Using collider tools to compute loop corrections to the equation of state of neutron stars

Samuel Abreu (CERN & The University of Edinburgh)

— ongoing work with Max Delto (CERN) and Aleksi Kurkela (Stavanger)

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Outline

- Introduction
- Aside: QCD corrections to the ρ parameter
- Loop corrections to the EOS of neutron stars
- Summary and Outlook

Introduction

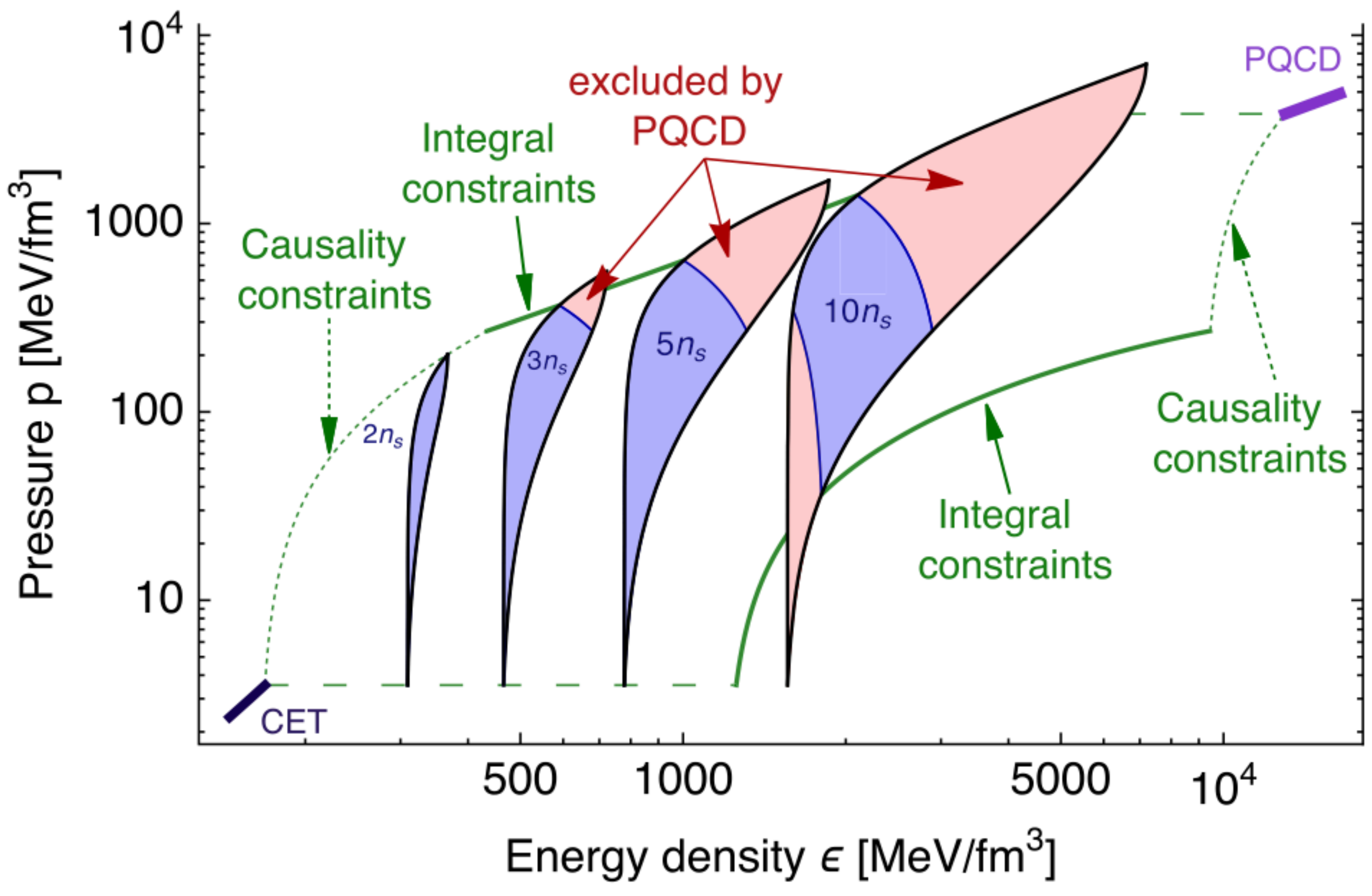
- Macroscopic properties of neutron stars coupled to microscopic properties of matter

$$R(M) \Leftrightarrow \text{EoS}$$

- Perturbative QCD imposes strong constraints on the EoS of neutron stars

- Consider EoS as:
  - function of chemical potential μ (instead of energy density)
  - at zero temperature, large density

- **Goal**: compute pressure p as function of μ, $p(\mu)$ at $T \to 0$



[Komoltsev, Kurkela, 2022]

Introduction — $p(\mu)$ and pQCD

- Recap: from Langrangian to EoS

$$\mathscr{L} = \mathscr{L}_{QCD} + \mu\ \bar{\psi}\gamma_0\psi \qquad \Omega(\mu, m) = -\ln Z \qquad p(\mu, m) = -\frac{\Omega(\mu, m)}{V}$$

- Perturbative expansion:

$$p = 1 + \alpha_s p_1^h + \alpha_s^2 p_2^h + \alpha_s^3 p_3^h$$

Hard contributions: loop integrals

$$+\alpha_s^2 p_2^s + \alpha_s^3 p_3^s$$

Soft contributions: resummation ✓ $p \sim g\mu$

$$+\alpha_s^3 p_3^m$$

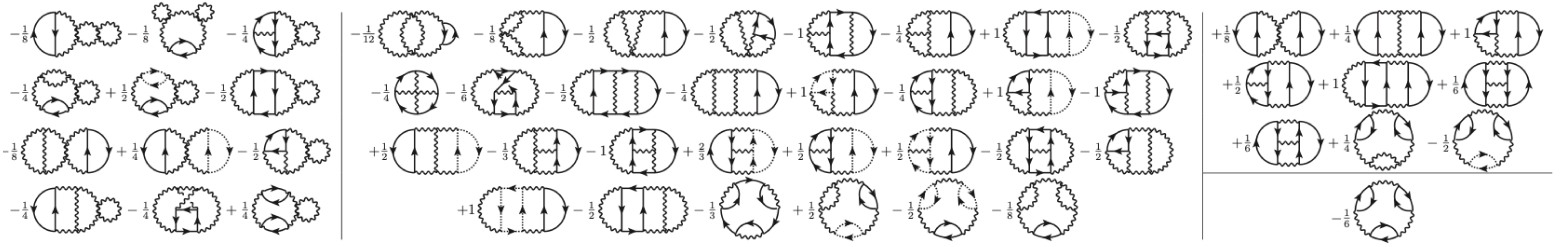
Mixed contributions ✓

[Gorda, Kurkela et al. 2018, 2021, 2023
Gorda, Paatelainen et al. 2023]

Introduction — α_s^3 corrections to EoS

$$p = 1 + \alpha_s p_1^h + \alpha_s^2 p_2^h + \alpha_s^3 p_3^h$$

- Calculation of four-loop Feynman integrals, with no external legs



- Some steps have been taken, but so far unknown:
  - very promising approach based on Loop-Tree-Duality
  - **Goal of this talk**: use tools from collider physics (where $\mu = 0$)

[Navarrete, Paatelainen, Seppänen, 2024]

[Kärkkäinen, Navarrete, Nurmela et al, 2025]

Aside: QCD corrections to ρ parameter

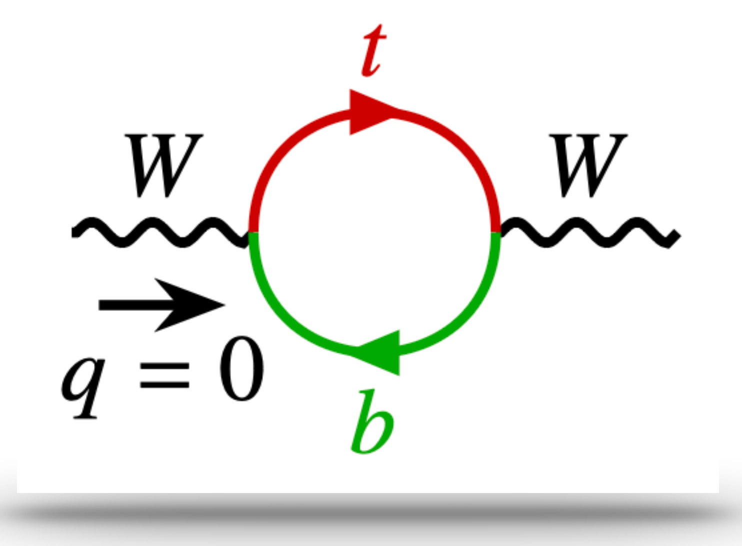
Aside: QCD corrections to ρ parameter

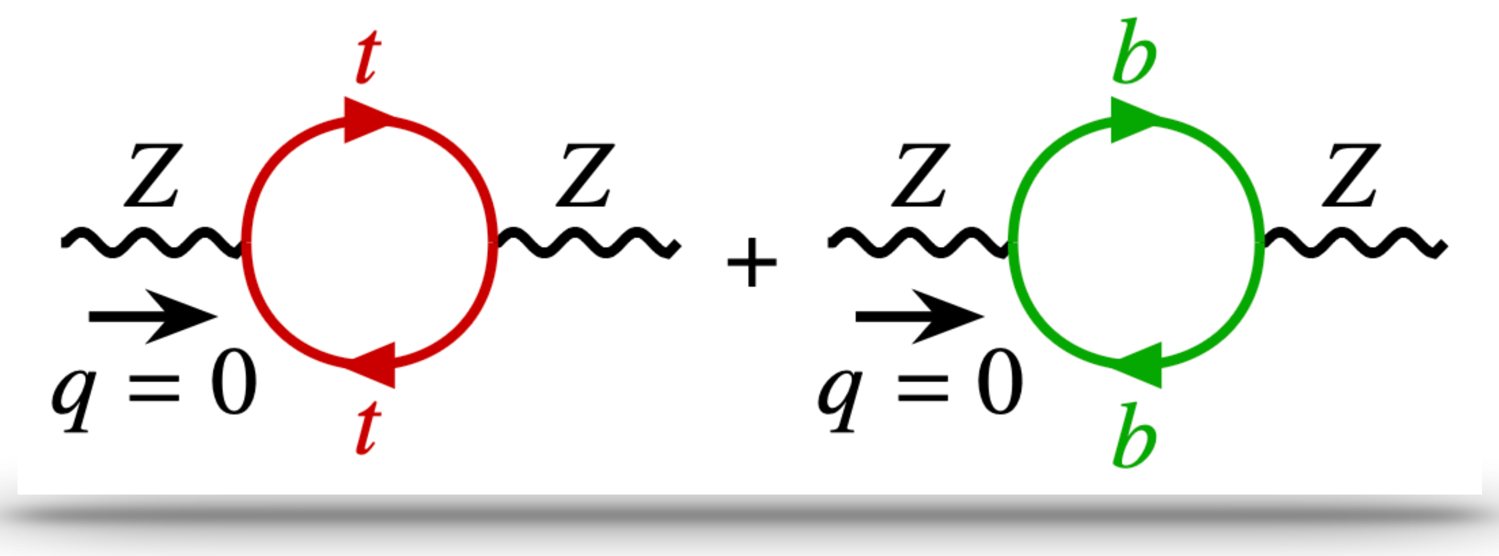
- Important precision observable in the Standard Model

[Ross, Veltmann 1975]

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 + \Delta\rho$$

- Related to W and Z self energies at zero momentum: $\Delta\rho = \frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2}$





- Function of two variables (if $m_b \neq 0$, one otherwise)

Aside: QCD corrections to ρ parameter

State of the art

Long history of higher-order corrections to the ρ parameter in the Standard Model

$O(\alpha)$	$O(\alpha\alpha_s)$	$O(\alpha\alpha_s^2)$	$O(\alpha\alpha_s^3)$
[Veltman '77]	[Djouadi, Vezegnassi '87] [Djouadi '87] [Kniehl et al. '88] [Kniehl '90]	$m_b=0$: [Anselm et al. '93] [Avdeev et al. '94] [Chetyrkin et al. '95] $m_b\neq0$: [Grigo et al. '12] [Blümlein et al. '18] [Abreu et al. '19]	$m_b=0$: [Steinhauser, Schröder '05] [Chetyrkin et al. '06] [Czakon, Boughezal '06]

$O(\alpha^2)$	$O(\alpha^2\alpha_s)$
[van der Bij, Hoogeveen '87] [Barbieri et al. '92] [Fleischer et al. '93]	[Faist et al. '03]

$O(\alpha^3)$
[van der Bij et al. '01] [Faist et al. '03]

Our goal: $O(\alpha\alpha_s^3)$ with $m_b\neq0$

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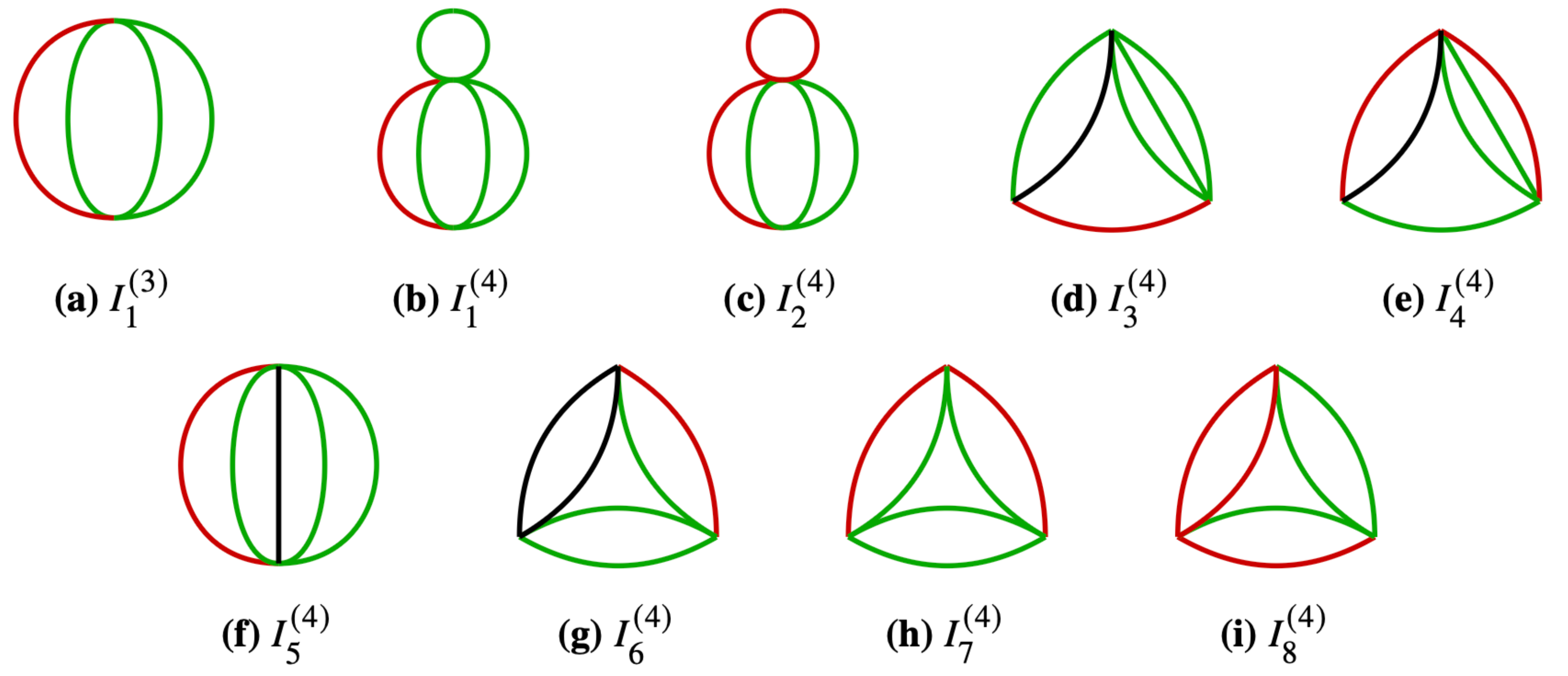
Talk by Arnd Behring at RadCor 23

- Calculation of four-loop Feynman integrals, with no external legs!

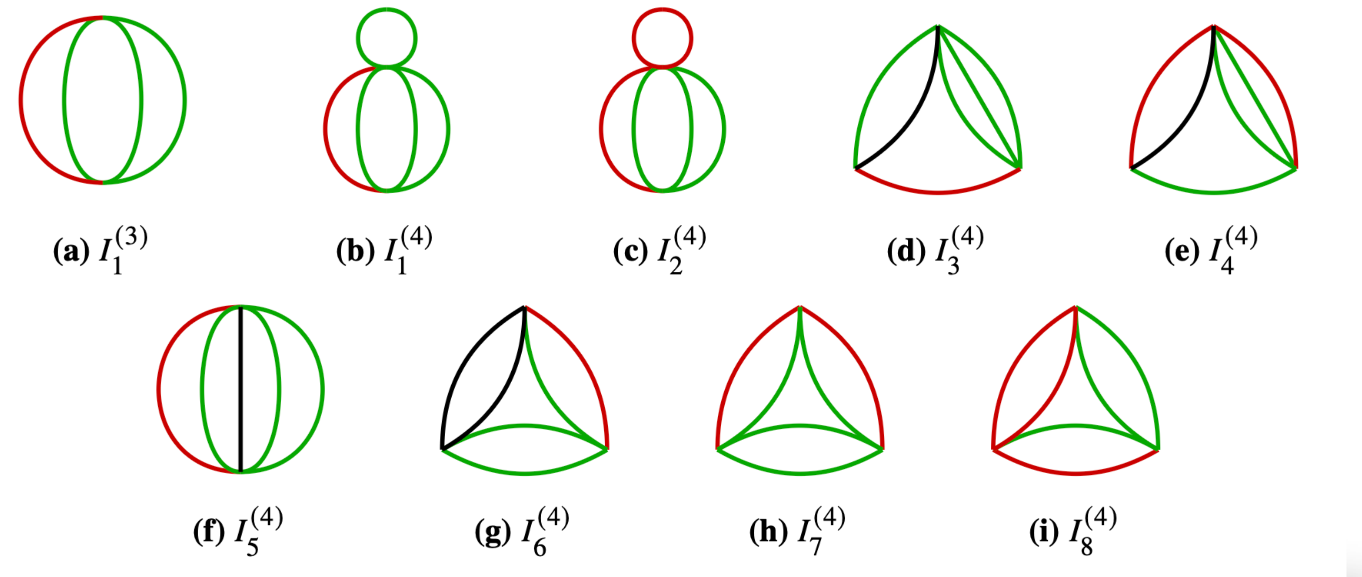


[Abreu, Behring, McLeod, Page, 24]

- Trivial to get number out, explore space of functions:
  - elliptic functions and generalisations

Aside: QCD corrections to ρ parameter

- Why is it trivial to get numbers out?
  - Integration-by-parts (IBP) relations: only compute small number of integrals
  - Differential equations: automates the calculation of integral at one point



- Feynman integrals and IBP relations:

[F. Tkachov 1981; K. Chetyrkin, F. Tkachov, 1981 ; Laporta, 2000]

$$I(p_1,\ldots,p_E;m_1^2,\ldots,m_p^2;\nu;D) = \int \left(\prod_{j=1}^{L} d^D k_j\right) \frac{\mathcal{N}(\{k_j\cdot k_l, k_j\cdot p_l\};D)}{\prod_{j=1}^{p}(m_j^2 - q_j^2 - i\varepsilon)^{\nu_j}}$$

$$\int d^D k_i \frac{\partial}{\partial k_i^\mu}\left[v^\mu \frac{\mathcal{N}(\{k_j\cdot k_l, k_j\cdot p_l\};D)}{\prod_{j=1}^{p}(m_j^2 - q_j^2 - i\varepsilon)^{\nu_j}}\right] = 0$$

  - Systematically build (linear) relations between integrals: drastic reduction in number of integrals
  - Compute basis of space (master integrals)
  - Behind any current loop calculation: automated in many public codes (fire, kira, reduze2, ...)

Aside: QCD corrections to ρ parameter

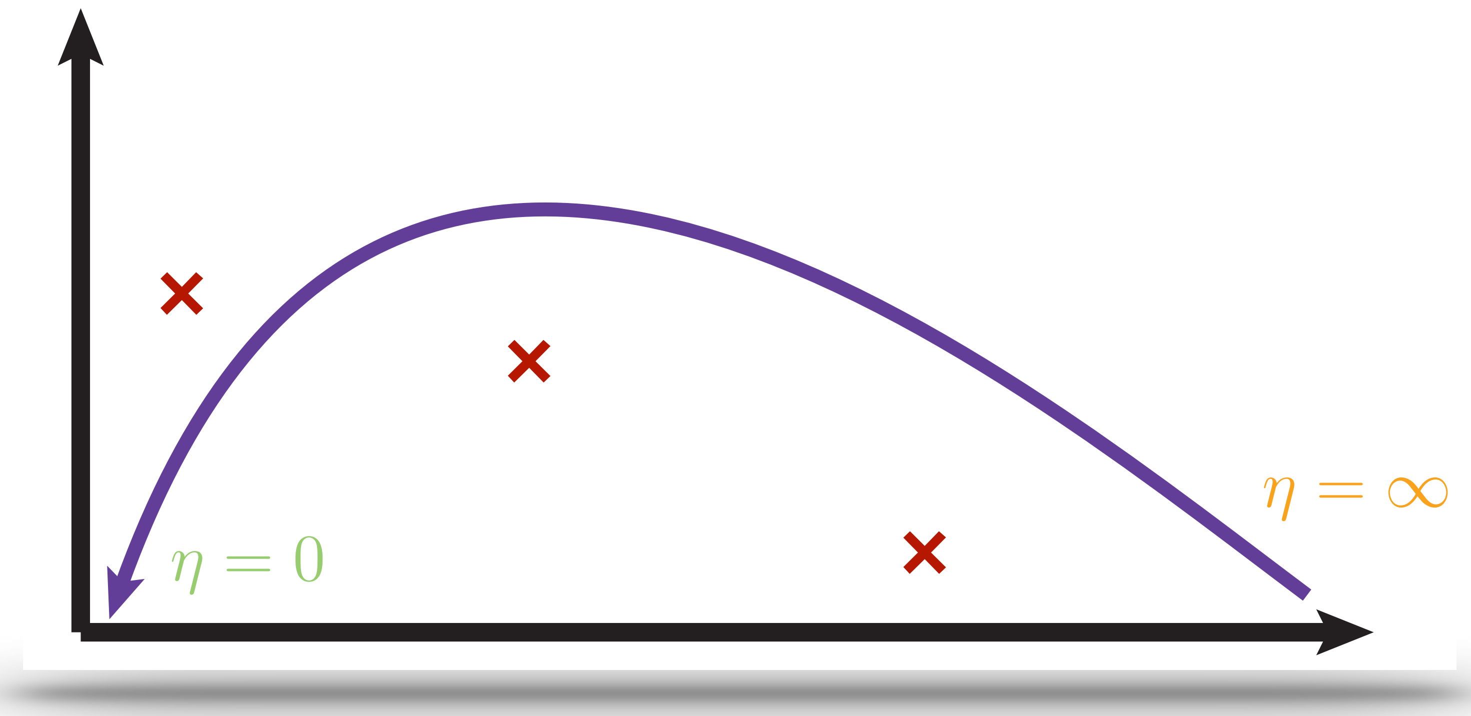
- Differential equations for Feynman integrals $d\vec{J} = M \cdot \vec{J}$
  - Master integrals are closed under differentiation
  - Compute them by solving system of first order differential equations

[A.V. Kotikov 1991]
[Z. Bern, L. Dixon, D. Kosower 1994]
[E. Remiddi 1997 ; E. Remiddi, T. Ghermann 2000]

- AMFlow approach

[X. Liu, Y. Ma 22]

  - Introduce fake mass η in all propagators, and take $\eta \to \infty$
  - Use DE in η to evolve down to $\eta = 0$, avoiding all singularities
  - Numerical evaluation at a single point to very high precision
  - ideal for numbers/one-variable functions!



Loop corrections to the EoS of neutron stars

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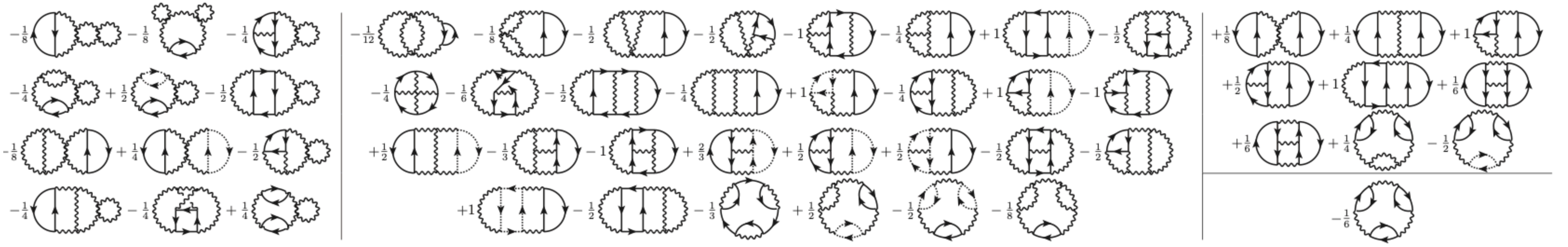
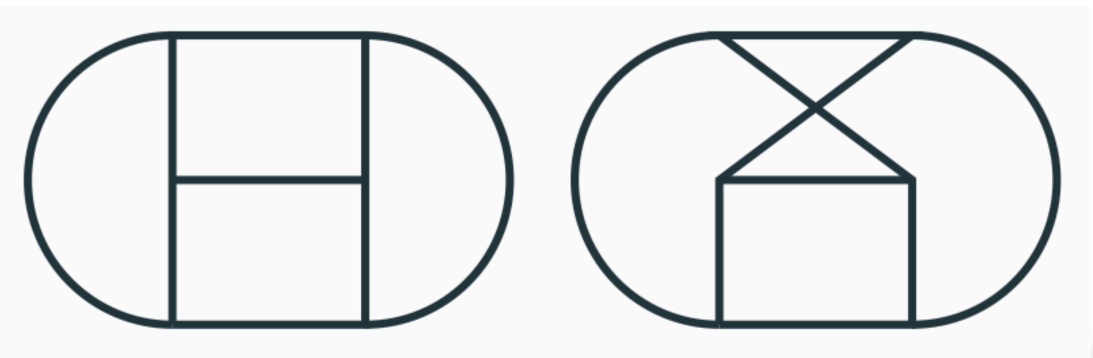
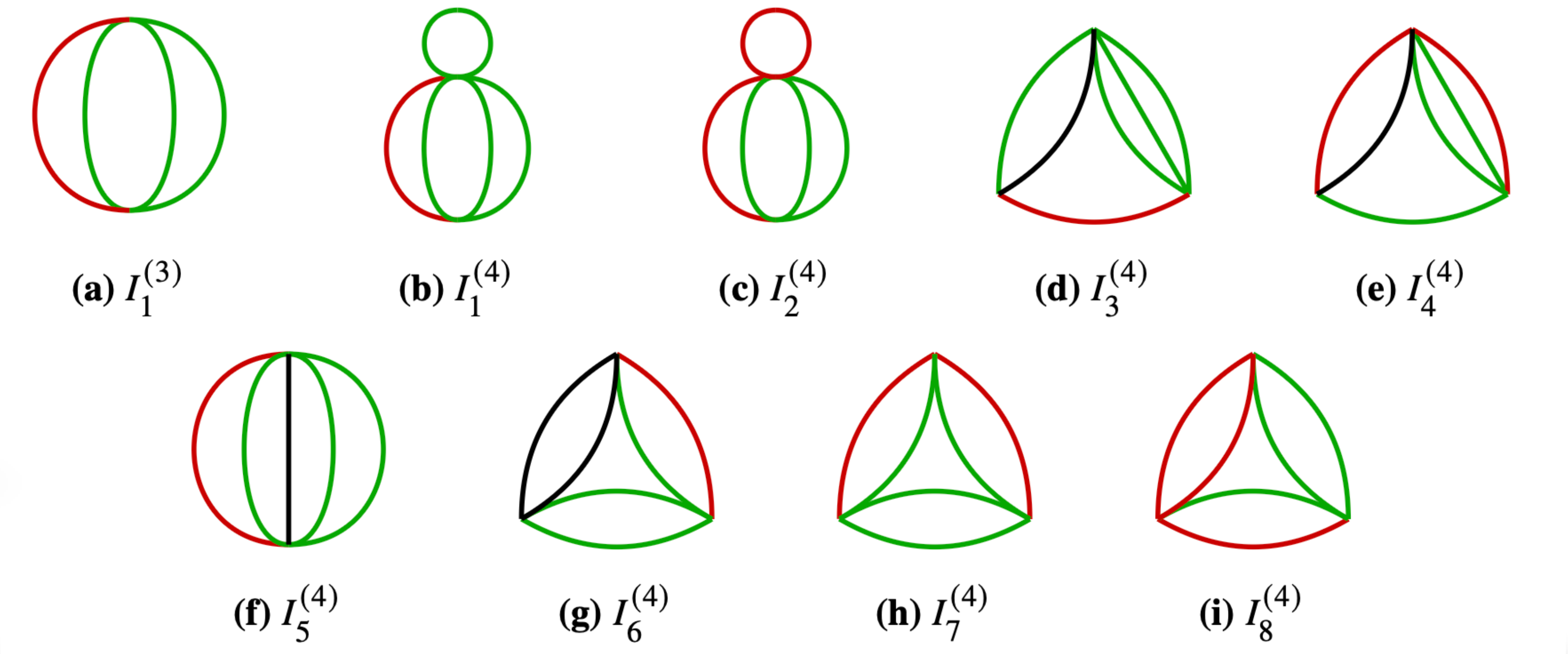




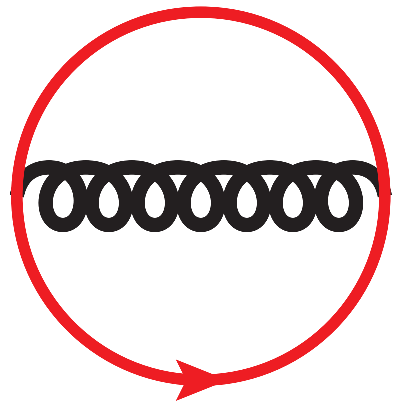


Same set of integrals:

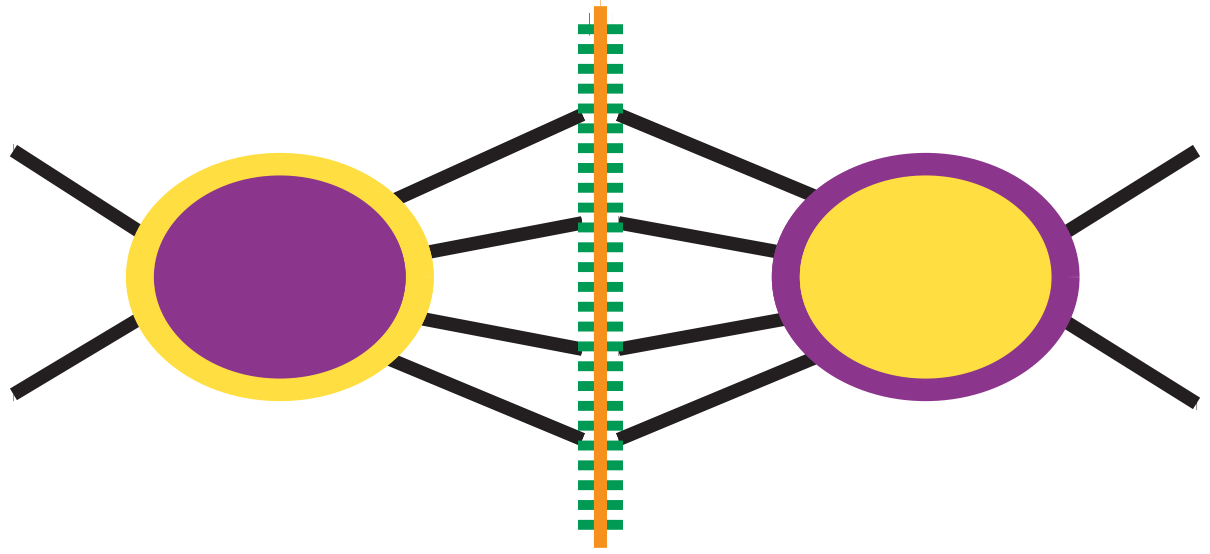
why are we not done yet!?

Loop corrections to the EoS of neutron stars

- Integrals depend on chemical potential μ



$$= \int d^D p \int d^D q \frac{1}{(p_0 + i\mu)^2 + \vec{p}^2 + m^2} \frac{1}{(q_0 + i\mu)^2 + \vec{q}^2 + m^2} \frac{1}{(p-q)^2}$$



- Standard loop techniques don’t work out of the box…
- Cutting rules: residue integration over energy components

$$\int d^D p \rightarrow \int \frac{d^{D-1}p}{2E} \theta(E-\mu) \rightarrow \int d^D k \; \delta(k^2 - m^2) \; \theta(k \cdot n - \mu)$$

[Ghisoiu, Gorda, Kurkela et al 2016]

$\Rightarrow$ very reminiscent of calculation of **observable integrated over phase-space**

see also LTD approach of [Kärkkäinen, Navarrete, Nurmela et al 2025]

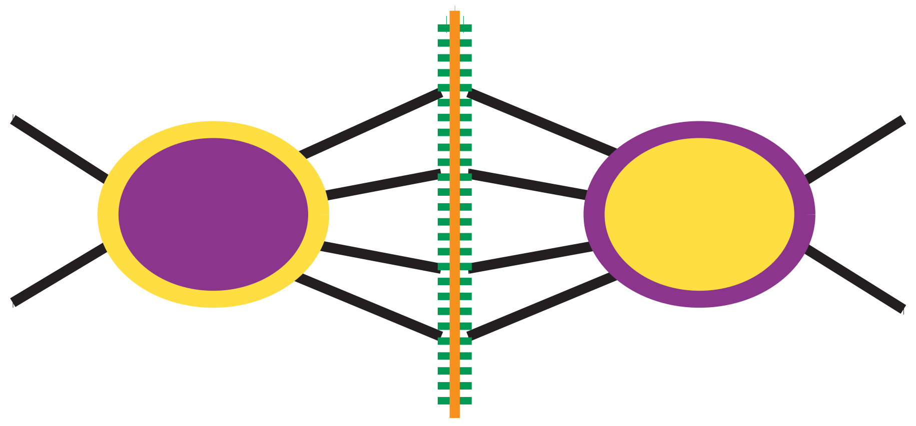
Loop corrections to the EoS of neutron stars

- Reverse unitarity $\delta(k^2 - m^2) = \frac{i}{2\pi}\left(\frac{1}{k^2 - m^2 + i0} - \frac{1}{k^2 - m^2 - i0}\right) \equiv \left(\frac{1}{k^2 - m^2 + i0}\right)_c$

[Anastasiou, Melnikov 2002]
[Anastasiou, Dixon, Melnikov, Petriello 2003]

  - Up to small subtleties (if cut propagator is absent, integral vanishes) just like any propagator
  - Cut propagators can be differentiated ⇒ use **IBPs and differential equations**!
  - Robust/well tested: e.g., Higgs production @ NN^3LO computed with this trick

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2015]



- Phase-space integrals with Heaviside functions

[Baranowski, **Delto**, Melnikov, Wang, 2023]

$$I\left[\theta(f), g\right] = \int d^D k\, \theta(f(k))\, g(k) \qquad 0 = I\left[\theta(f), \partial_\mu(v^\mu g)\right] + I\left[\delta(f), g v^\mu \partial_\mu f\right]$$

  - New **boundary term in the IBP relation**

see also related work in [Osterman, Schicho, Vuorinen, 2023]

  - Just a delta function: if $f(k)$ polynomial in loop momentum, use trick above!
  - Approach developed for calculation of zero-jettiness soft function, $\theta(k_1 \cdot n - k_1 \cdot \bar{n})$

Loop corrections to the EoS of neutron stars

- Two-loops
  - Reproduce all known results, with or without masses
  - No non-trivial integrals: only squares of one-loop ⇒ fully analytic

- Three-loops massless
  - Reproduce known results [Vuorinen, 2003]
  - 5 non-trivial integrals to compute: use high-precision numerics to get analytic expression

- Three-loops massive [Kurkela, Romatschke, Vuorinen, 2010]
  - Less than 100 integrals to compute, automated setup
  - Differential equations available, determining boundary conditions
  - Goal: obtain series expansion around $m = 0$

- Four-loops massless: **next target!**

Summary and Outlook

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- Four-loop pQCD corrections to the EoS have important phenomenological impact
- New approaches developed in the world of collider physics can be used
  - make the calculation as algorithmic as possible to handle increased complexity with loops/scales
- Two completely different approaches (at least) being explored
  - completely different ways to organise calculation, important for cross-check of results
- Beyond this example: very mature tools to compute Feynman integrals
  - if you are stuck with this type of problem, get in touch!

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Thank you