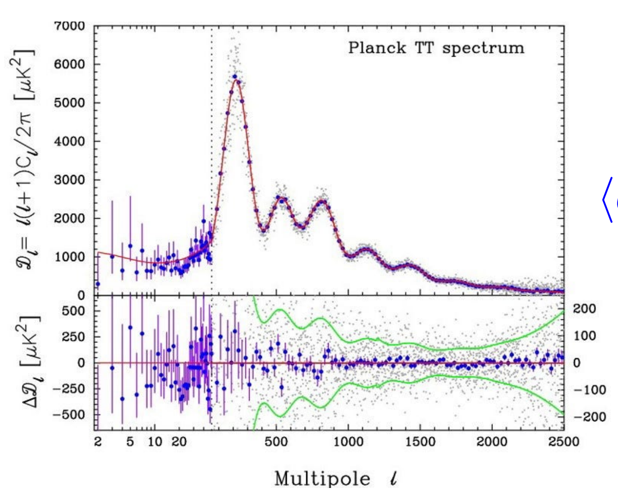
Correlation function



CMB



One event at a lepton collider
(generated with Pythia)



$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \cdots \zeta_{\vec{k}_n} \rangle$

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\hat{n})$$

$$\mathcal{E}(\hat{n})|X\rangle = \sum_a E_a \delta^{(2)}\left(\Omega_{\vec{p}_a} - \Omega_{\hat{n}}\right)|X\rangle$$

$$\langle \mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_N)\rangle \equiv \langle \Psi|\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_N)|\Psi\rangle$$

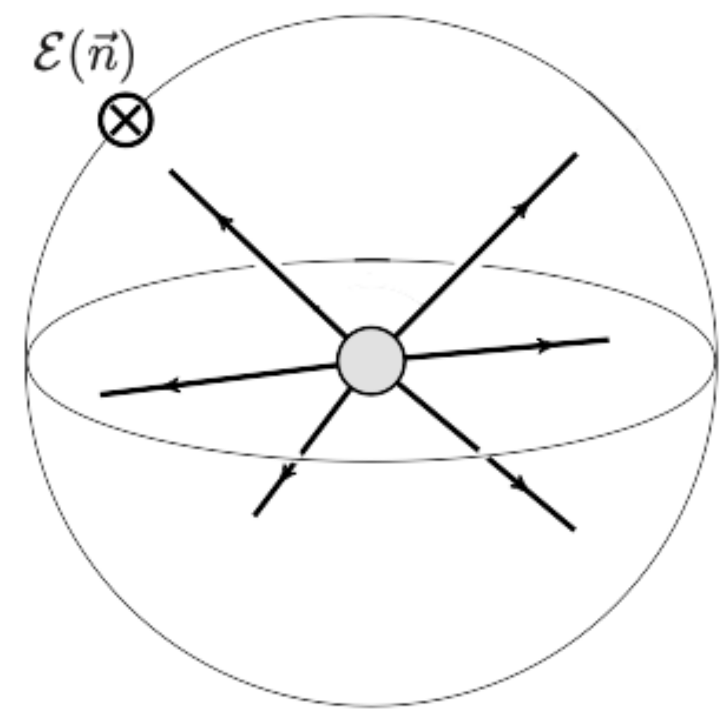
Introduction to Energy Correlators

- Energy flux & One point energy correlator [Sterman, `75]

states. To make this idea more quantitative we define for any state a, an "angular energy current" in the e^+e^- CM frame:

$$j_a(\Omega) = \sum_{i=1}^{n_a} \eta_i \delta(\Omega - \omega_i) \qquad (1)$$

where the sum is over the n_a massless particles in a, with energies $\{\eta_i\}$ and momentum directions $\{\omega_i\}$ (ω_i stands for angles θ_i and φ_i). Jet-

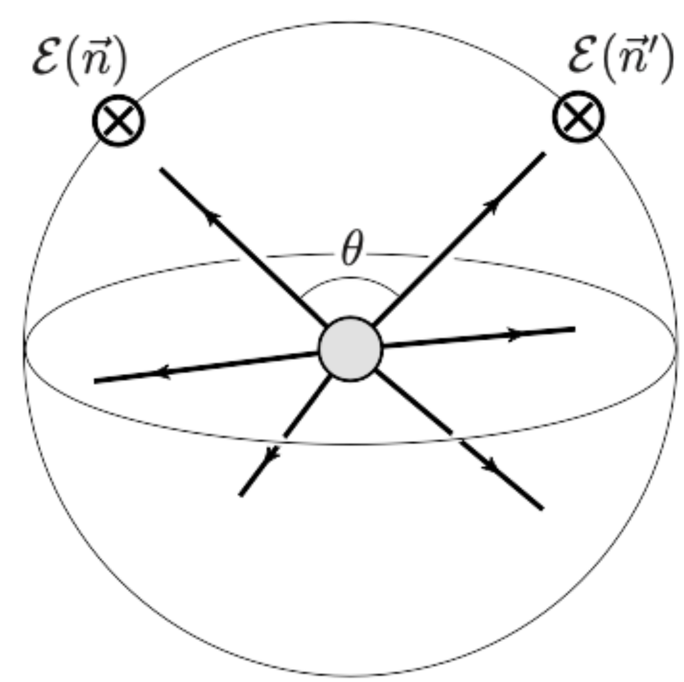


- The Energy-Energy Correlator (EEC)

[Basham, Brown, Ellis, Love, `78]

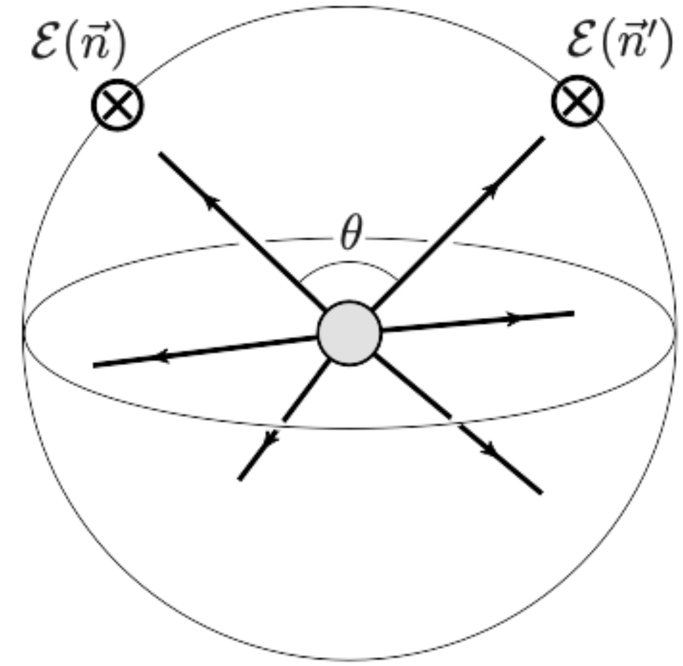
$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle$$

$$\frac{d\sigma}{d\cos\theta} = \sum_X \int d\sigma_{e^+e^-\to X} \sum_{a,b\in X} \frac{E_a E_b}{Q^2} \delta(\cos(\theta_{ab}) - \cos(\theta))$$



Introduction to Energy Correlators

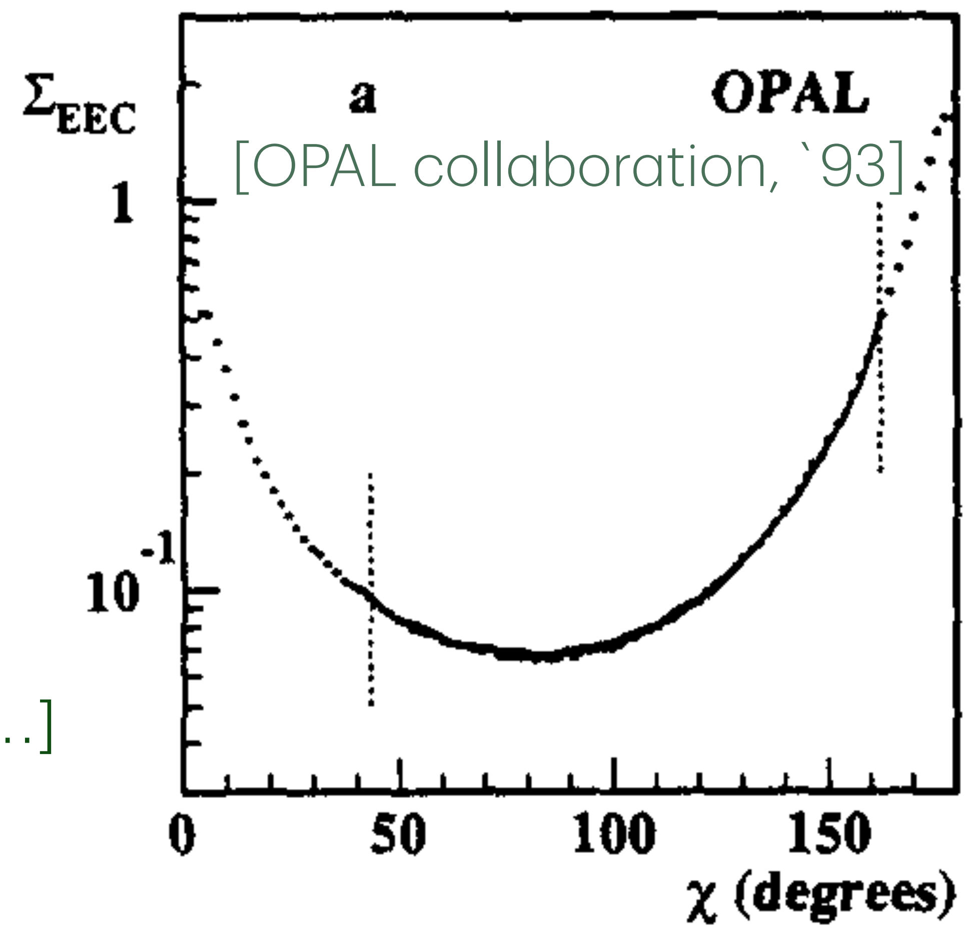
$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle$$



- The Energy-Energy Correlator (EEC)

[Basham, Brown, Ellis, Love, `78]

$$\frac{d\sigma}{d\cos\chi} = \sum_X \int d\sigma_{e^+e^-\to X} \sum_{a,b\in X} \frac{E_a E_b}{Q^2}\delta(\cos(\theta_{ab}) - \cos(\chi))$$



Collinear limit

[Dixon, Moult, Zhu, `19; …]

Back-to-back limit

[Moult, Zhu, `18; …]

N^4LL [Duhr, Mistlberger, Vita, `22]

Fixed order

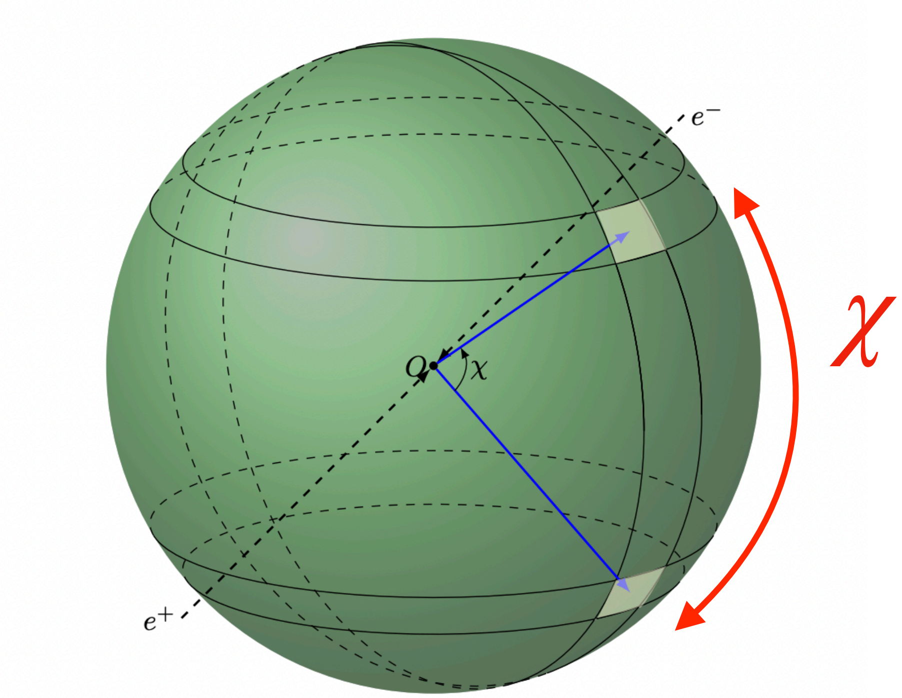
Analytic NLO [Dixon, Luo, Shtabovenko, Yang, Zhu, `18]

Numerical NNLO [Del Duca, Duhr, Kardos, Somogyi, Trócsányi, `16]

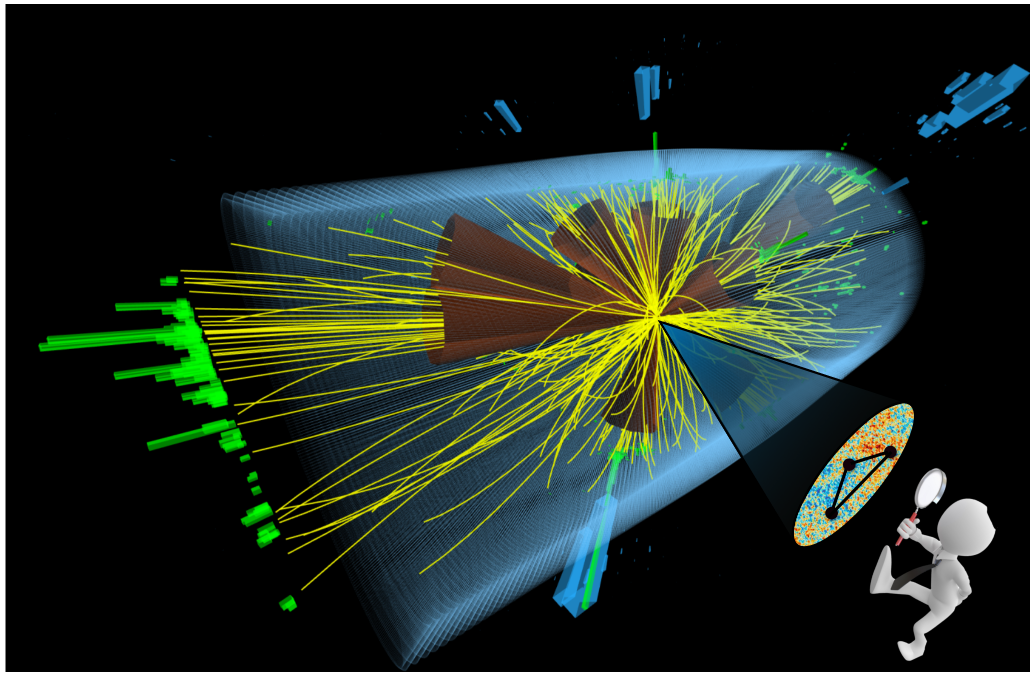
Introduction to Energy Correlators

- The Energy-Energy Correlator (EEC) [Basham, Brown, Ellis, Love, `78]

$$\frac{d\sigma}{d\cos\chi} = \sum_X \int d\sigma_{e^+e^-\to X} \sum_{a,b\in X} \frac{E_a E_b}{Q^2} \delta(\cos(\theta_{ab}) - \cos(\chi))$$



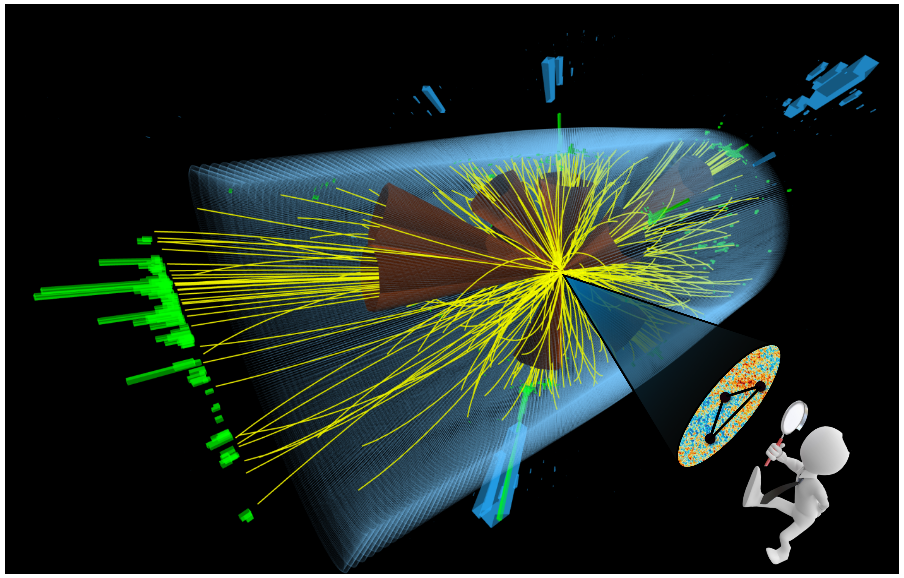
- *Extensions*: transverse EEC, multi-point energy correlators, different processes, ...
- *Exciting for theory:* perturbative calculations, non-perturbative effects, jet substructure, connection to CFT, gravity...
- *Probe for phenomena*: SM parameters like α_s and masses, perturbative QCD dynamics, confinement, heavy-ion physics, nuclear physics, ...

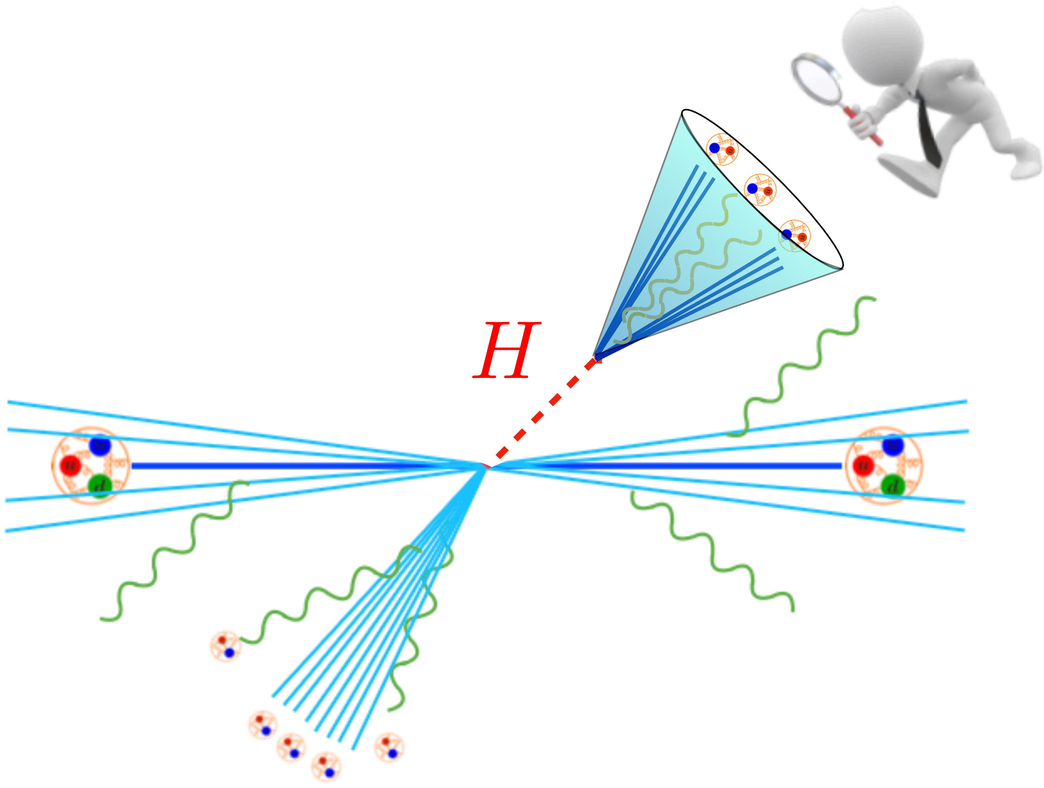


Motivation

Jets substructure for heavy particle through ECs

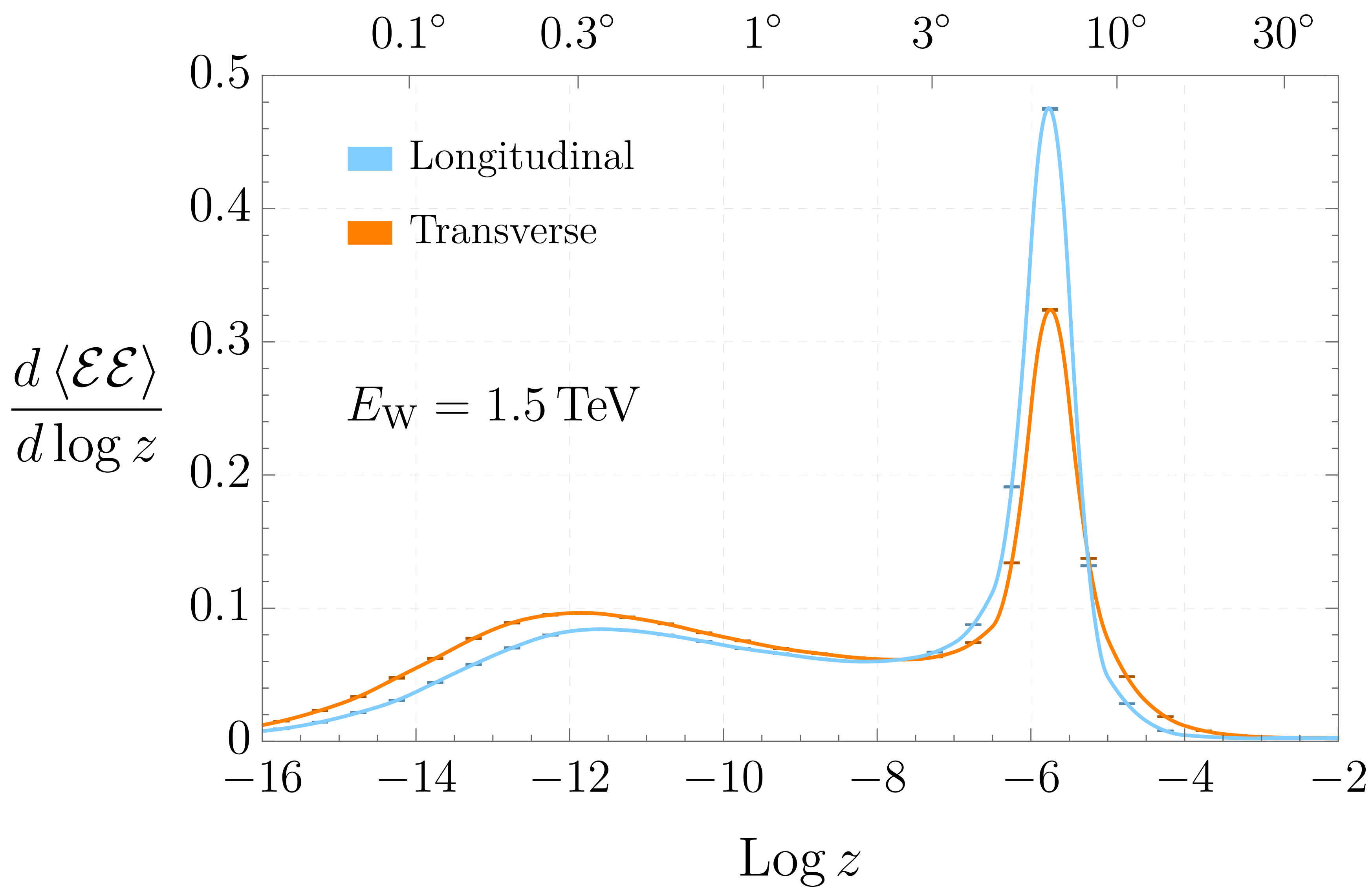
- Want to study the decay of heavy particles (H, Z, W, t) in high energy colliders (LHC, future e+e- collider, …)
- Why? Precision QCD & EW, constraining BSM, searching for new physics





Motivation

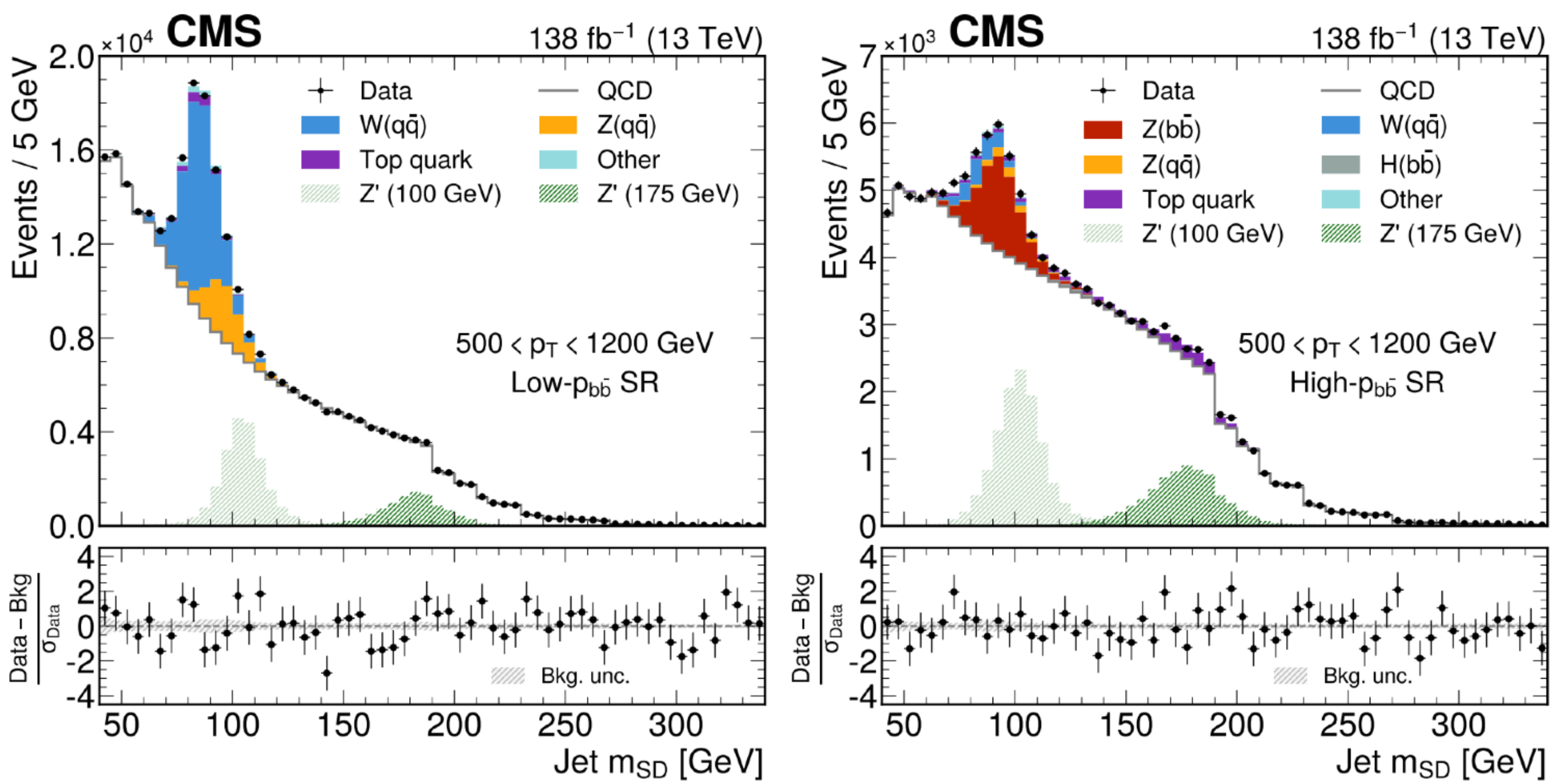
- Sensitive to polarization of W



[figure from Ricci, Riembau, `22]

Motivation

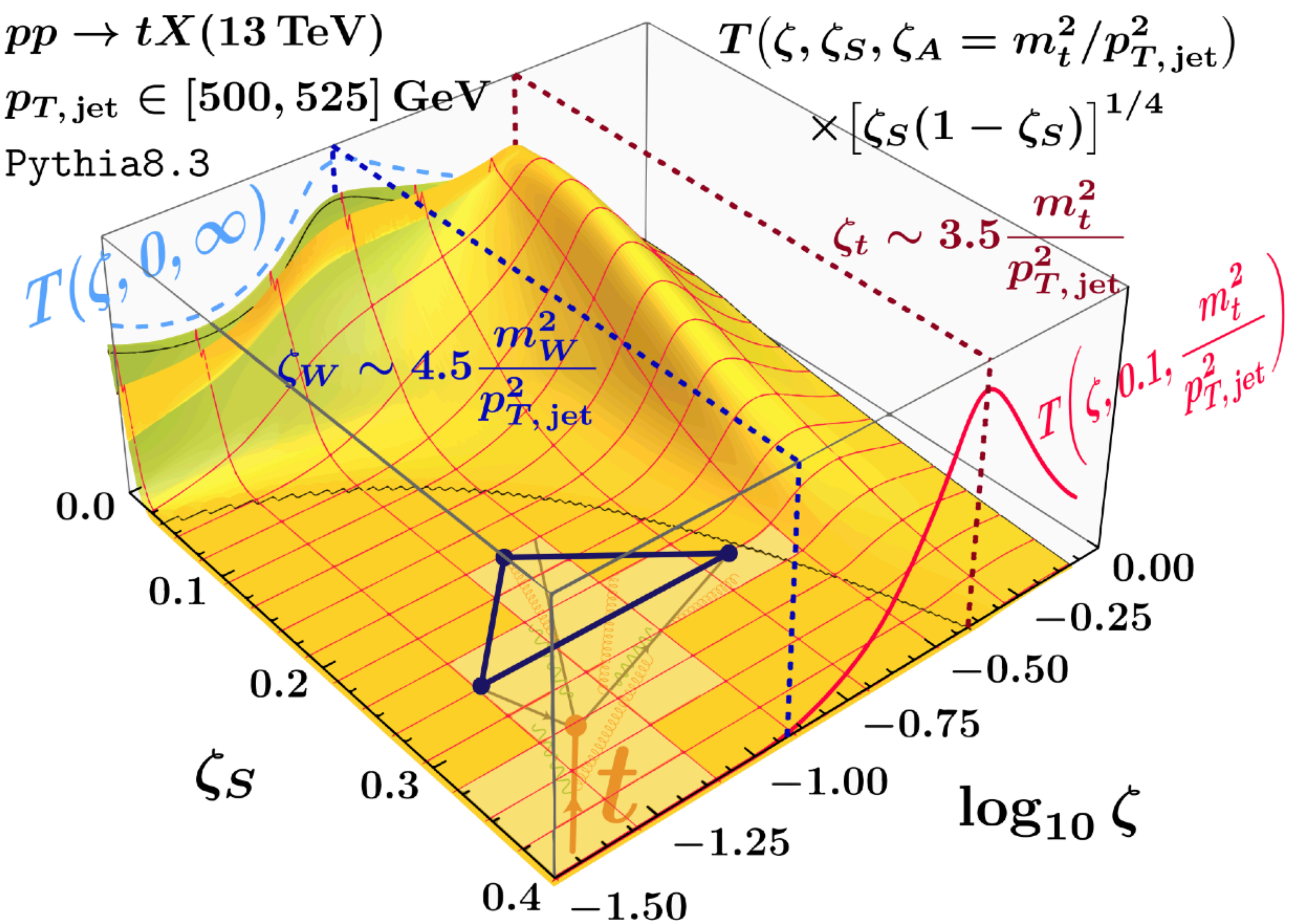
- Setting bounds to BSM models?

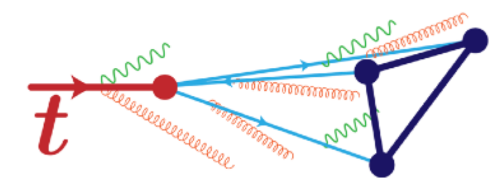


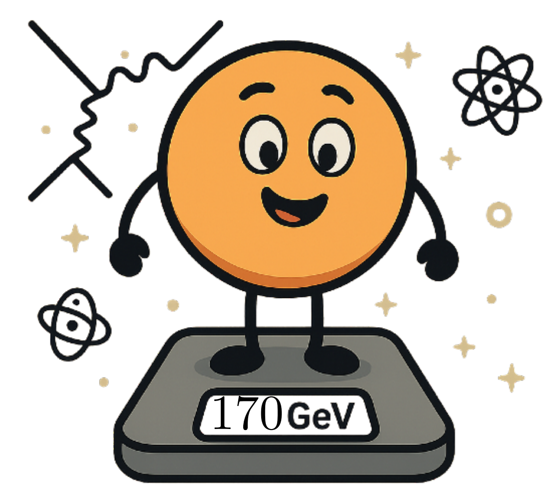
[CMS, 26`]

Motivation

- Improve top mass measurement using EEEC at the LHC



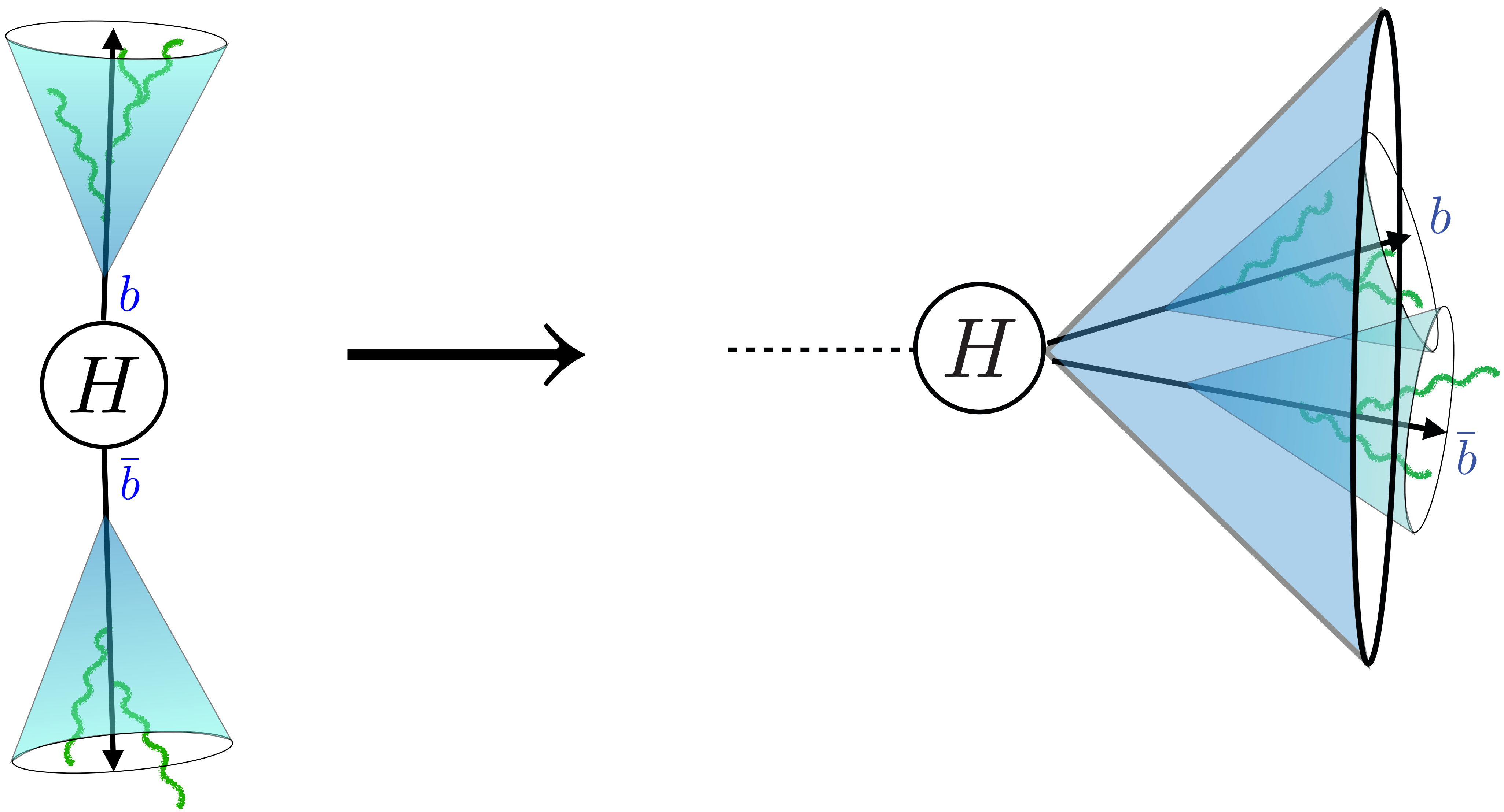




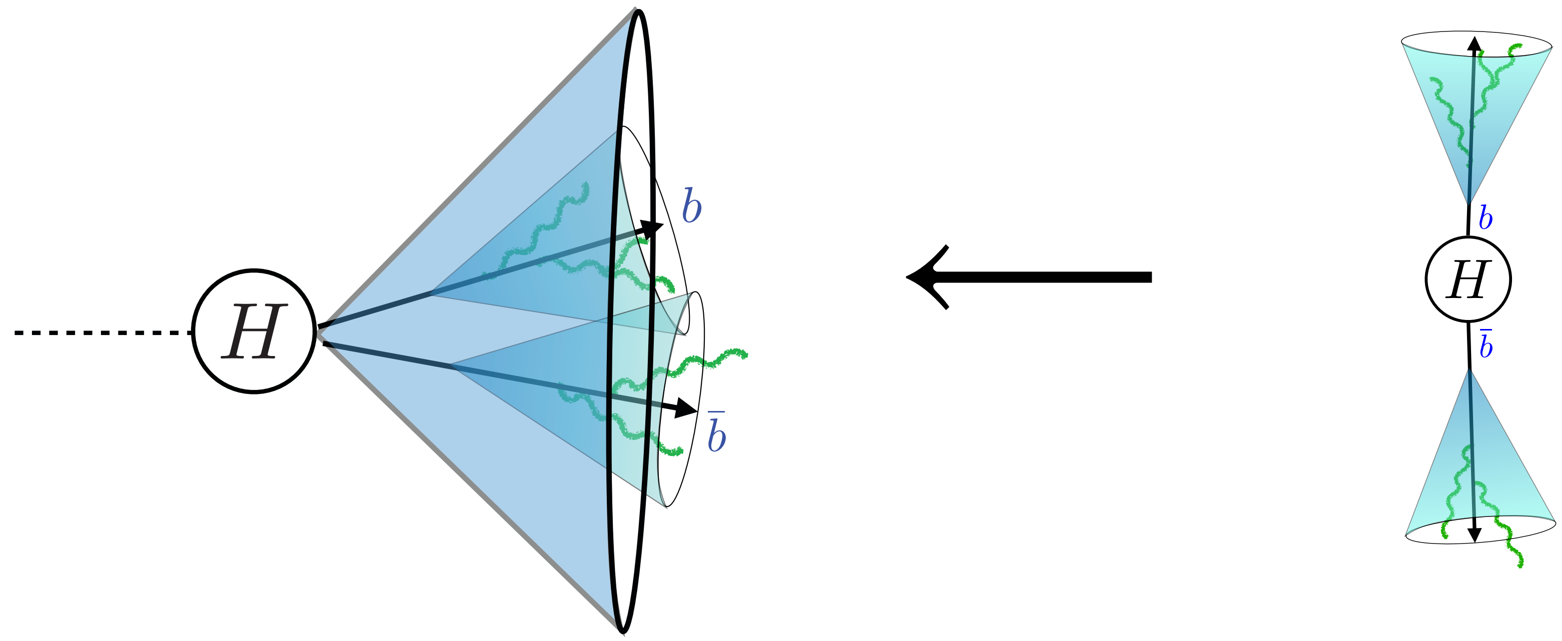
[Holguin, Moult, Pathak, Procura, `22
Holguin, Moult, Pathak, Procura, Schöfbeck, Schwarz, `23, `24]

Boost for Higgs EEC

$$\int d\theta_{\rm rest}\,\mathcal{K}_\gamma(\theta_{\rm rest},\theta)\,\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta_{\rm rest}} = \frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta}$$



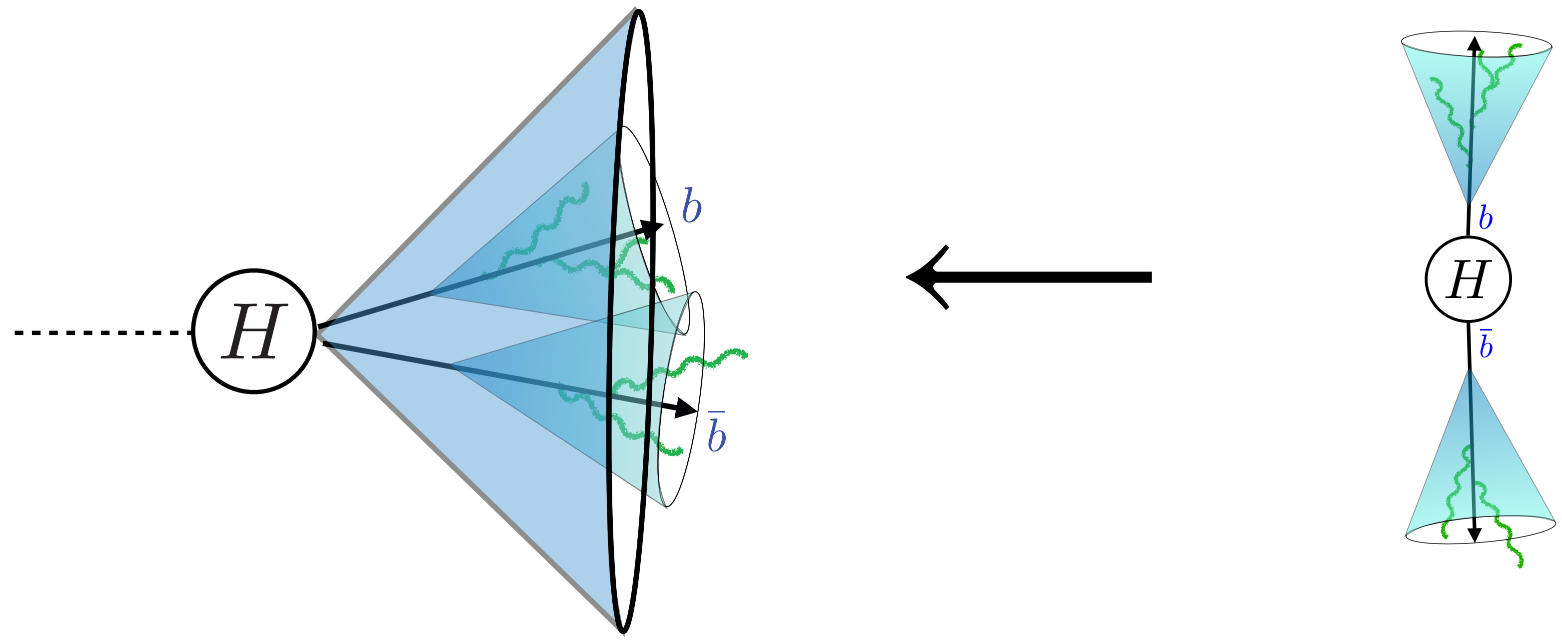
Outline



$$\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta} = \int d\theta_{\rm rest}\,\mathcal{K}_\gamma(\theta_{\rm rest},\theta)\,\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta_{\rm rest}}$$

- Higgs EEC
  - Deriving boost kernel
  - Rest-frame EEC
  - Boosted EEC
- Extensions: vector bosons (Z/W), multi-point energy correlators

Outline



$$\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta} = \int d\theta_{\rm rest}\, \mathcal{K}_\gamma(\theta_{\rm rest}, \theta)\, \frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta_{\rm rest}}$$

- Higgs EEC
  - Deriving boost kernel
  - Rest-frame EEC
  - Boosted EEC
- Extensions: vector bosons (Z/W), multi-point energy correlators

$$\textbf{EEC}(z) \xrightarrow{\textbf{boost}} \textbf{EEC}'(z')$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz} = \frac{1}{\sigma}\sum\!\!\!\!\!\!\int_X \sum_{i,j\in X} \mathrm{d}\sigma_{H(p_H)\to X(\{p_i\})} \frac{E_i E_j}{(\sum_{i\in X} E_i)^2}\delta(z - n_i \cdot n_j/2)$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz'} = \frac{1}{\sigma}\sum\!\!\!\!\!\!\int_X \sum_{i,j\in X} \mathrm{d}\sigma_{H(p'_H)\to X(\{p'_i\})} \frac{E'_i E'_j}{(\sum_{i\in X} E'_i)^2}\delta(z' - n'_i \cdot n'_j/2)$$

$$z = \frac{1-\cos\theta}{2} = \frac{n_i \cdot n_j}{2}$$

What is changed under boost?

EEC$(z) \xrightarrow{\textbf{boost}}$ **EEC**$'(z')$

$$\sum\!\!\!\!\!\!\int_X \sum_{i,j\in X} \mathrm{d}\sigma_{H(p'_H)\to X(\{p'_i\})} \frac{E'_i E'_j}{(\sum_{i\in X} E'_i)^2}\, \delta(z' - n'_i \cdot n'_j/2)$$

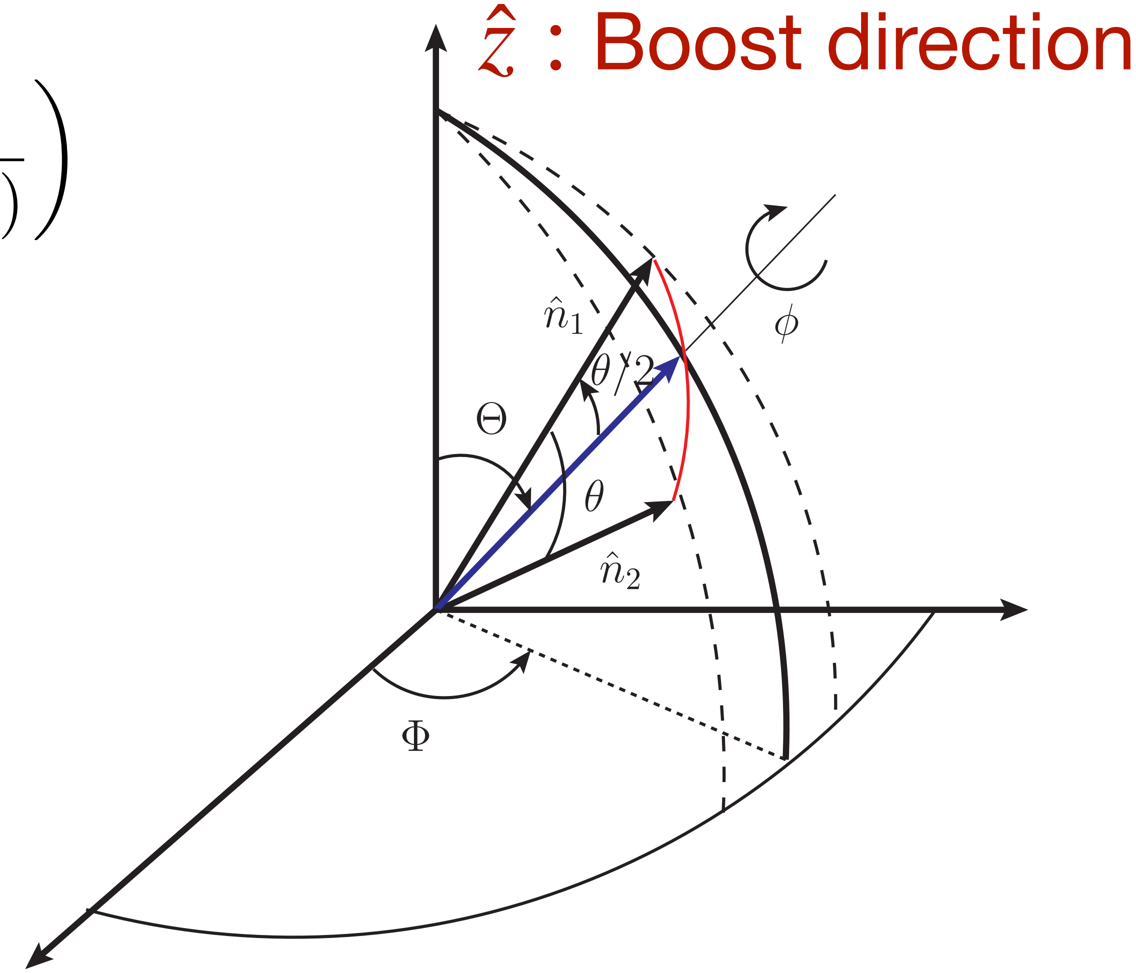
- Under boost Λ, $n_i \to n'_i = \dfrac{\Lambda \cdot n_i}{\lambda(\vec{n}_i)}$, where $\lambda(\vec{n}_i) = (\Lambda \cdot n_i)^0 = \gamma(1 + \vec{\beta} \cdot \vec{n}_i)$
- $z = \dfrac{1 - \cos\theta}{2} = \dfrac{n_i \cdot n_j}{2} \;\to\; z' = \dfrac{1 - \cos\theta'}{2} = \dfrac{n'_i \cdot n'_j}{2} = \dfrac{z}{\lambda(\vec{n}_i)\lambda(\vec{n}_j)}$
- Energy weighting $E'_i E'_j = \lambda(\vec{n}_i)\lambda(\vec{n}_j) E_i E_j = \dfrac{z}{z'} E_i E_j$
- Energy sum $\sum_{i\in X} E'_i = (p'_H)^0 = \big(\Lambda \cdot p_H\big)^0 = \gamma m_H$

EEC$(z) \xrightarrow{\textbf{boost}}$ **EEC**(z')

- Need to integrate over $d\Omega = d\phi\, d\cos\Theta$
- In the rest frame, Higgs EEC depends only on θ

$$\mathcal{K}_\gamma(z, z') = \frac{1}{\gamma^2}\int \frac{d^2\Omega}{4\pi}\frac{z}{z'}\delta\left(z' - \frac{z}{\lambda(\vec{n}_1)\lambda(\vec{n}_2)}\right)$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz'} = \int dz\, K_\gamma(z, z')\frac{1}{\Gamma}\frac{d\Gamma}{dz}$$



[figure from Riembau, Son `25]

Boost kernel

$$\mathcal{K}_\gamma(z,z') = \frac{1}{\gamma^2}\int \frac{d^2\Omega}{4\pi}\,\frac{z}{z'}\delta\left(z' - \frac{z}{\lambda(\vec{n}_1)\lambda(\vec{n}_2)}\right)$$

$$= \frac{z^{3/2}\,\Theta(m(z,z'))}{2\pi\,\gamma^2\sqrt{\gamma^2-1}\;z'^{5/2}\;\sqrt[4]{(1-z)(1-z')}} \times \begin{cases} K\big(\sqrt{m(z,z')}\big)\,, & 0 \le m(z,z') < 1\,, \\ \dfrac{1}{\sqrt{m(z,z')}}\,K\big(1/\sqrt{m(z,z')}\big)\,, & m(z,z') > 1\,, \end{cases}$$

$$K(k) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)\,(1-k^2\,t^2)}}$$ Elliptic integral of the first kind

$$m(z,z') = \frac{2\sqrt{1-z}\sqrt{1-z'} + z + z' + zz'(\gamma^2-1) - 2}{4\sqrt{1-z}\sqrt{1-z'}}$$

- Very symmetric under $z \leftrightarrow z'$
- Two branches (one closed curve v.s. two curves)

Aside: EEC Sum Rules

- EEC sum rules (energy & momentum conservation)

$$\int_0^1 dz\,\frac{1}{\Gamma}\frac{d\Gamma}{dz} = 2\int_0^1 dz\,z\,\frac{1}{\Gamma}\frac{d\Gamma}{dz} = 1$$

- It relates $z \to 0$ with $z \to 1$

$$\frac{1}{\sigma_0}\frac{d\sigma(z,\mu=Q)}{dz}\bigg|_{a_s^1} = \left[\frac{1}{2}j_1^q + h_1^q + h_1^g\right]\delta(z) + C_F\left\{(-2\zeta_2-4)\delta(1-z) + \frac{3}{2}\frac{1}{[z]_+} - 2\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{[1-z]_+}\right.$$
$$\left. + \frac{1}{2z^5}\left[-9z^4 - 6z^3 - 42z^2 + 36z + 4(-z^4 - z^3 + 3z^2 - 15z + 9)\ln(1-z)\right]\right\}.$$

[Dixon, Moult, Zhu, `19]

EEC Sum Rules

- EEC sum rules (energy & momentum conservation)

$$\int_0^1 dz\, \frac{1}{\Gamma}\frac{d\Gamma}{dz} = 2\int_0^1 dz\, z\, \frac{1}{\Gamma}\frac{d\Gamma}{dz} = 1$$

- First sum rule: obvious from definition,

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz} = \frac{1}{\sigma} \sum\!\!\!\!\!\!\int_X \sum_{i,j\in X} \mathrm{d}\sigma_{H(p_H)\to X(\{p_i\})} \frac{E_i E_j}{(\sum_{i\in X} E_i)^2} \delta(z - n_i\cdot n_j/2)$$

- Second sum rule : replace $z = \dfrac{n_i \cdot n_j}{2} = \dfrac{p_i \cdot p_j}{2(E_i \cdot E_j)}$, then $\displaystyle\sum_{i,j\in X} p_i \cdot p_j = m_H^2$

- Boosted EEC sum rules

$$\int_0^1 dz'\, \frac{1}{\Gamma}\frac{d\Gamma}{dz'} = 2\gamma^2 \int_0^1 dz'\, z'\, \frac{1}{\Gamma}\frac{d\Gamma}{dz'} = 1$$

Can our boost kernel reproduce it?

Boosted EEC Sum Rules Checked!

- Want to check $\int_0^1 dz' \, \frac{1}{\Gamma} \frac{d\Gamma}{dz'} = 2\gamma^2 \int_0^1 dz' \, z' \, \frac{1}{\Gamma} \frac{d\Gamma}{dz'} = 1$

- Compute

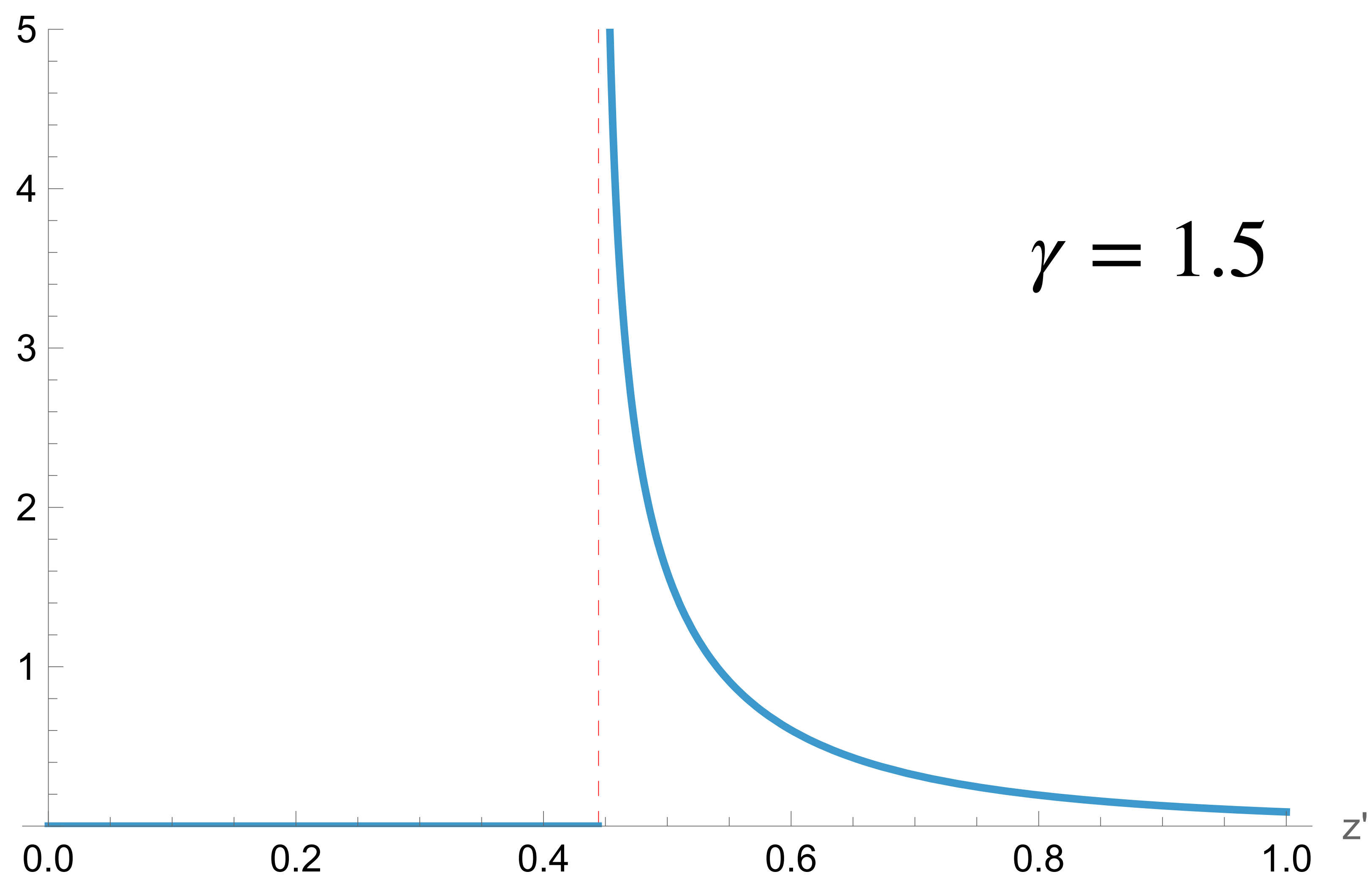
$$\int_0^1 dz' \frac{1}{\Gamma} \frac{d\Gamma}{dz'} = \int_0^1 dz \, [dz' \mathcal{K}_\gamma(z, z')] \, \frac{1}{\Gamma} \frac{d\Gamma}{dz} = \int_0^1 dz \left[-\frac{2(\gamma^2 - 1)}{3\gamma^2} \big(z - \frac{1}{2}\big) + 1 \right] \frac{1}{\Gamma} \frac{d\Gamma}{dz} = 1 \,,$$

$$\int_0^1 dz' z' \, \frac{1}{\Gamma} \frac{d\Gamma}{dz'} = \int_0^1 dz \, [dz' \, z' \, \mathcal{K}_\gamma(z, z')] \, \frac{1}{\Gamma} \frac{d\Gamma}{dz} = \int_0^1 dz \frac{z}{\gamma^2} \frac{1}{\Gamma} \frac{d\Gamma}{dz} = \frac{1}{2\gamma^2} \,,$$

Boosting fixed-order EEC

- At born level (back-to-back decay), $\frac{1}{\Gamma}\frac{d\Gamma}{dz} = \frac{1}{2}\left[\delta(z) + \delta(1-z)\right]$

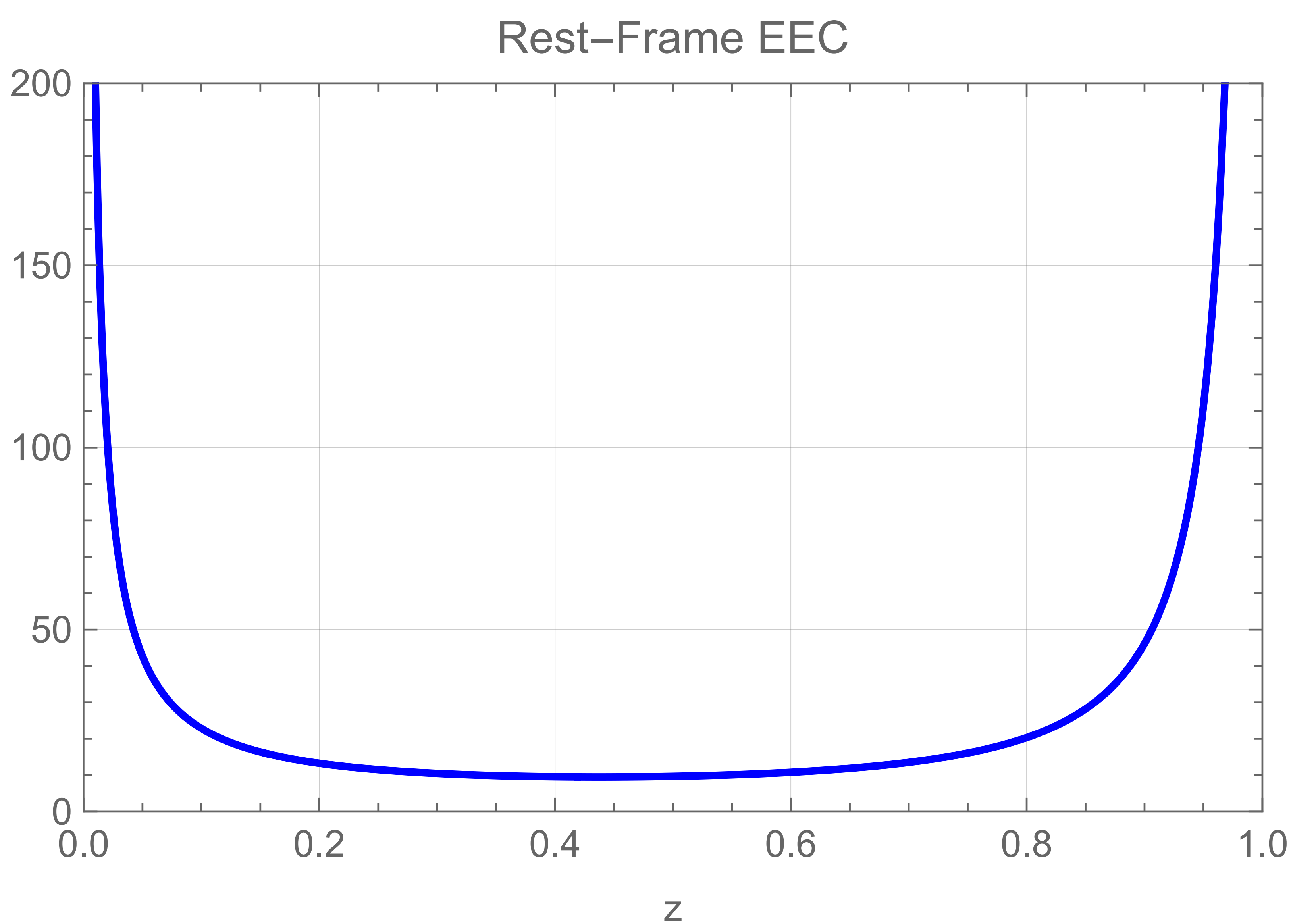
$\delta(1-z)$ contributes to $\frac{1}{2}\mathcal{K}_\gamma(z=1, z') = \frac{\Theta(\gamma^2 z' - 1)}{4\gamma^2\sqrt{\gamma^2-1}\, z'^{5/2}\sqrt{\gamma^2 z' - 1}}$



Boosting fixed-order EEC

- At one loop $$\frac{1}{\sigma_0}\frac{d\sigma(z,\mu=Q)}{dz}\bigg|_{a_s^1} = \left[\frac{1}{2}j_1^q + h_1^q + h_1^g\right]\delta(z) + C_F\Bigg\{(-2\zeta_2-4)\delta(1-z) + \frac{3}{2}\frac{1}{[z]_+} - 2\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{[1-z]_+} + \frac{1}{2z^5}\left[-9z^4-6z^3-42z^2+36z+4(-z^4-z^3+3z^2-15z+9)\ln(1-z)\right]\Bigg\}.$$

[Dixon, Moult, Zhu, `19]

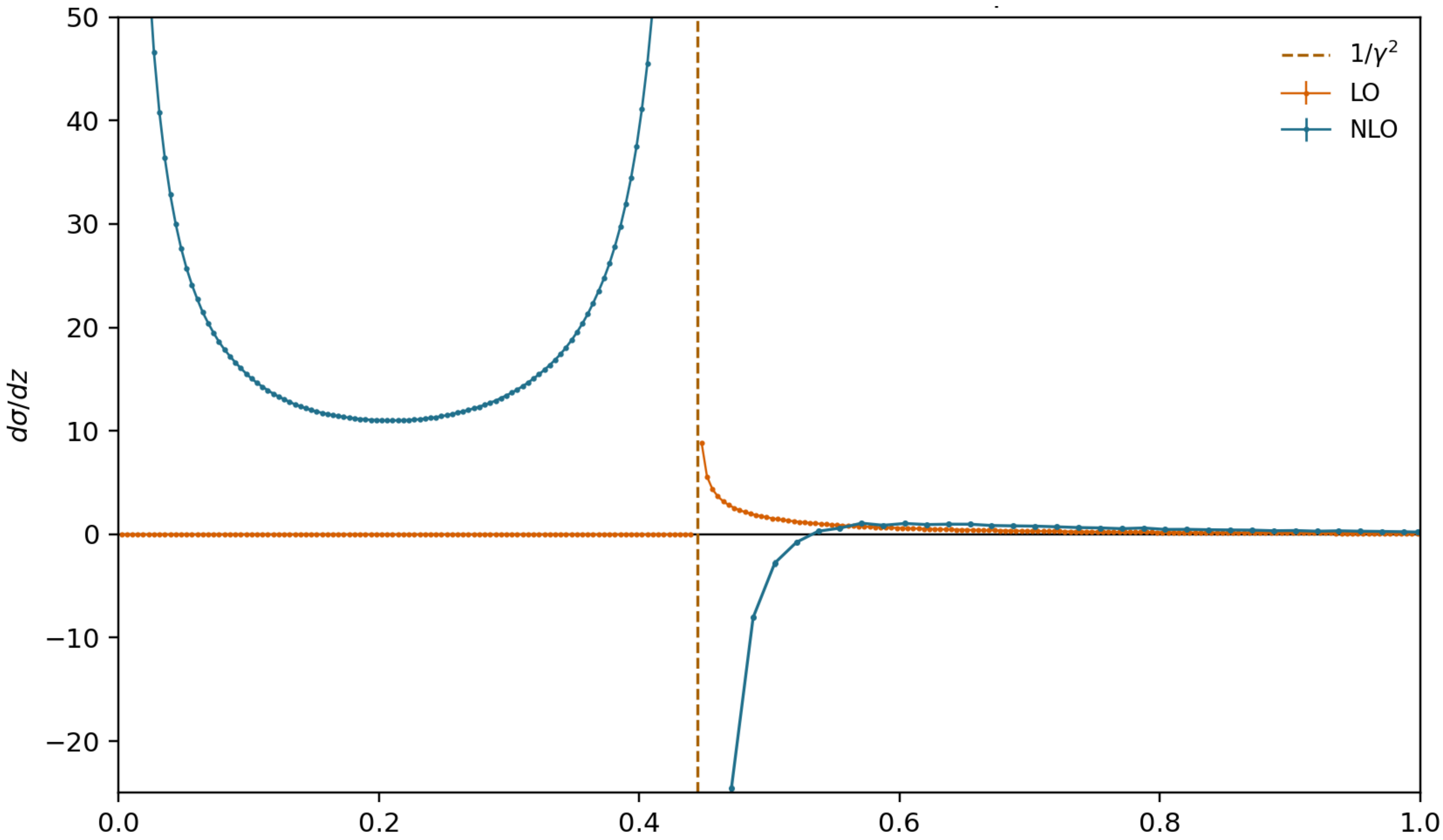


Boosting fixed-order EEC

- At one loop $$\frac{1}{\sigma_0}\frac{d\sigma(z,\mu=Q)}{dz}\bigg|_{a_s^1} = \left[\frac{1}{2}j_1^q + h_1^q + h_1^g\right]\delta(z) + C_F\bigg\{(-2\zeta_2 - 4)\delta(1-z) + \frac{3}{2}\frac{1}{[z]_+} - 2\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{[1-z]_+}$$ $$+ \frac{1}{2z^5}\left[-9z^4 - 6z^3 - 42z^2 + 36z + 4(-z^4 - z^3 + 3z^2 - 15z + 9)\ln(1-z)\right]\bigg\}.$$

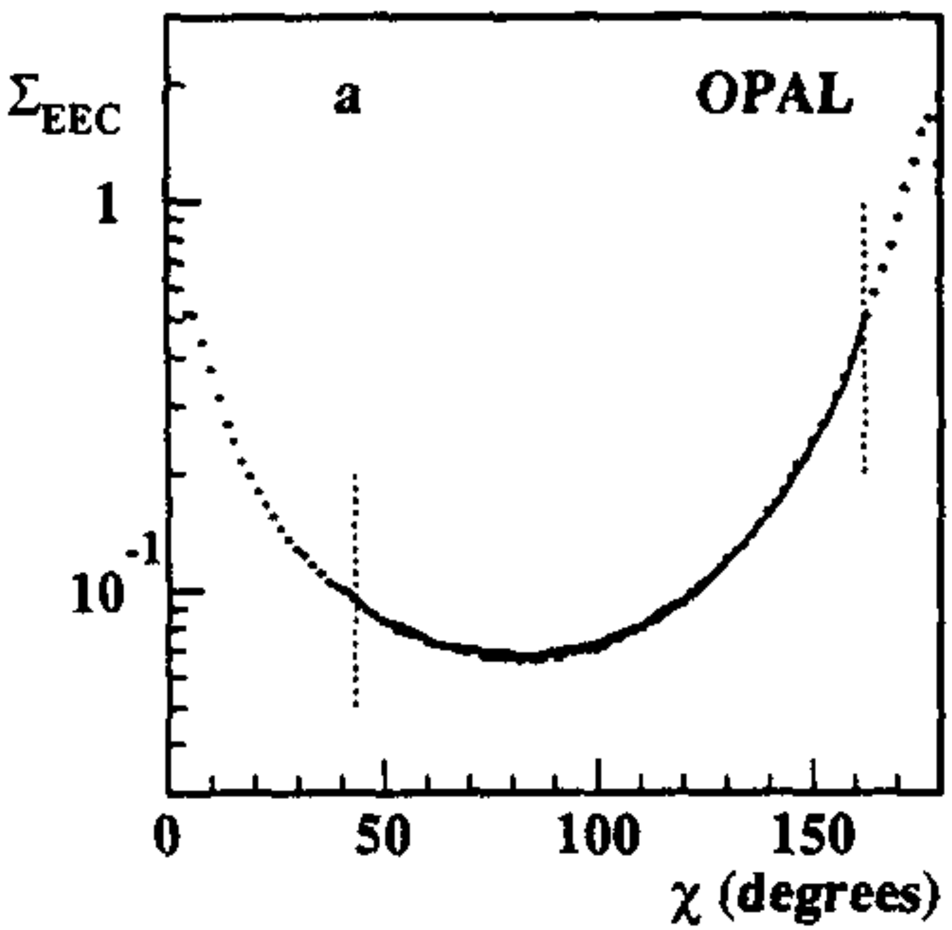
[Dixon, Moult, Zhu, `19]

- After boost

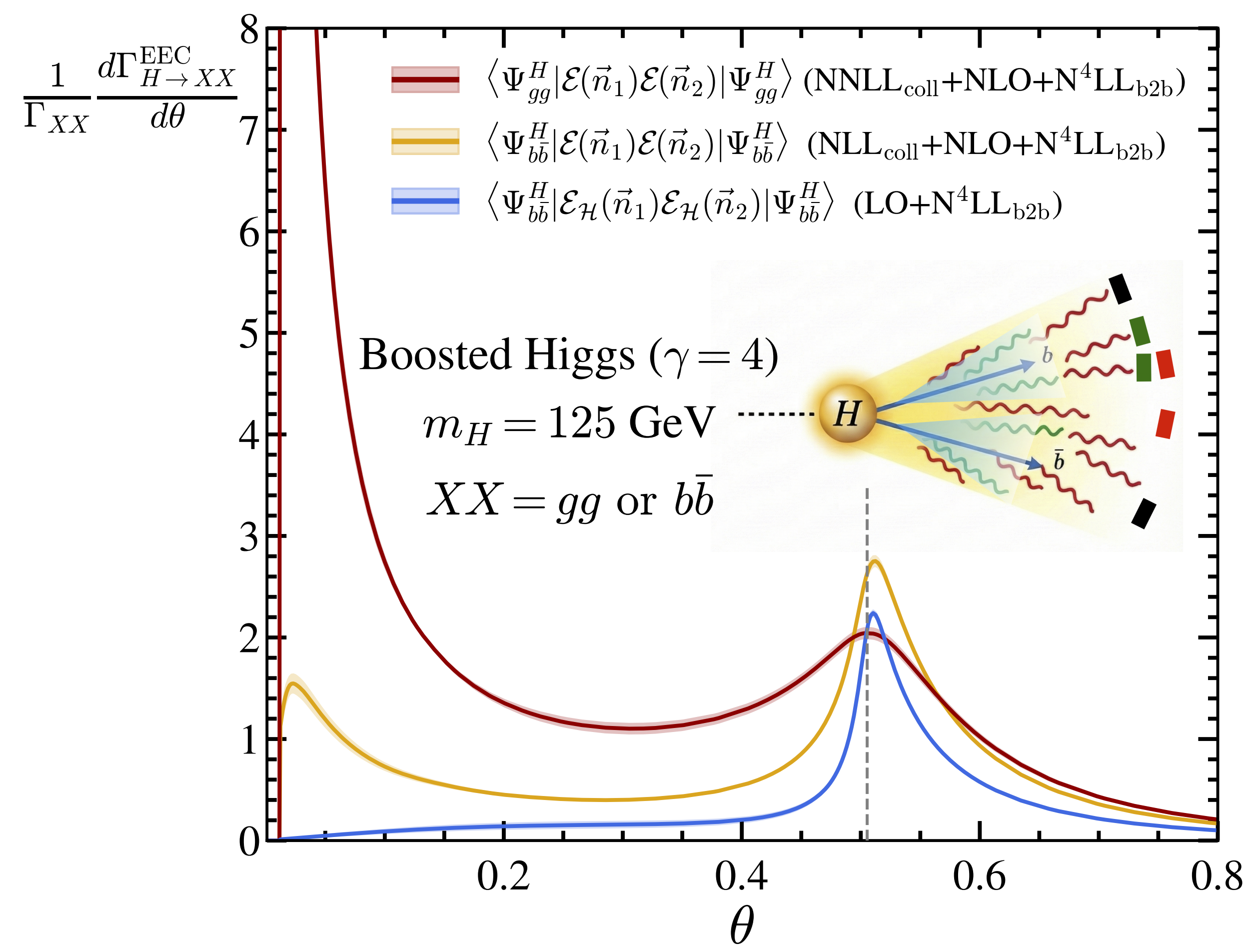


Back-to-Back Peak from the Boost kernel

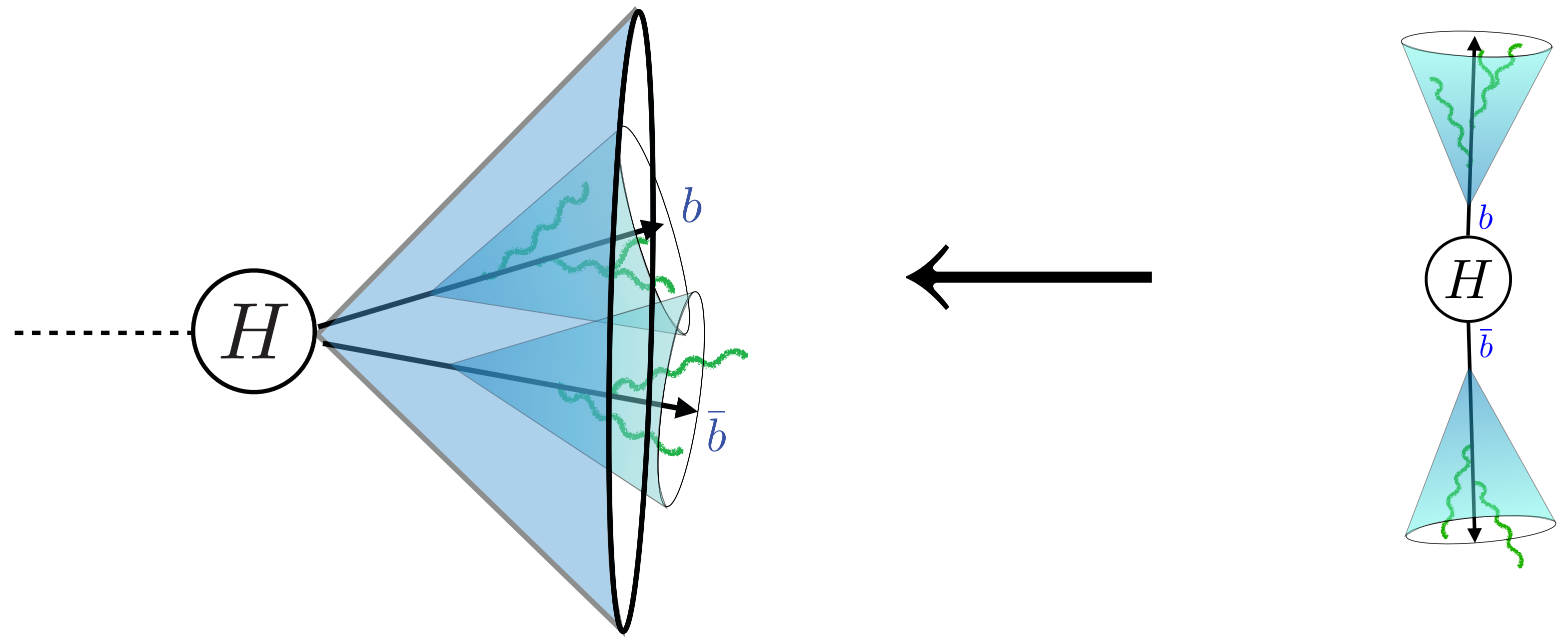
Back-to-back peak → peak near $z' = 1/\gamma^2$



→



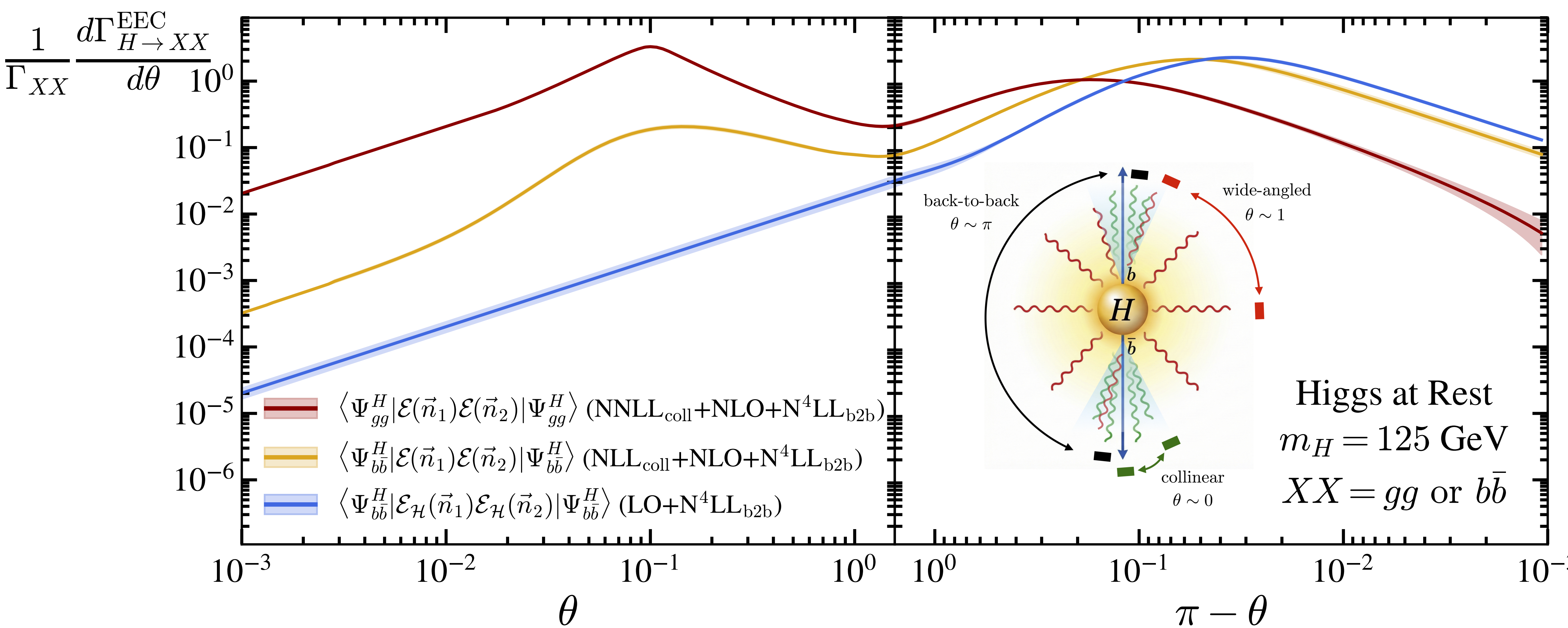
Outline



$$\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\mathrm{EEC}}_{H\to XX}}{d\theta} = \int d\theta_{\mathrm{rest}}\, \mathcal{K}_{\gamma}(\theta_{\mathrm{rest}}, \theta)\, \frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\mathrm{EEC}}_{H\to XX}}{d\theta_{\mathrm{rest}}}$$

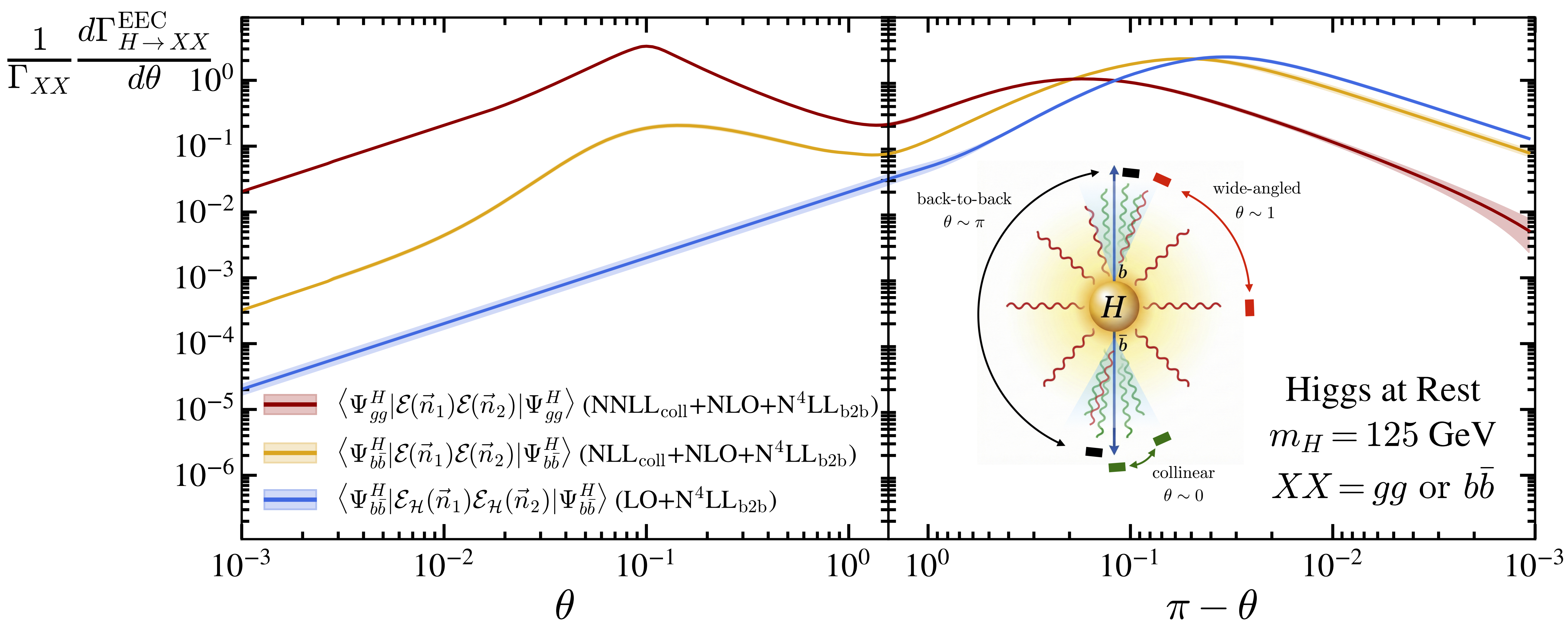
- Higgs EEC
  - Deriving boost kernel
  - Rest-frame EEC
  - Boosted EEC
- Extensions: vector bosons (Z/W), multi-point energy correlators

Rest Frame Higgs EEC



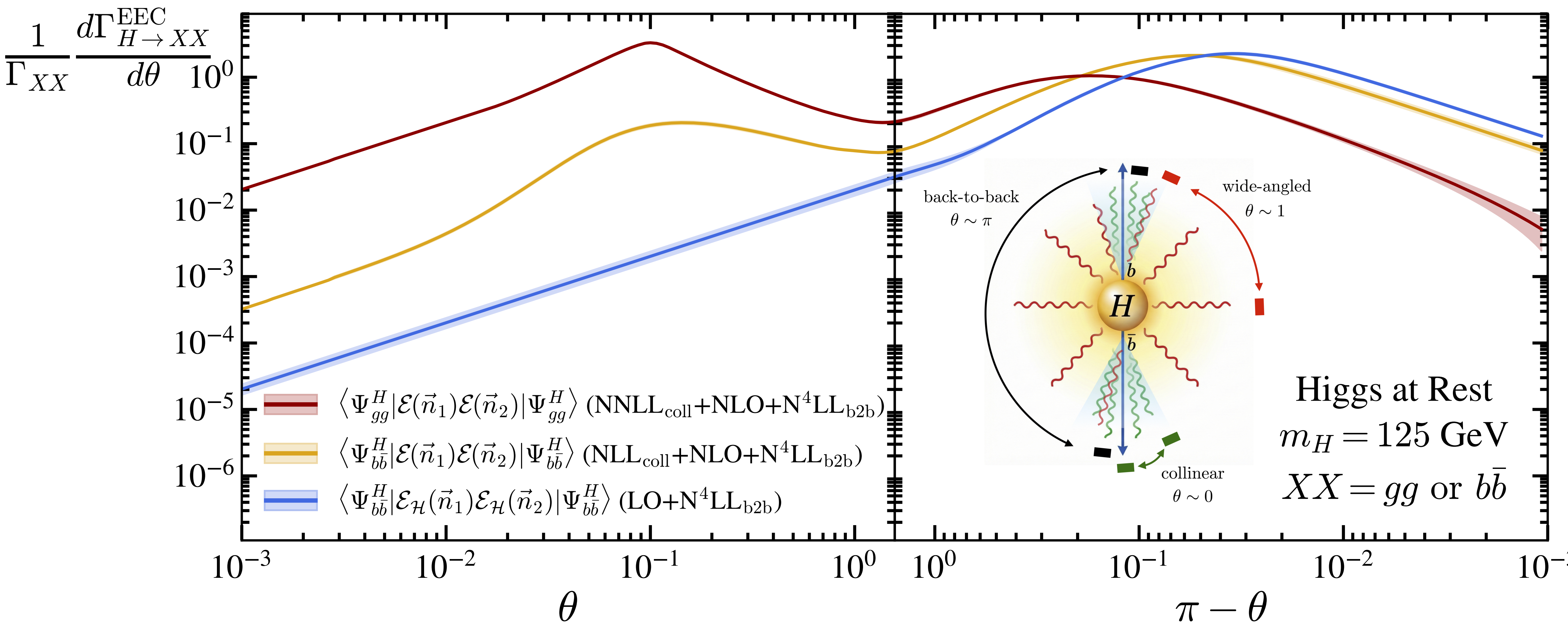
- Hard channels $H \to gg, H \to b\bar{b}$
- Inclusive energy detectors $\mathscr{E}$ v.s. heavy flavor detector $\mathscr{E}_{\mathscr{H}}$ (using heavy flavor track function)

Rest Frame Higgs EEC

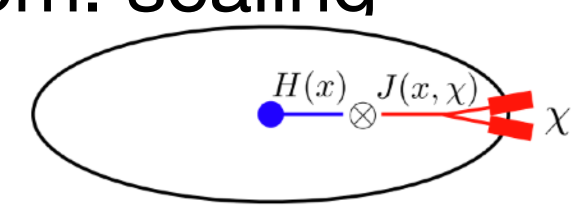


- Wide-angle region $\theta \sim 1, \mu_{\text{EEC}} \sim m_H$, fixed-order $\mathcal{O}(\alpha_s^2)$
- Collinear regime $\theta \ll 1, \quad \mu_{\text{EEC}} \sim m_H\theta/2 \ll m_H$
- Back-to-back regime $\pi - \theta \ll 1, \quad \mu_{\text{EEC}} \sim m_H(\pi - \theta)/2$

Rest Frame Higgs EEC

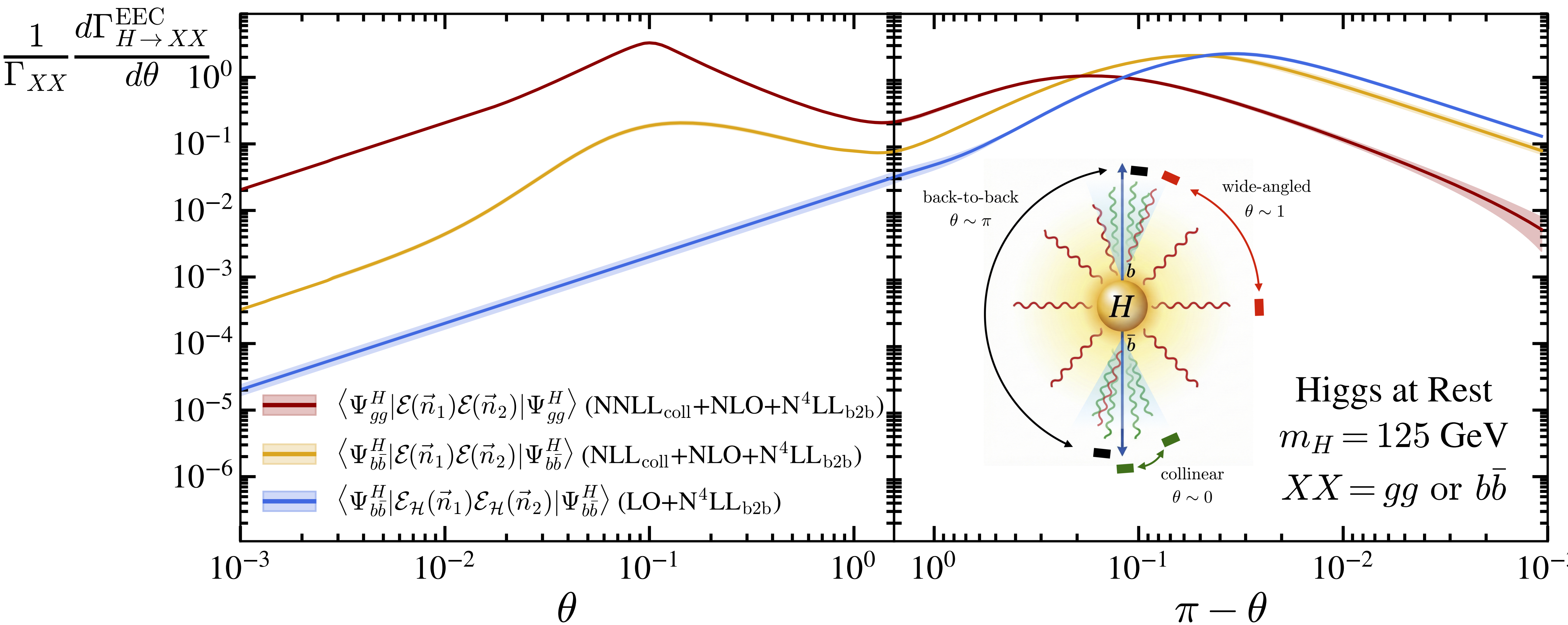


- Collinear regime $\theta \ll 1$, $m_b, \Lambda_{\rm QCD} \ll \mu_{\rm EEC} \sim m_H\theta/2 \ll m_H$
  - Classical $1/\theta$ scaling + DGLAP res. $\log\theta$, anom. scaling
  - NNLL for $H \to gg$, NLL for $H \to b\bar{b}$



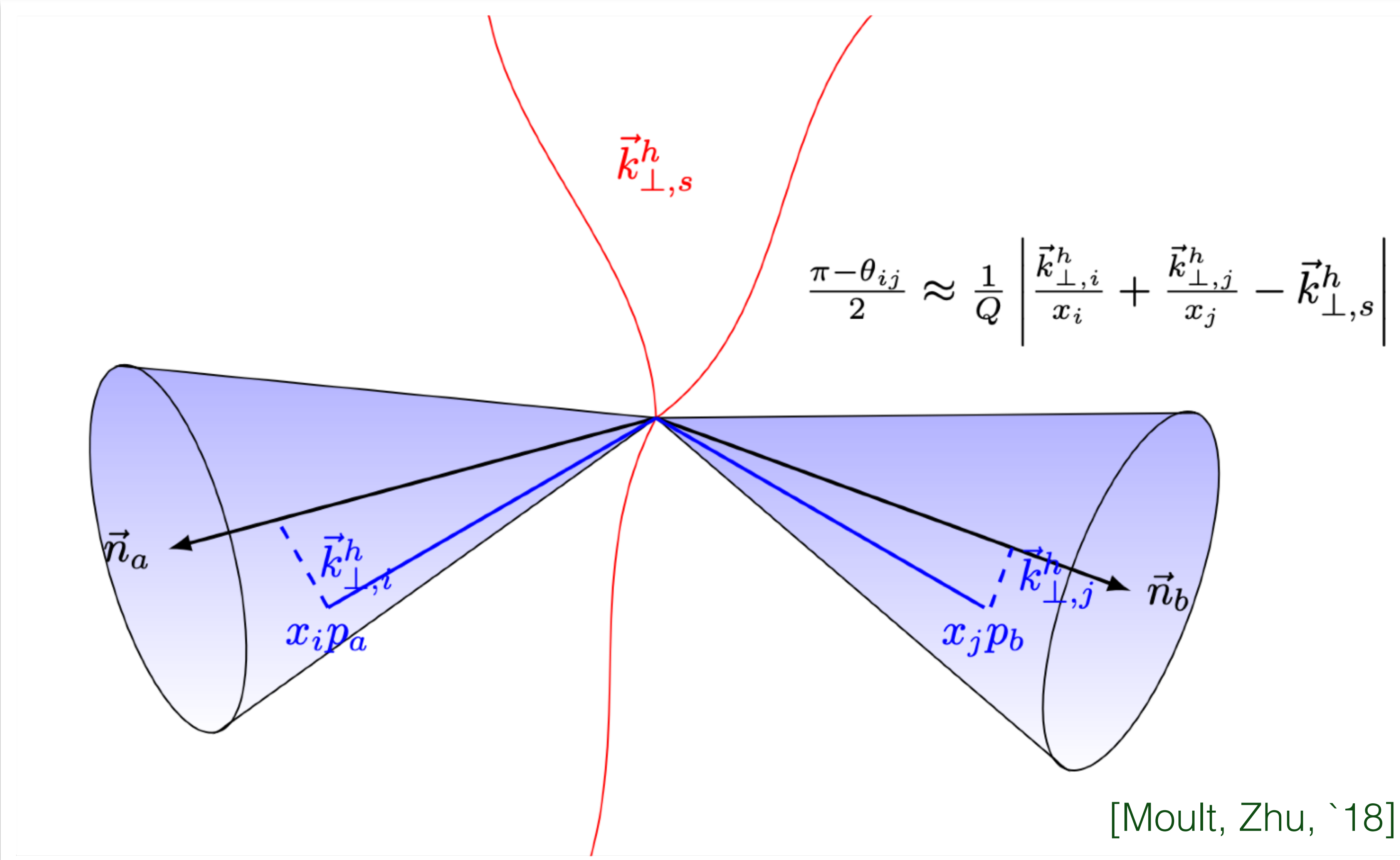
[Dixon, Moult, Zhu, `19]

Rest Frame Higgs EEC



- Collinear regime $\theta \ll 1$, $\mu_{\rm EEC} \sim m_H\theta/2$
  - $\mu_{\rm EEC} \sim m_b$, massive emitter supp., "deadcone effect", $\frac{d\Gamma^{\rm EEC}}{d\theta} \propto \text{const}\times\theta$
  - $\mu_{\rm EEC} \sim \Lambda_{\rm QCD}$, confinement, $\frac{d\Gamma^{\rm EEC}}{d\theta} \propto \text{const}\times\theta$

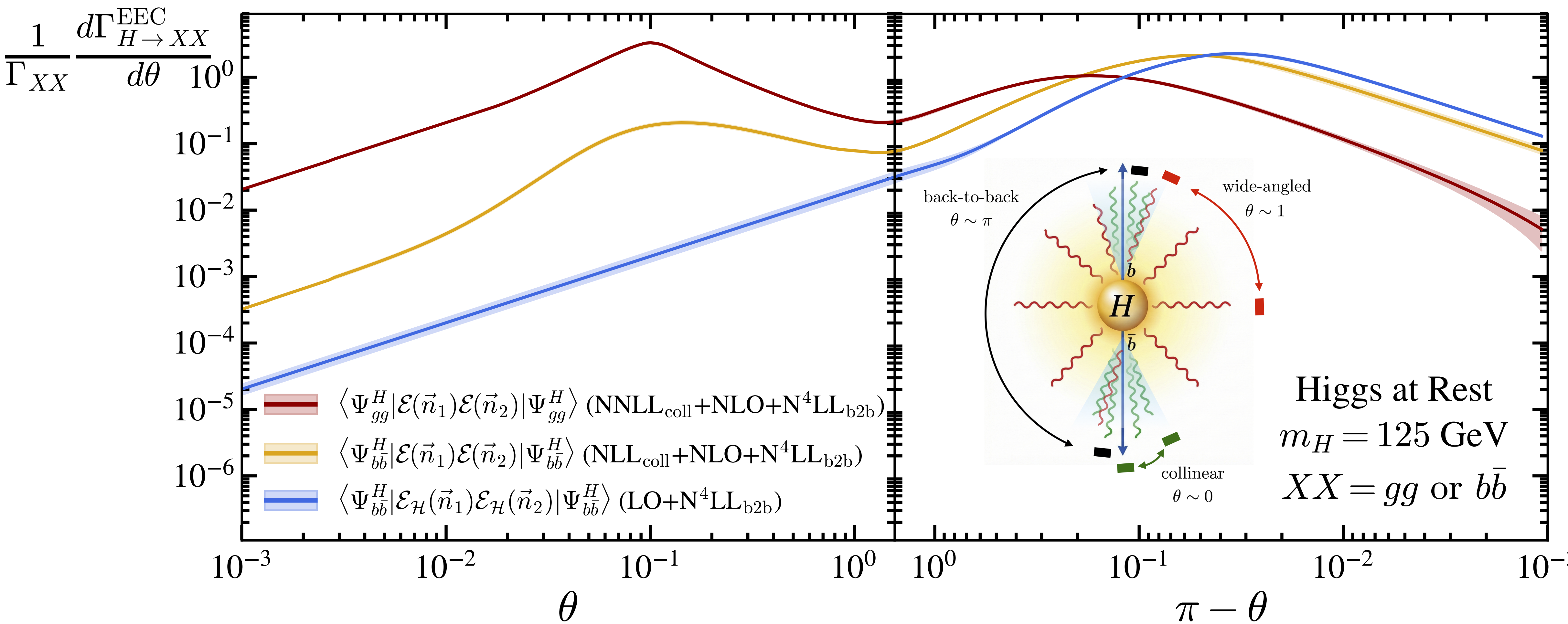
Rest Frame Higgs EEC



[Moult, Zhu, `18]

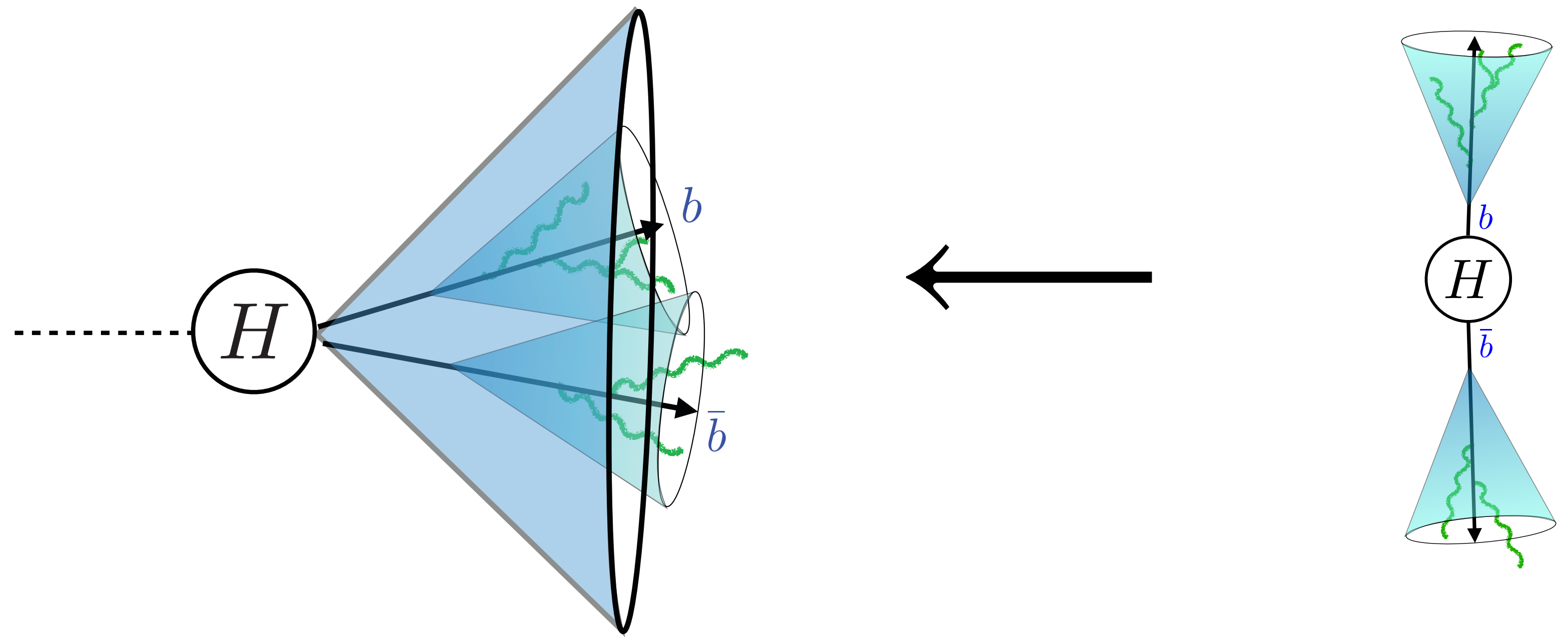
- Back-to-back regime $\pi - \theta \ll 1$, double log series $\alpha_s^n \log^{2n}(\pi - \theta)$
- TMD resummation to N^{4}LL

Rest Frame Higgs EEC



- Back-to-back regime $\pi - \theta \ll 1$, double log series $\alpha_s^n \log^{2n}(\pi - \theta)$
- TMD resummation to N^{4}LL
- For B-correlation, use track function

Outline



$$\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta} = \int d\theta_{\rm rest}\, \mathcal{K}_\gamma(\theta_{\rm rest}, \theta)\,\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta_{\rm rest}}$$

- Higgs EEC
  - Deriving boost kernel
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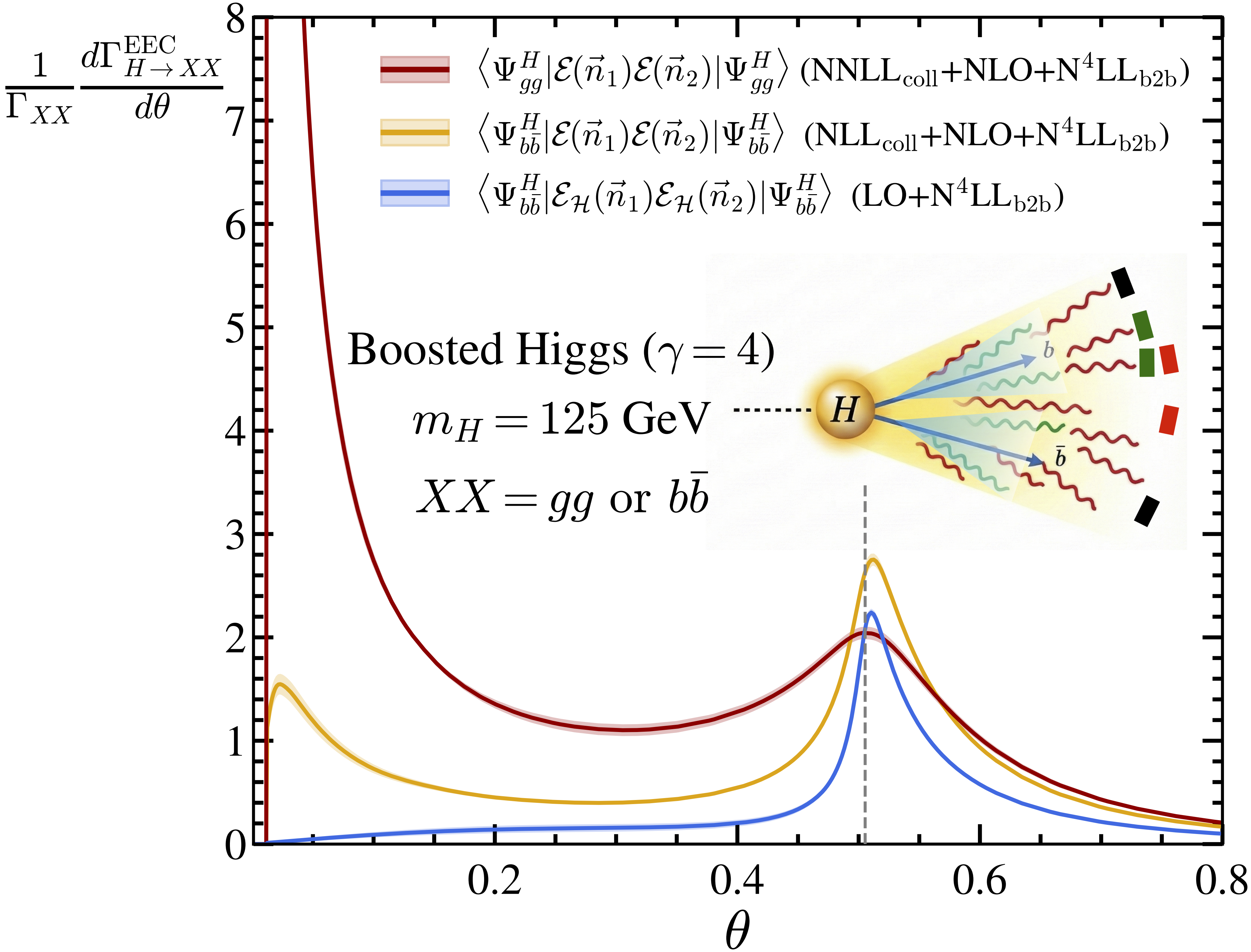
  - Boosted EEC
- Extensions: vector bosons (Z/W), multi-point energy correlators

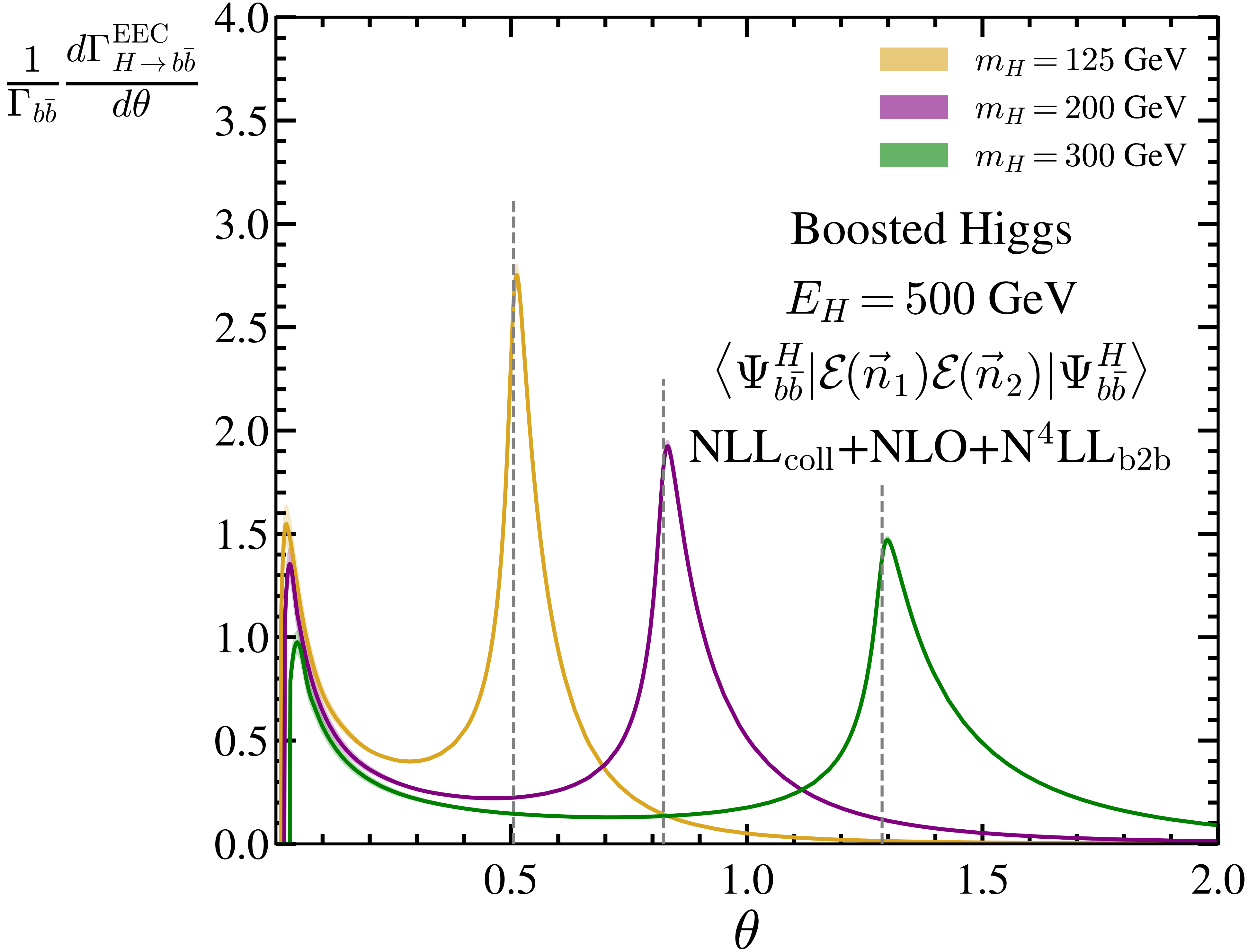
Boosted EEC



$\frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\mathrm{EEC}}_{H\to XX}}{d\theta}$
$\langle\Psi^H_{gg}|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|\Psi^H_{gg}\rangle$ (NNLL$_{\mathrm{coll}}$+NLO+N^{4}LL$_{\mathrm{b2b}}$)
$\langle\Psi^H_{b\bar{b}}|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|\Psi^H_{b\bar{b}}\rangle$ (NLL$_{\mathrm{coll}}$+NLO+N^{4}LL$_{\mathrm{b2b}}$)
$\langle\Psi^H_{b\bar{b}}|\mathcal{E}_{\mathcal{H}}(\vec{n}_1)\mathcal{E}_{\mathcal{H}}(\vec{n}_2)|\Psi^H_{b\bar{b}}\rangle$ (LO+N^{4}LL$_{\mathrm{b2b}}$)
Boosted Higgs ($\gamma = 4$)
$m_H = 125$ GeV
$XX = gg$ or $b\bar{b}$
H
b
$\bar{b}$
0
1
2
3
4
5
6
7
8
0.2
0.4
0.6
0.8
θ

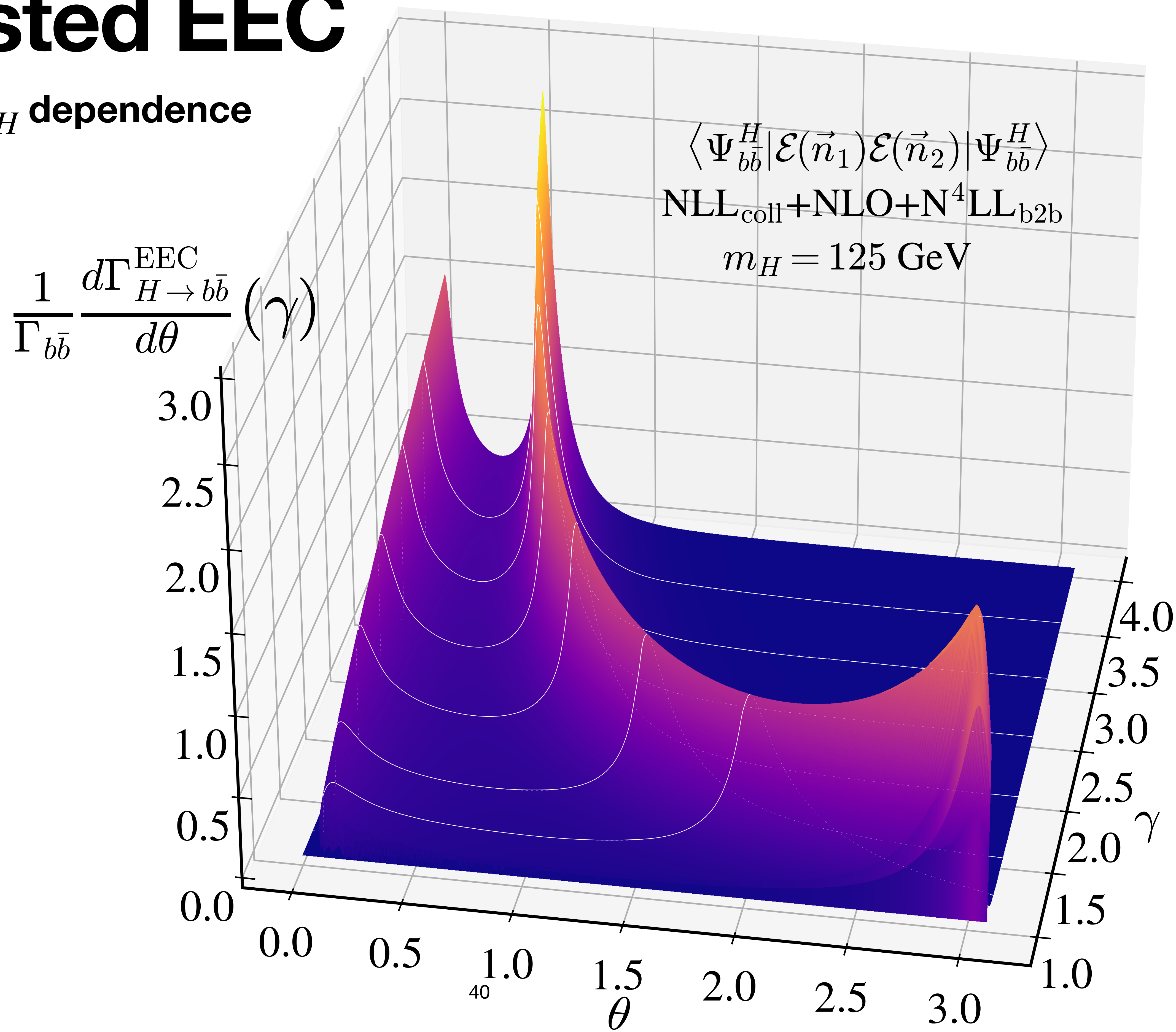
Boosted EEC

Mass sensitivity



Boosted EEC

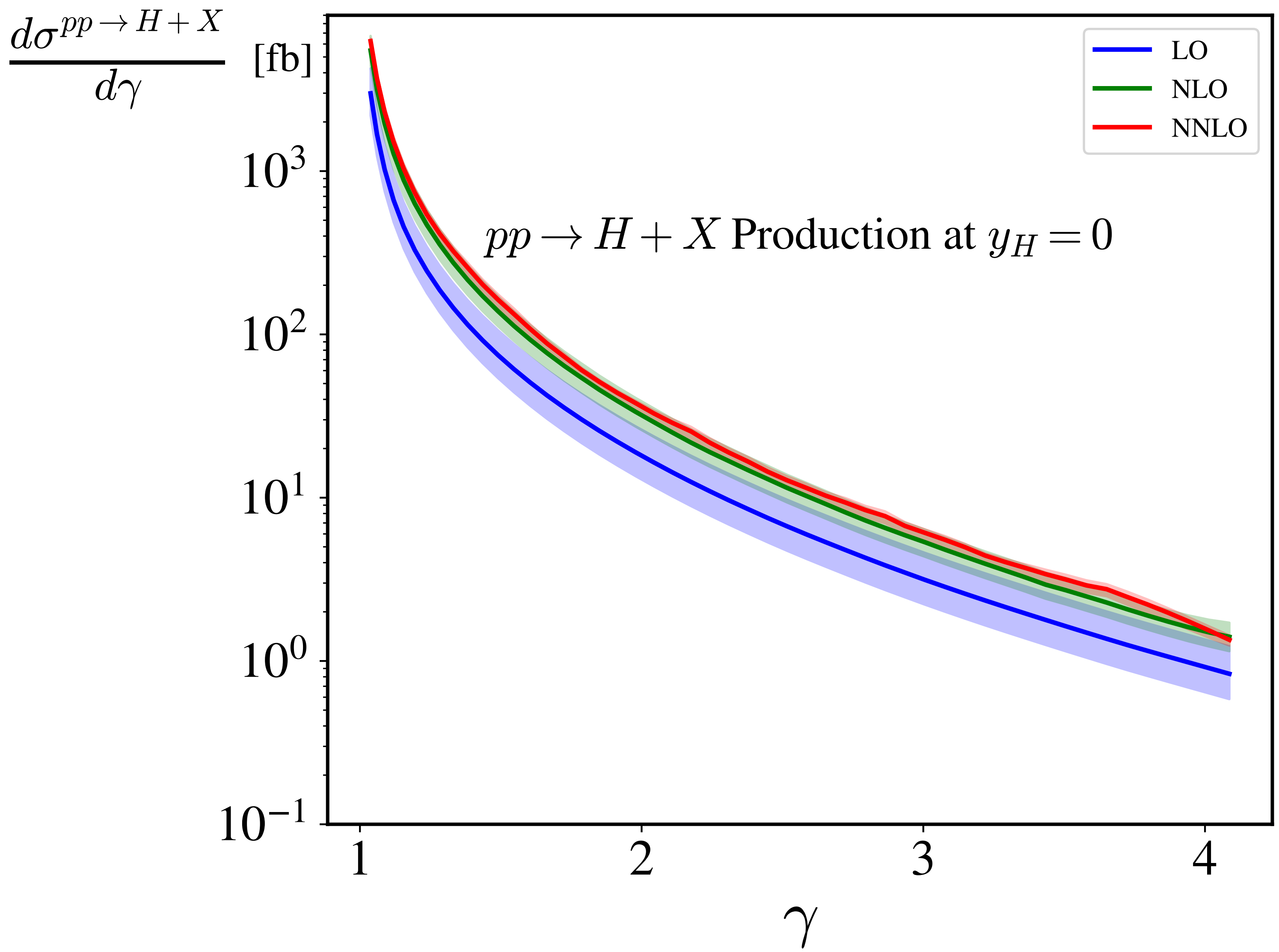
$\gamma = E_H/m_H$ **dependence**



Higgs Production

Separation of production with the EEC

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{\rm EEC}_{pp\to H\to XX}}{d\theta} \approx \sigma^{pp\to H} \times \frac{1}{\Gamma_{XX}}\frac{d\Gamma^{\rm EEC}_{H\to XX}}{d\theta}$$



As an example, we used NNLOJET program

Outline

- Higgs EEC
    - Deriving boost kernel
    - Rest-frame EEC
    - Boosted EEC
- Extensions: vector bosons (Z/W), multi-point energy correlators

Extension to Vector Bosons

- Need to include different polarization contributions
- The most generic EEC hadronic tensor [Riembau, Son `25]

$$H^{\mu\nu}_{\mathcal{E}^{(N)}} = \int d^4x\, e^{iq\cdot x} \langle J^\mu(x)\, \mathcal{E}_{n_1} \ldots \mathcal{E}_{n_N}\, J^\nu(0)\rangle$$

$$H^{ab}_{\mathcal{E}\mathcal{E}} = H^{\rm iso}_{\mathcal{E}\mathcal{E}}(z)\,\frac{\delta^{ab}}{3} + H^{c}_{\mathcal{E}\mathcal{E}}(z)\left(\frac{1}{1-z}\frac{n_1^a+n_2^a}{2}\frac{n_1^b+n_2^b}{2} - \frac{\delta^{ab}}{3}\right)$$
$$+ H^{b}_{\mathcal{E}\mathcal{E}}(z)\left(\frac{1}{z}\frac{n_1^a-n_2^a}{2}\frac{n_1^b-n_2^b}{2} - \frac{\delta^{ab}}{3}\right) .$$

Isotropic part $H^{\rm iso}_{\mathcal{E}\mathcal{E}}(z)$, two anisotropic ratios

$$a^{(2,0)}(z) = \frac{H^{c}_{\mathcal{E}\mathcal{E}}(z) - \frac{1}{2}H^{b}_{\mathcal{E}\mathcal{E}}(z)}{H^{\rm iso}_{\mathcal{E}\mathcal{E}}(z)}\,, \qquad a^{(2,2)}(z) = \frac{H^{b}_{\mathcal{E}\mathcal{E}}(z)}{H^{\rm iso}_{\mathcal{E}\mathcal{E}}(z)}$$

Extension to Vector Bosons

- Need to include different polarization contributions
- In helicity basis, define $\mathcal{W}_{\lambda\lambda'}(\vartheta, \varphi, \Phi; z) \equiv \dfrac{\langle V, \lambda | \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) | V, \lambda' \rangle}{H^{\rm iso}_{\mathcal{E}\mathcal{E}}(z)}$

$$\mathcal{W}_{00} = 1 + a^{(2,0)}(z)\big(3\cos^2\vartheta - 1\big) + \frac{3}{2}a^{(2,2)}(z)\sin^2\vartheta\cos 2\varphi\,,$$

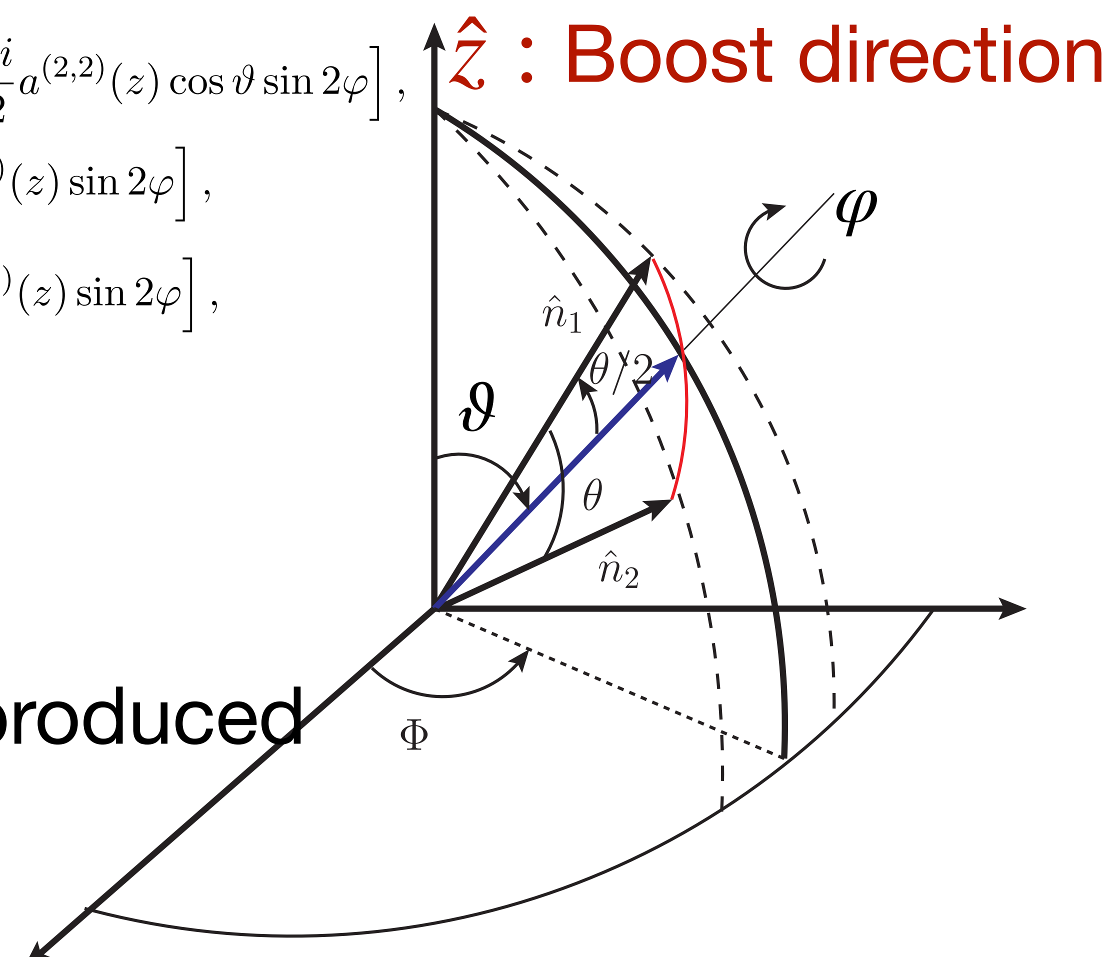
$$\mathcal{W}_{++} = \mathcal{W}_{--} = 1 + a^{(2,0)}(z)\left(\frac{3}{2}\sin^2\vartheta - 1\right) - \frac{3}{4}a^{(2,2)}(z)\sin^2\vartheta\cos 2\varphi\,,$$

$$\mathcal{W}_{+-} = e^{-2i\Phi}\Big[-\frac{3}{2}a^{(2,0)}(z)\sin^2\vartheta - \frac{3}{4}a^{(2,2)}(z)\big(1+\cos^2\vartheta\big)\cos 2\varphi + \frac{3i}{2}a^{(2,2)}(z)\cos\vartheta\sin 2\varphi\Big]\,,$$

$$\mathcal{W}_{+0} = e^{-i\Phi}\frac{\sin\vartheta}{\sqrt{2}}\Big[3a^{(2,0)}(z)\cos\vartheta - \frac{3}{2}a^{(2,2)}(z)\cos\vartheta\cos 2\varphi + \frac{3i}{2}a^{(2,2)}(z)\sin 2\varphi\Big]\,,$$

$$\mathcal{W}_{-0} = -e^{i\Phi}\frac{\sin\vartheta}{\sqrt{2}}\Big[3a^{(2,0)}(z)\cos\vartheta - \frac{3}{2}a^{(2,2)}(z)\cos\vartheta\cos 2\varphi - \frac{3i}{2}a^{(2,2)}(z)\sin 2\varphi\Big]\,,$$

- The full EEC = $\rho_{\lambda\lambda'}\mathscr{W}_{\lambda\lambda'}$
- $\rho_{\lambda\lambda'}$ describes polarization states produced

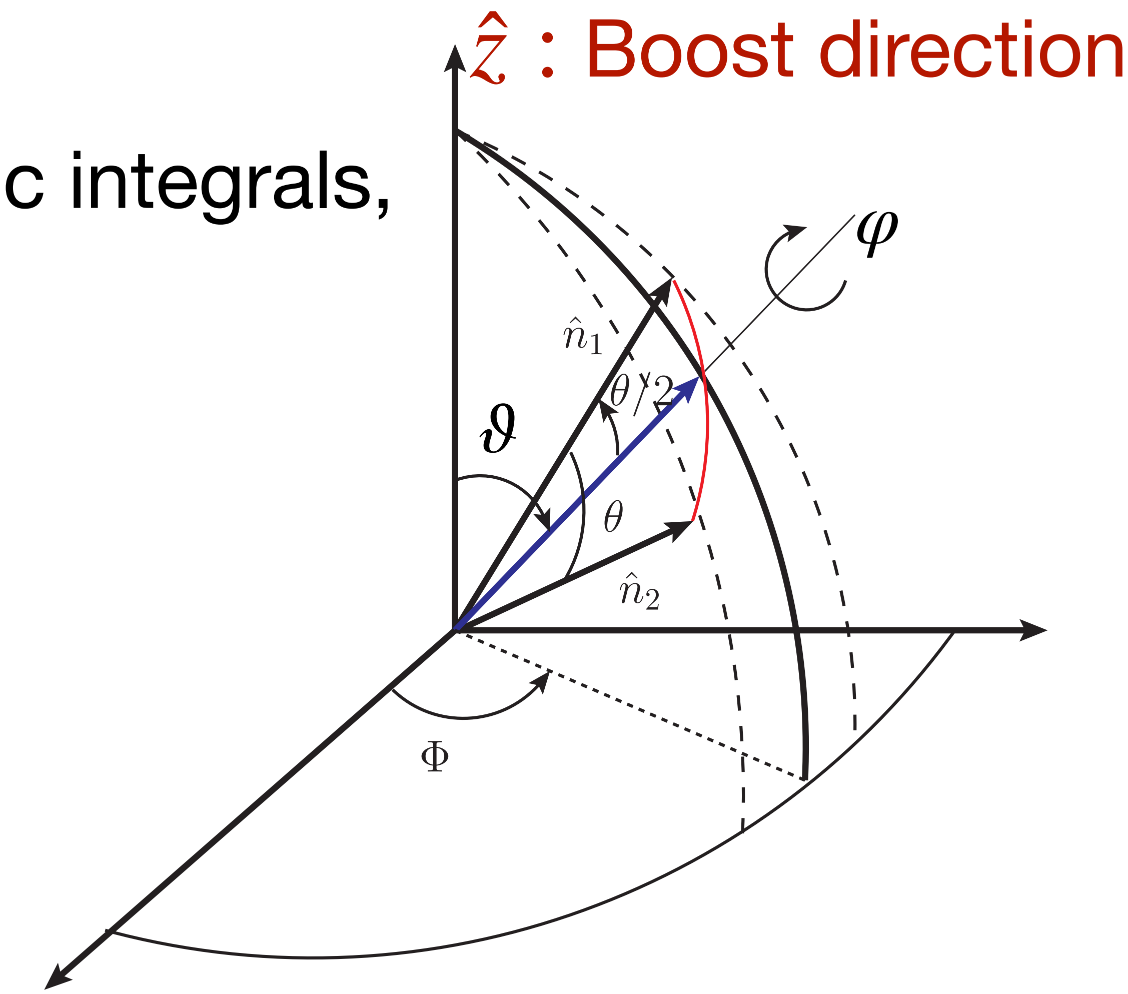


[figure adopted from Riembau, Son `25]

Extension to Vector Bosons

$$\mathcal{K}^{[w]}_{\gamma}(z, z') = \frac{1}{\gamma^2} \int \frac{d^2\Omega}{4\pi} \frac{z}{z'} \delta\left(z' - \frac{z}{\lambda(\vec{n}_1)\lambda(\vec{n}_2)}\right) w(\varphi, \vartheta)$$

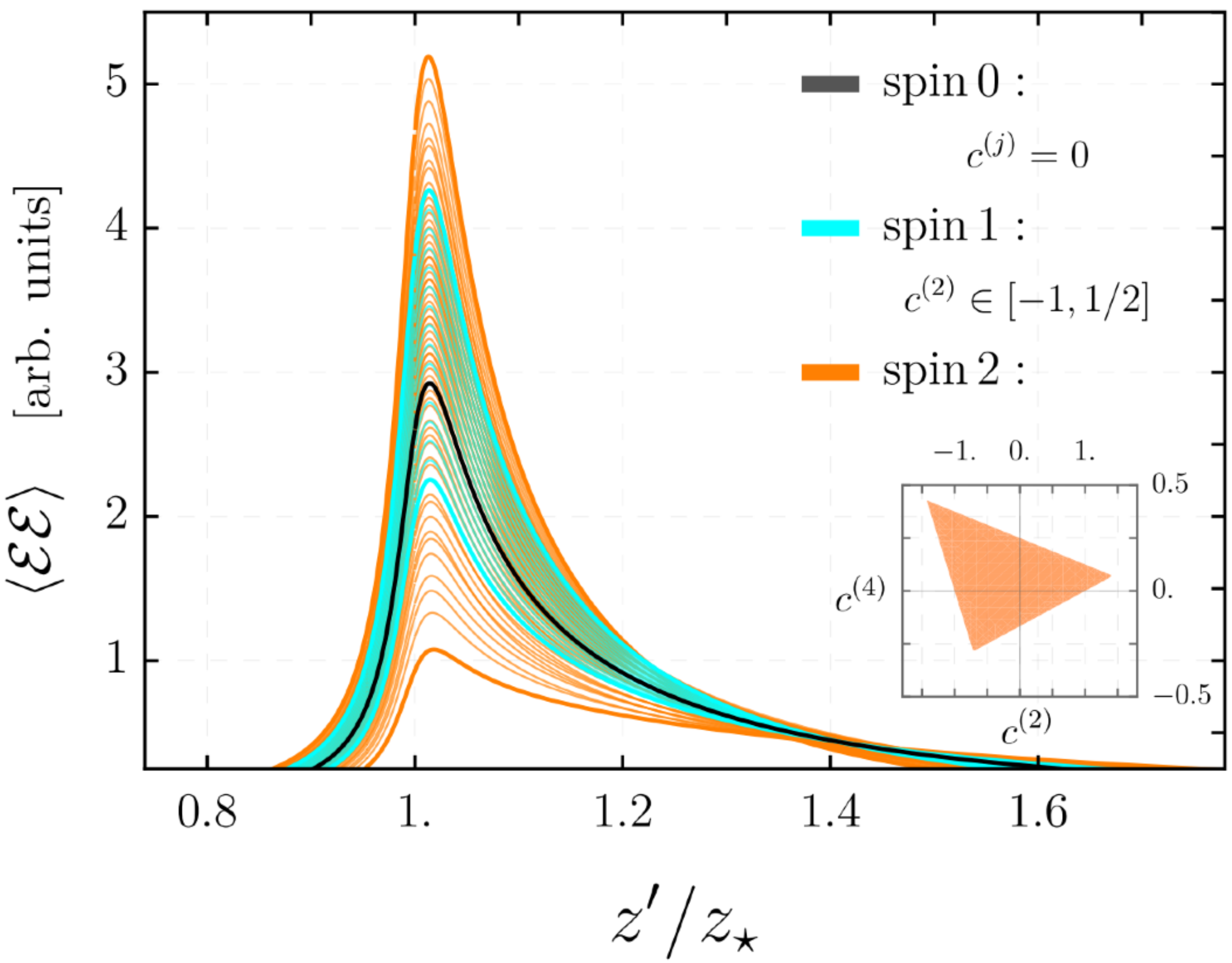
- Need three kinds of complete elliptic integrals,
- and Hyperelliptic integrals

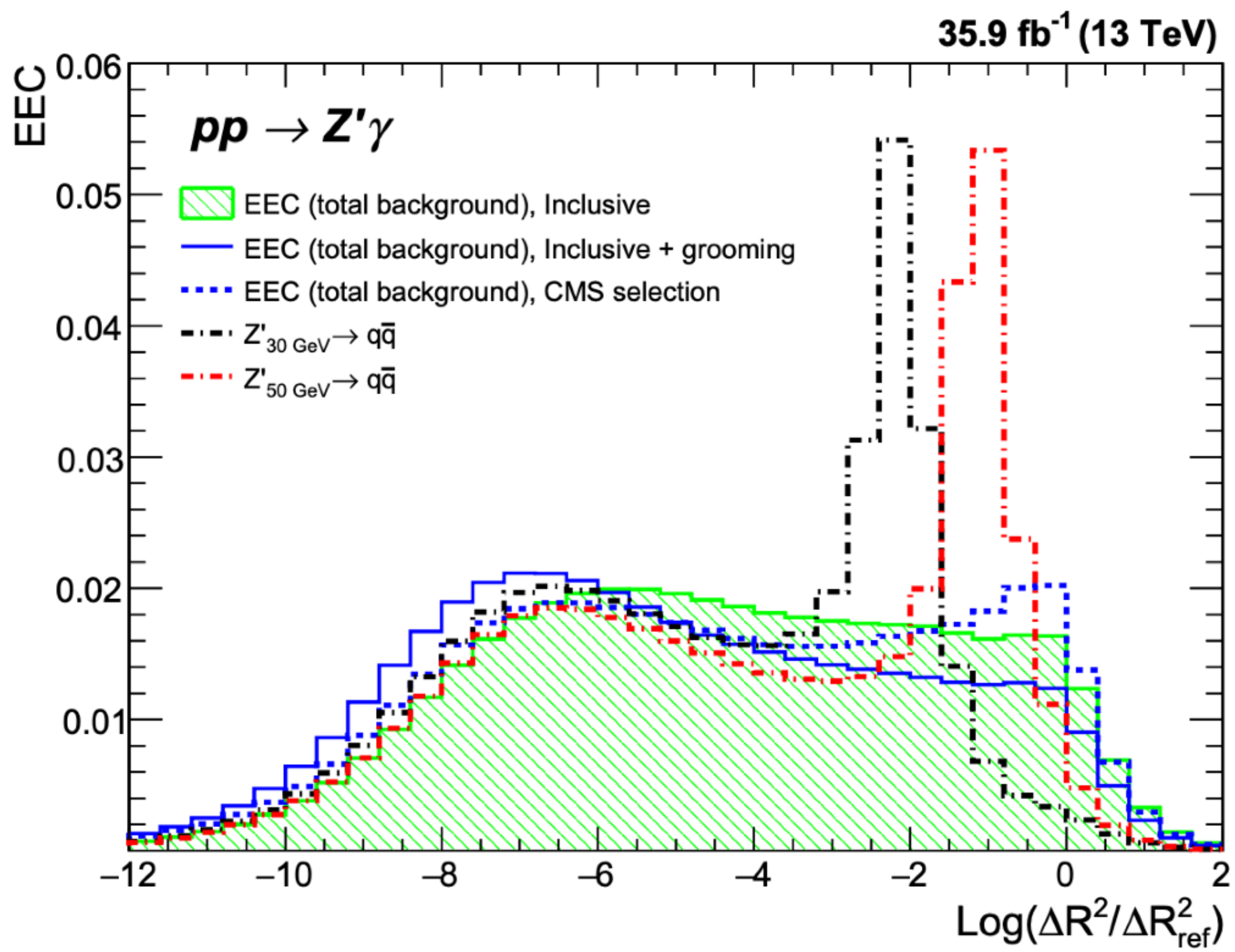


[figure adopted from Riembau, Son `25]

Extension to Vector Bosons

- Search for new physics?





[Ricci, Riembau, Son `26]

Extension to Multi-Point Correlator

N-point energy correlator depends on $2N$–3 independent angular variables

$$\mathcal{I}_N \equiv \{12, 23, \ldots, N\!-\!1\,N\} \cup \{13, 14, \ldots, 1N\}$$

$$z_{ij} = \frac{1-\cos\theta_{ij}}{2}\,, \qquad z'_{ij} = \frac{1-\cos\theta'_{ij}}{2} = \frac{z_{ij}}{\lambda(\vec{n}_i)\lambda(\vec{n}_j)}$$

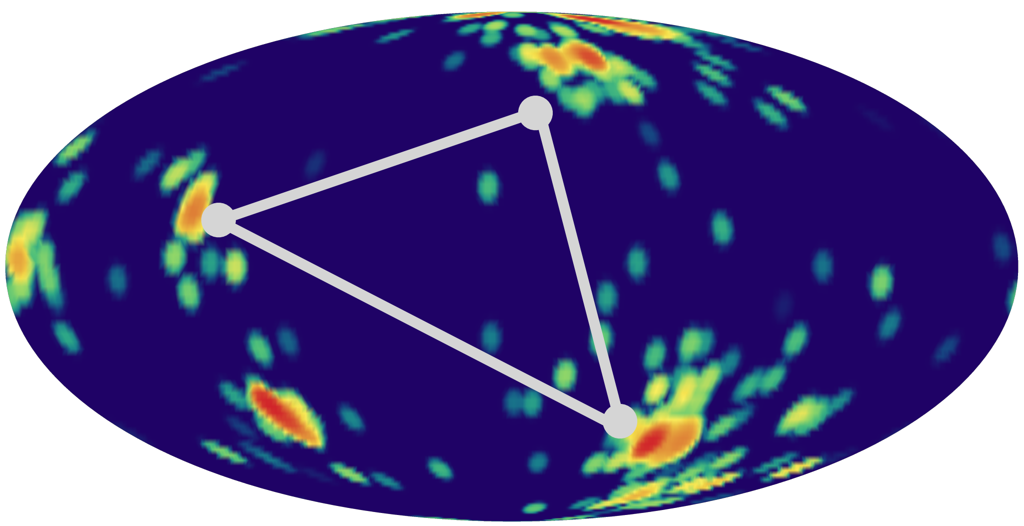
Boost delta function

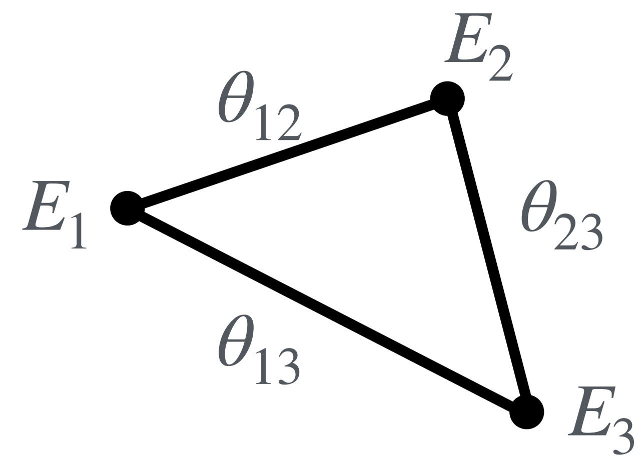
$$\hat{\delta}_{2N-3} \equiv \prod_{ij\in\mathcal{I}_N} \delta\left(z'_{ij} - \frac{z_{ij}}{\lambda(\vec{n}_i)\lambda(\vec{n}_j)}\right)$$

Boost kernel

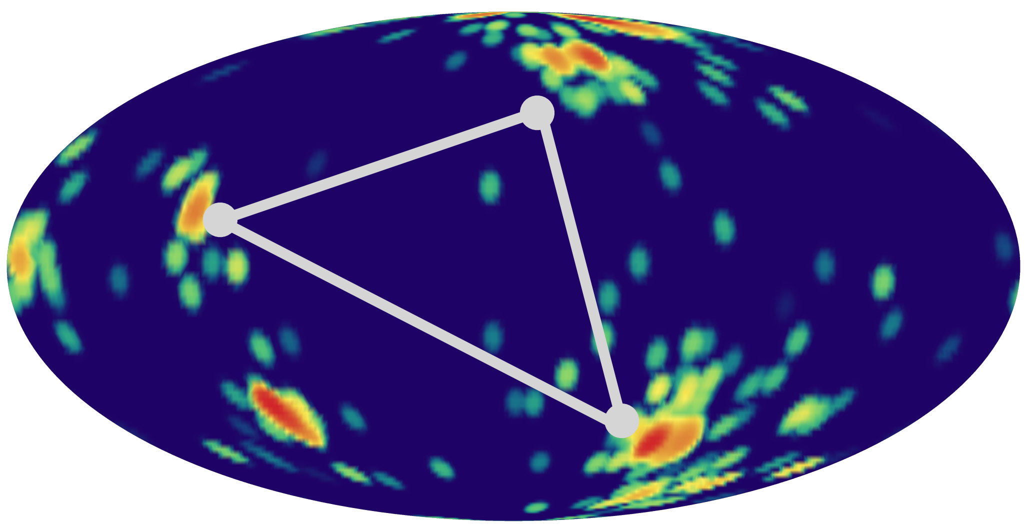
$$\mathcal{K}^N_\gamma(\{z_{ij}\},\{z'_{ij}\}) = \frac{1}{\gamma^2}\int \frac{d^2\Omega}{4\pi}\prod_{i=1}^{N}\lambda(\vec{n}_i)\,\hat{\delta}_{2N-3}$$

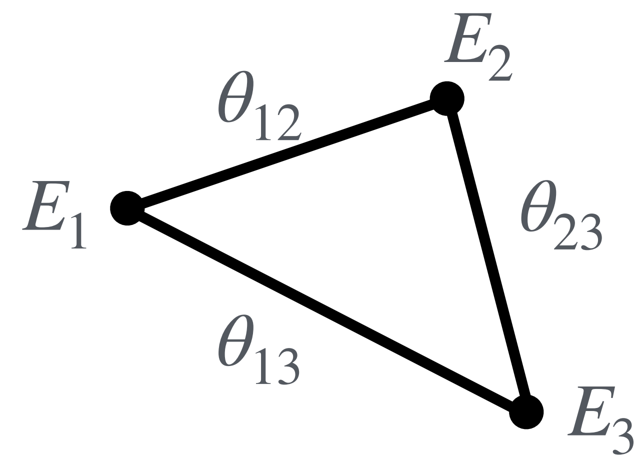
Three-Point Correlator (EEEC)





Three-Point Correlator (EEEC)





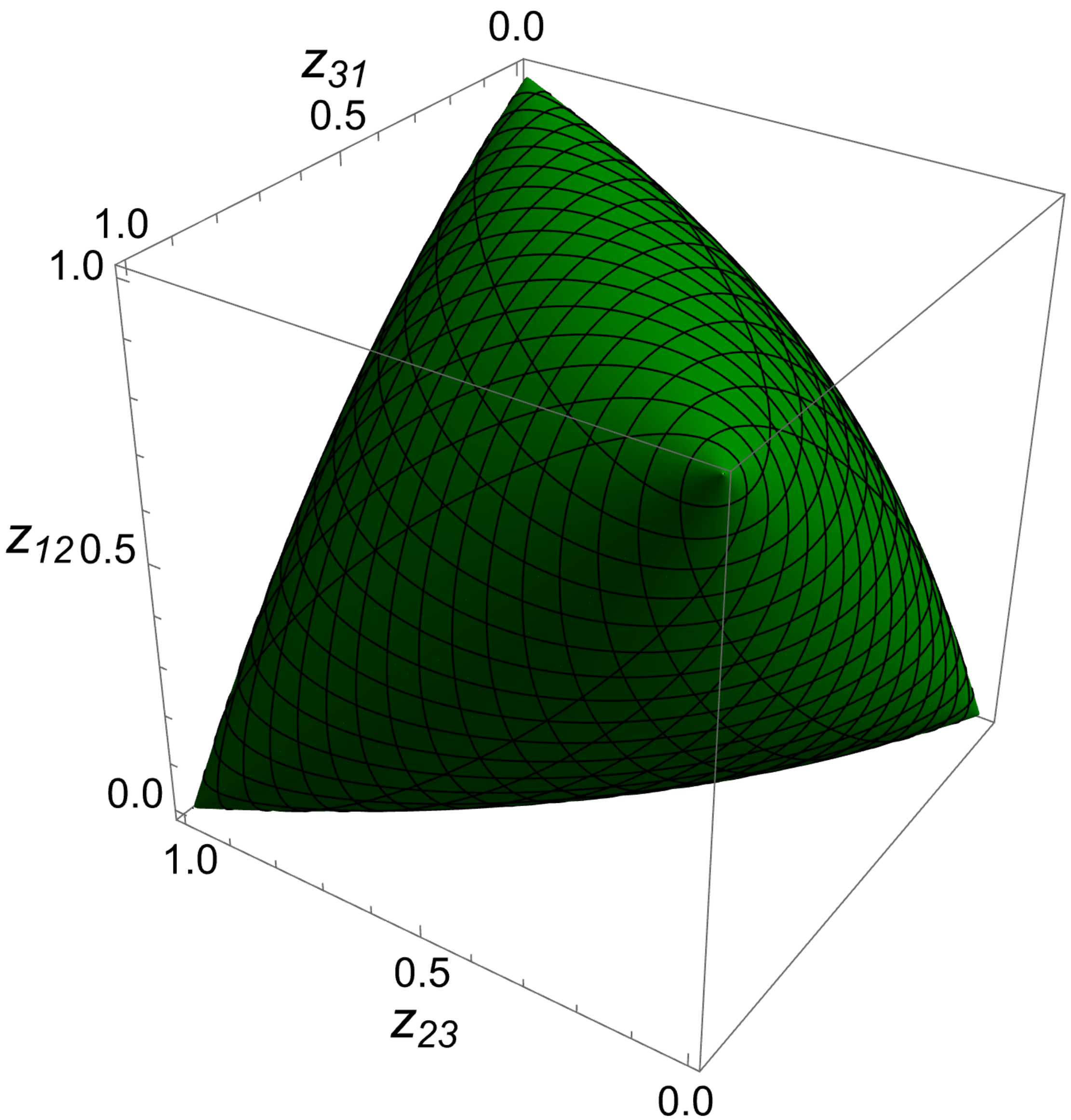
$$\frac{d\sigma}{dz_{12}dz_{23}dz_{31}} \equiv \sum_m \sum_{1\leq i_1,\cdots i_3 \leq m} \int d\sigma_m \times \prod_{1\leq k\leq 3} \frac{E_{i_k}}{Q} \prod_{1\leq j<l\leq 3} \delta\left(z_{jl} - \frac{1-\cos\theta_{i_j i_l}}{2}\right)$$

Coordinate of the EEEC

[Yang, Zhang `22]

$$z_{ij} = \frac{1-\cos\theta_{ij}}{2} = \sin^2\frac{\theta_{ij}}{2}$$

- Allowed region: Gram determinant $\det\begin{pmatrix} 1 & \cos\theta_{12} & \cos\theta_{13} \\ \cos\theta_{12} & 1 & \cos\theta_{23} \\ \cos\theta_{13} & \cos\theta_{23} & 1 \end{pmatrix} \geq 0$



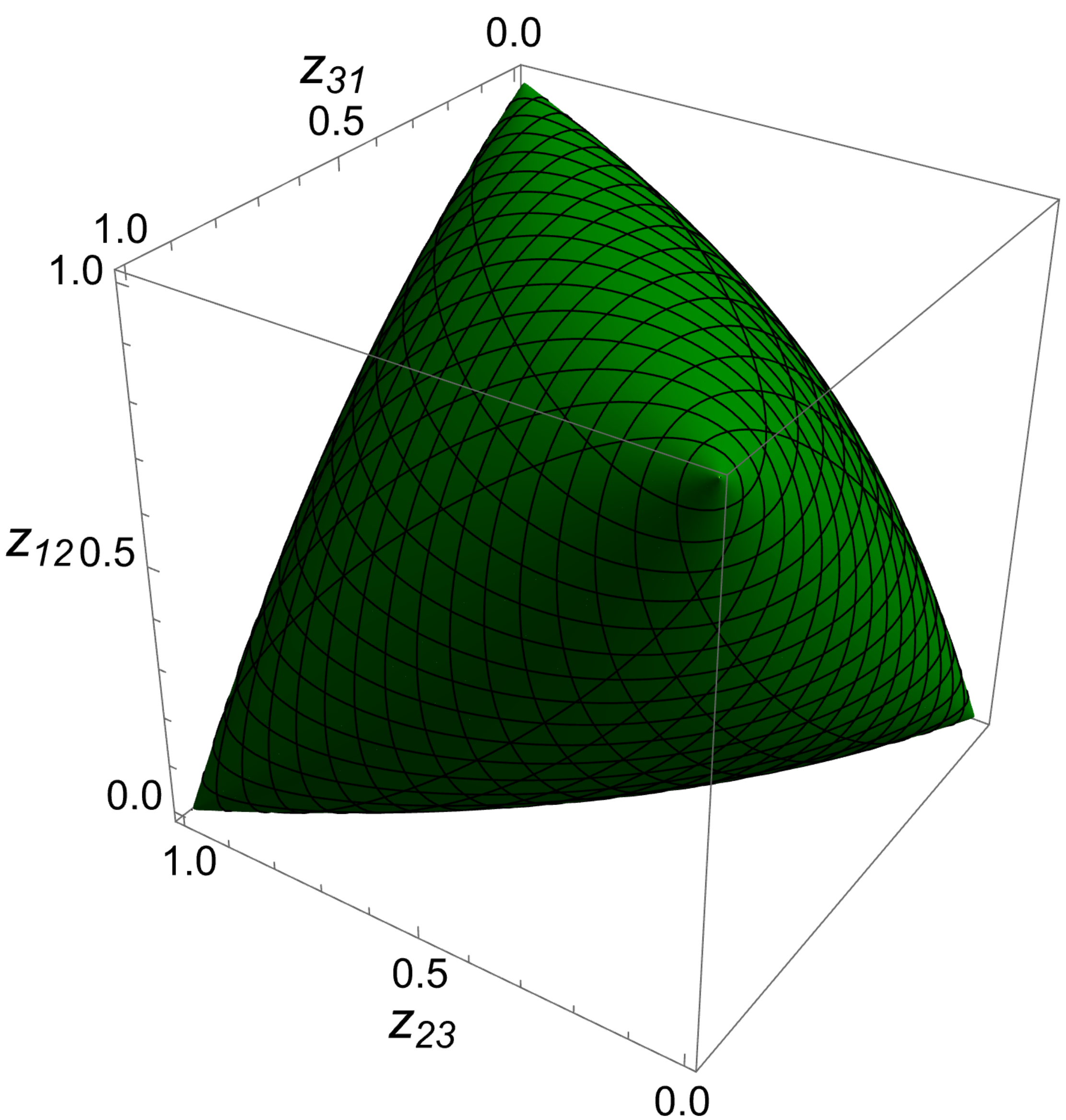
Coordinate of the EEEC

[Yang, Zhang `22]

$$z_{ij} = \frac{1 - \cos\theta_{ij}}{2} = \sin^2\frac{\theta_{ij}}{2}$$

- Allowed region: Gram determinant

$$\det\begin{pmatrix} 1 & \cos\theta_{12} & \cos\theta_{13} \\ \cos\theta_{12} & 1 & \cos\theta_{23} \\ \cos\theta_{13} & \cos\theta_{23} & 1 \end{pmatrix} \geq 0$$



粽子 zong-zi

Celestial Parametrization for the EEEC

$$z_{ij} = \sin^2 \frac{\theta_{ij}}{2}$$

$$z_{23} = \kappa(s)\ \sin^2 \frac{\phi_1}{2}\,, \quad z_{31} = \kappa(s)\ \sin^2 \frac{\phi_2}{2}\,, \quad z_{12} = \kappa(s)\ \sin^2 \frac{\phi_1 + \phi_2}{2}$$

$$\kappa(s) = \frac{4s}{(1+s)^2}$$

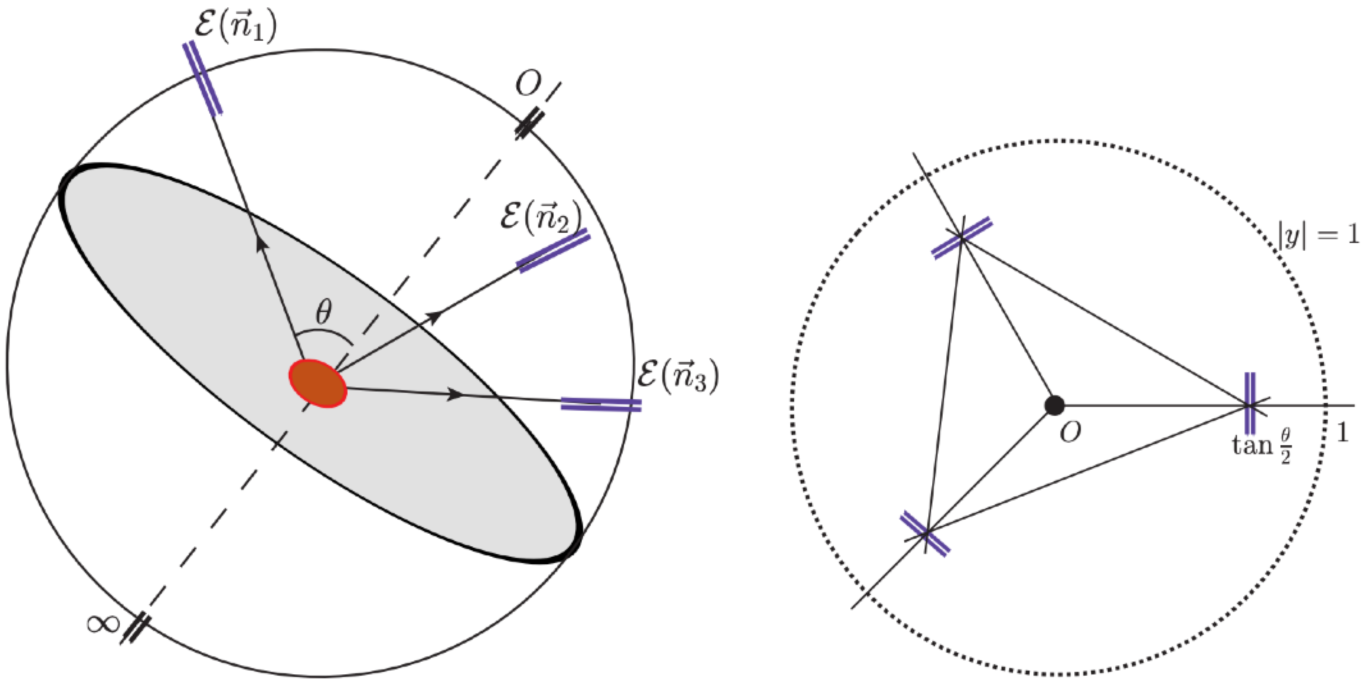
Celestial Parametrization for the EEEC

$$z_{ij} = \sin^2 \frac{\theta_{ij}}{2}$$

$$z_{23} = \kappa(s)\ \sin^2 \frac{\phi_1}{2}, \quad z_{31} = \kappa(s)\ \sin^2 \frac{\phi_2}{2}, \quad z_{12} = \kappa(s)\ \sin^2 \frac{\phi_1 + \phi_2}{2}$$

$$\kappa(s) = \frac{4s}{(1+s)^2}$$

$$z_{ij} = \sin^2 \frac{\theta_{ij}}{2} = \frac{|y_i - y_j|^2}{(1+|y_i|^2)(1+|y_j|^2)}$$

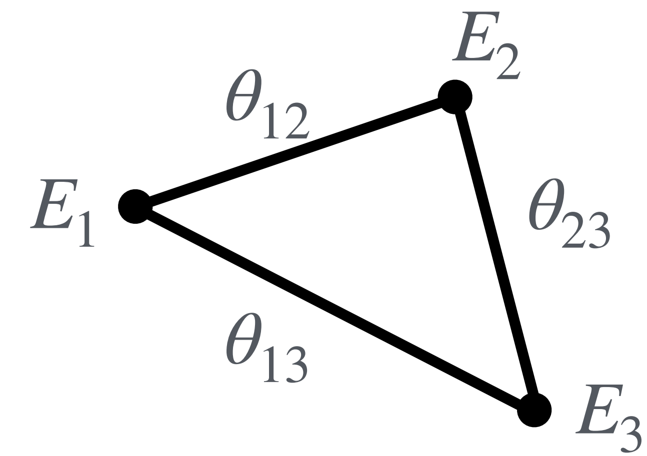


$$y_1 = \sqrt{s}e^{i\phi_1}, \quad y_2 = \sqrt{s}e^{i(\phi_1+\phi_2)}, \quad y_3 = \sqrt{s}$$

$$\{\theta_{12}, \theta_{23}, \theta_{13}\} \to \{y_1, y_2, y_3\} \to \{\phi_1, \phi_2, s\}$$

Boosting EEEC

$$K_\gamma^{(3)}(\{z_{ij}\},\{z'_{ij}\}) = \frac{\sqrt{\Delta_G}}{16\pi\gamma^2\sqrt{\gamma^2-1}}\,\frac{z_{12}z_{23}z_{31}}{z'_{12}z'_{23}z'_{31}}\delta(\mathcal{P})$$



- One delta function left
- $\mathscr{P}$ symmetric under $z_{ij} \leftrightarrow z'_{ij}$
- Boost $(s, \phi_1, \phi_2) \to (s', \phi'_1, \phi'_2)$, rewrite $\mathcal{K}_\gamma^{(3)}(s,\phi_1,\phi_2;s',\phi'_1,\phi'_2)$

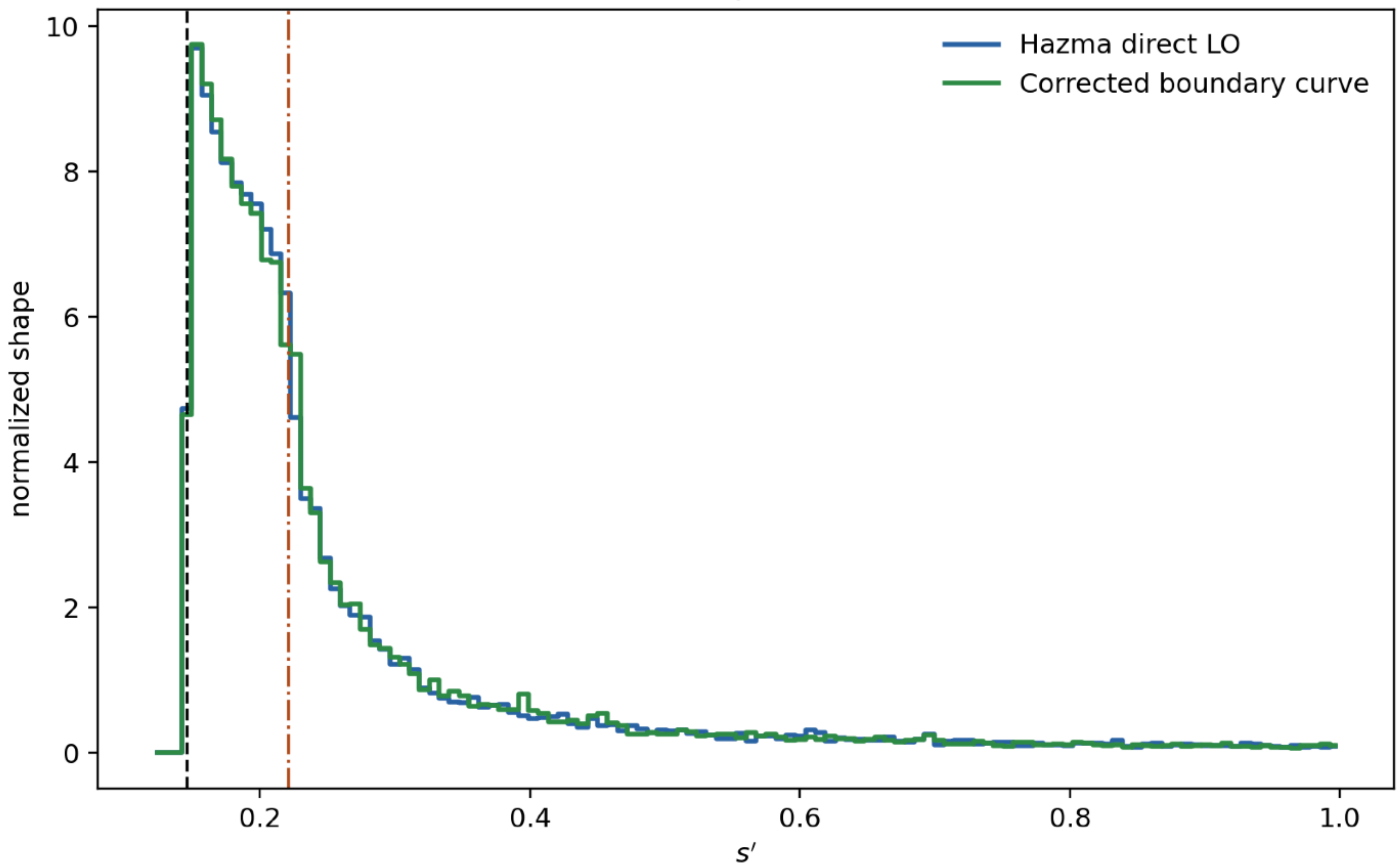
$$\begin{aligned}\mathcal{P} = {} & (z_{12}z_{23}z_{31})\,Q(z'_{12},z'_{23},z'_{31}) + (z'_{12}z'_{23}z'_{31})\,Q(z_{12},z_{23},z_{31}) \\ & - 4(\gamma^2-1)z_{12}z_{23}z_{31}z'_{12}z'_{23}z'_{31} + (d_1d_2+d_2d_3+d_3d_1) \\ & + 2\gamma\sqrt{z_{12}z_{23}z_{31}z'_{12}z'_{23}z'_{31}}\Big[(z_{12}+z_{23}+z_{31})(z'_{12}+z'_{23}+z'_{31}) \\ & \qquad - 2(z_{12}z'_{12}+z_{23}z'_{23}+z_{31}z'_{31})\Big],\end{aligned}$$

$$Q(z_{12},z_{23},z_{31}) = z_{12}^2+z_{23}^2+z_{31}^2-2z_{12}z_{23}-2z_{23}z_{31}-2z_{31}z_{12},$$

$$d_1 = z_{23}z'_{31}-z_{31}z'_{23}, \qquad d_2 = z_{31}z'_{12}-z_{12}z'_{31}, \qquad d_3 = z_{12}z'_{23}-z_{23}z'_{12}.$$

$$\Delta_G = -4(z_{12}^2+z_{23}^2+z_{31}^2)+8(z_{12}z_{23}+z_{23}z_{31}+z_{31}z_{12})-16z_{12}z_{23}z_{31}$$

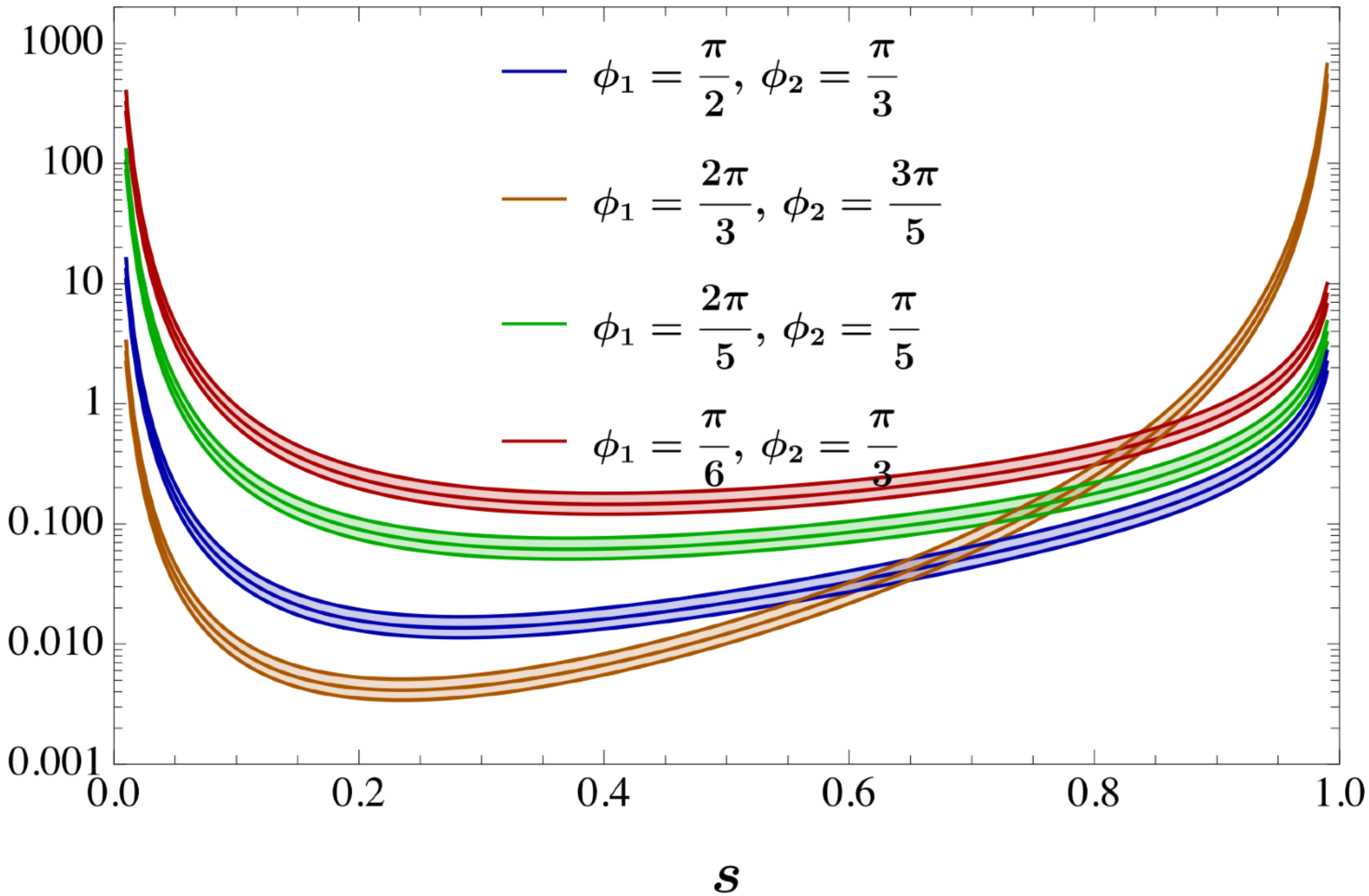
Boosted LO EEEC



- Plug in $\delta(1-s)f(\phi_1, \phi_2)$ and boost
- Fixed ϕ_1', ϕ_2' (e.g., $= 2\pi/3$)
- Lower bound on $s'(\gamma) = (\gamma - \sqrt{\gamma^2 - 1})^2 = 0.1459$ for $\gamma = 1.5$

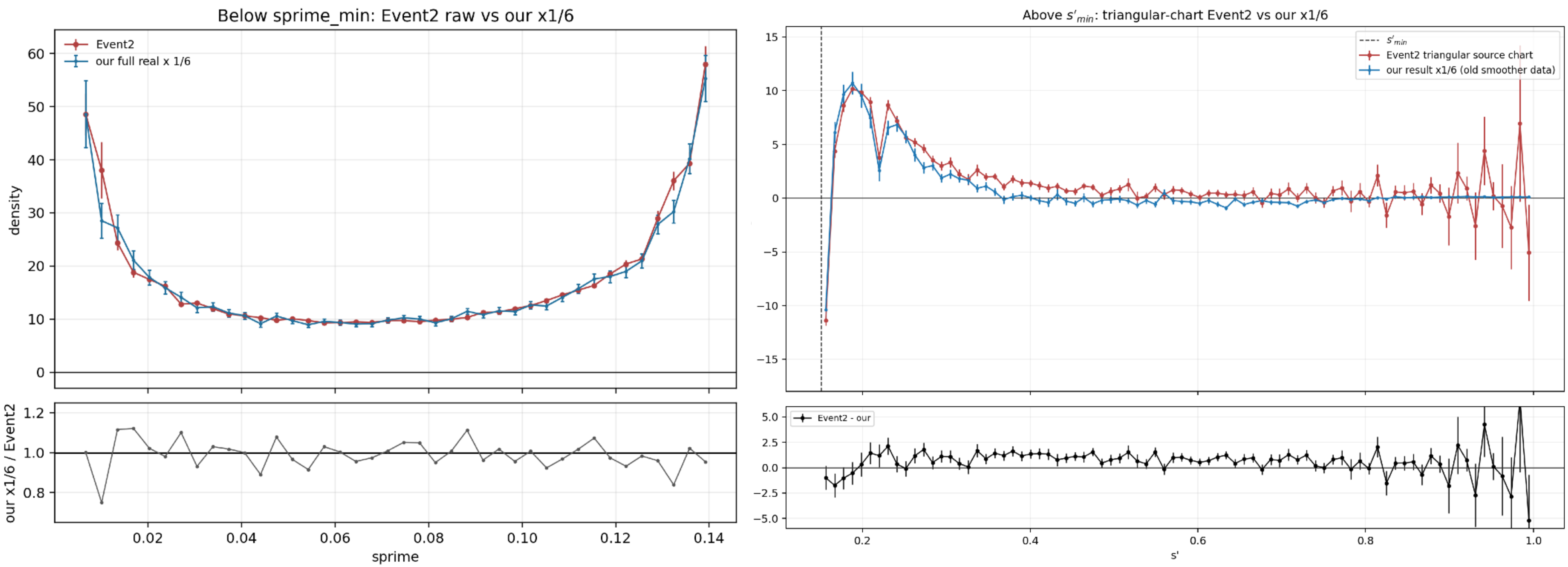
Boosted NLO EEEC

- In the rest frame [Yang, Zhang `24]



Boosted NLO EEEC

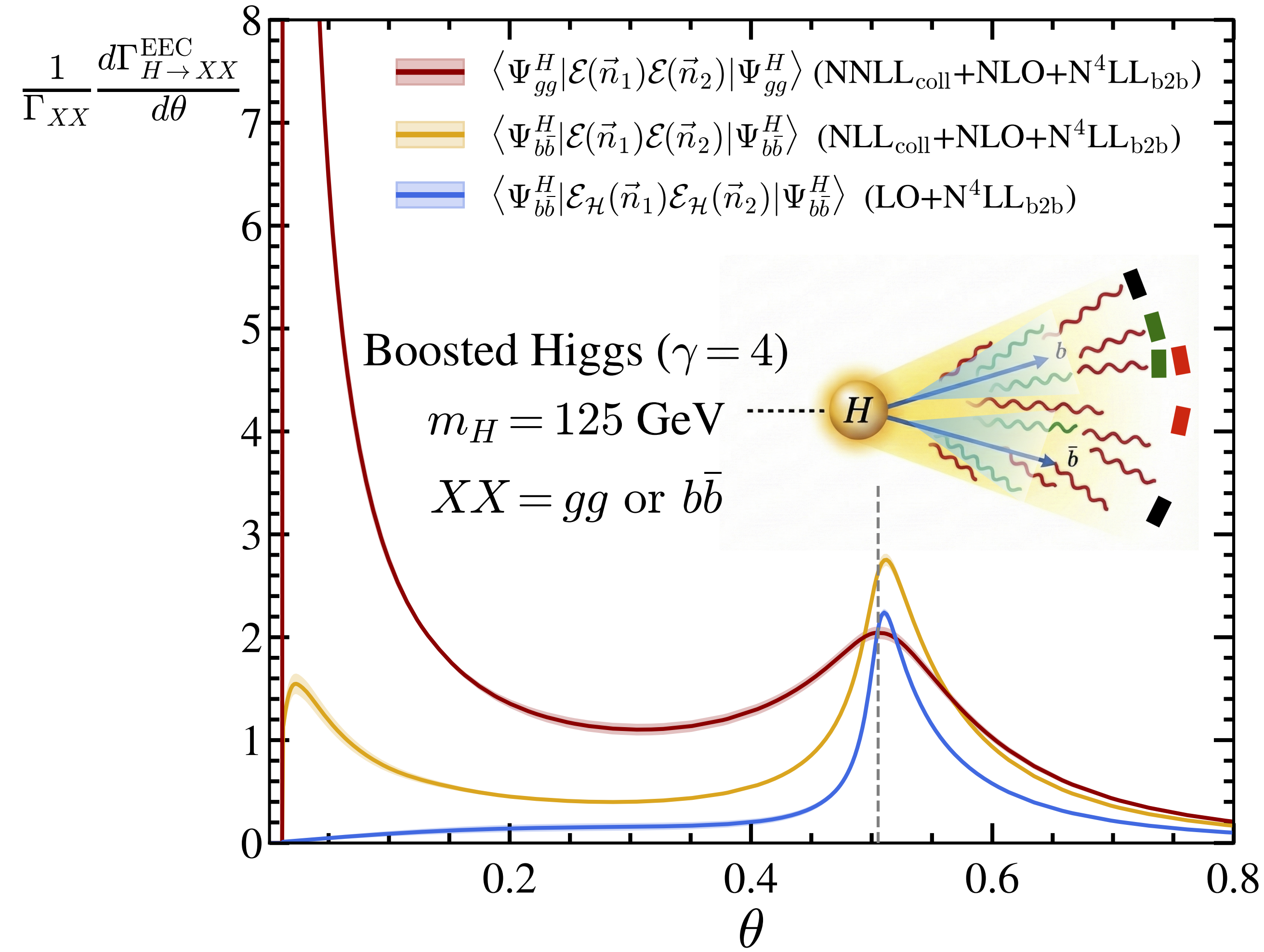
- After boost ($\gamma = 1.5$, $\phi_1' = \phi_2' = \frac{2\pi}{3}$, $s_*' = (\gamma - \sqrt{\gamma^2 - 1})^2 = 0.1459$)



- Need to work out mismatch (bug?) for $s' > s_*'$

- Need to include resummation at $s \to 1$ ($s' \sim s_*'$) [**AG**, Yang, Zhang `24]

Conclusions



- Higgs EEC
  - Boost kernel
  - Rest-frame Higgs EEC
  - Boosted Higgs EEC
- Boosting EEC for vector bosons (Z/W)
- How to boost for multi-point energy correlators