



Strong-Field QED Limitations on TeV-class Plasma Wakefield Accelerators^[1]

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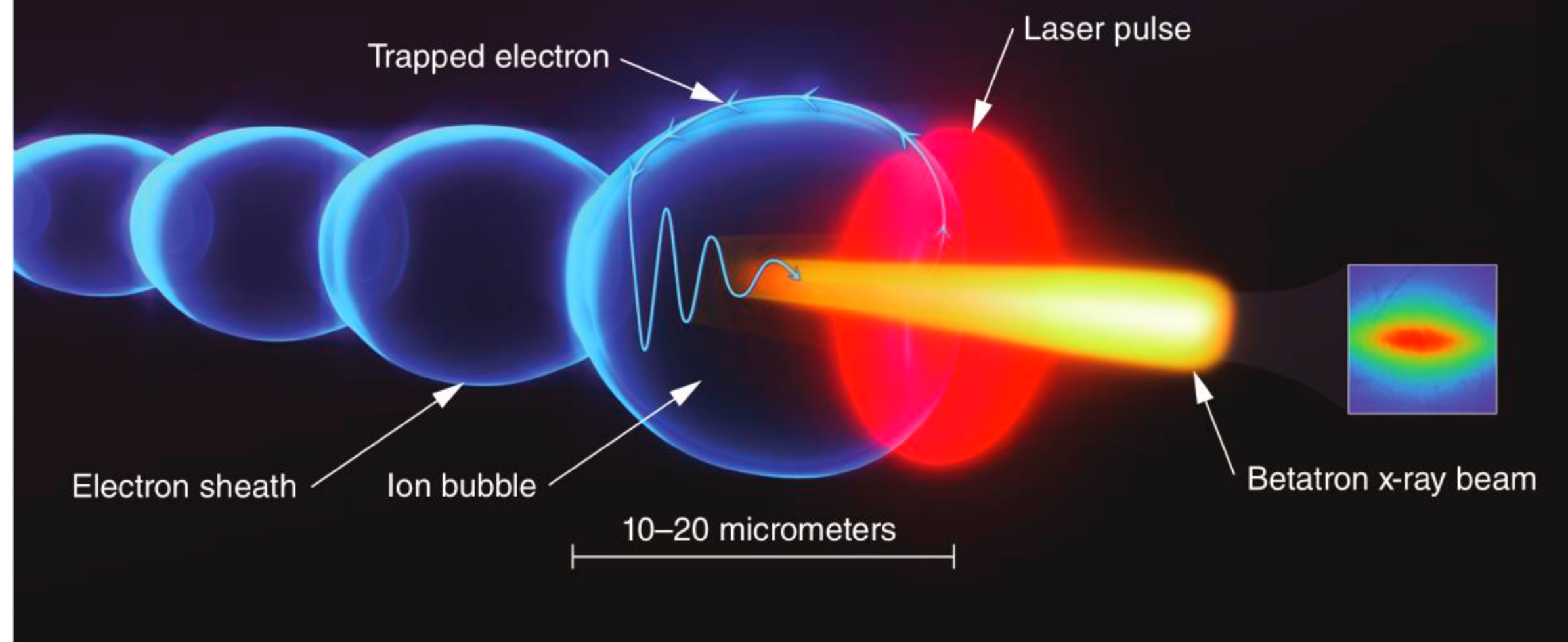
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F. Albert PPCF 2014



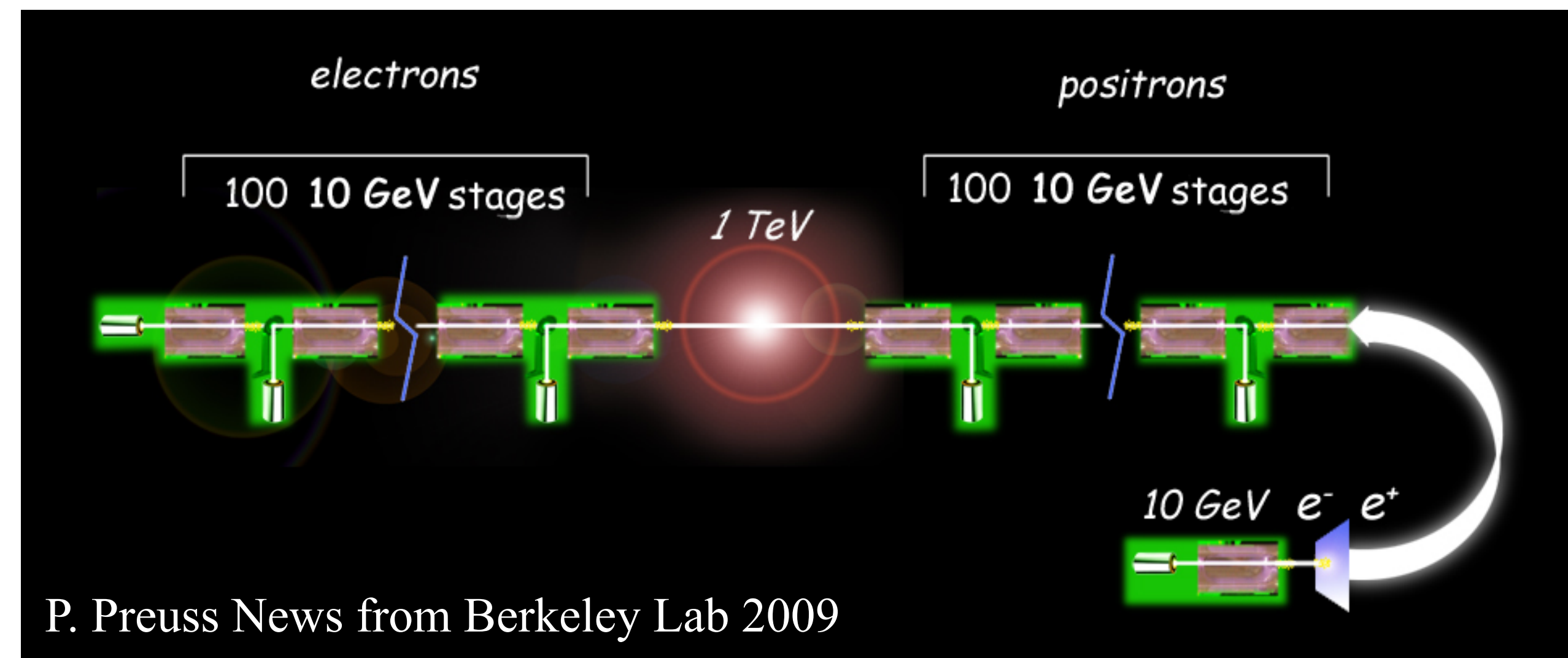
LWFA based accelerator has the advantages:

- Carrying strong longitudinal fields and transverse focusing fields ($\sim 100\text{GV/m}$).
- Can serve as a high gradient lepton accelerator and a wiggler for x-ray/ γ ray generation.

- Beam quality is important:

Energy spread and Emittance

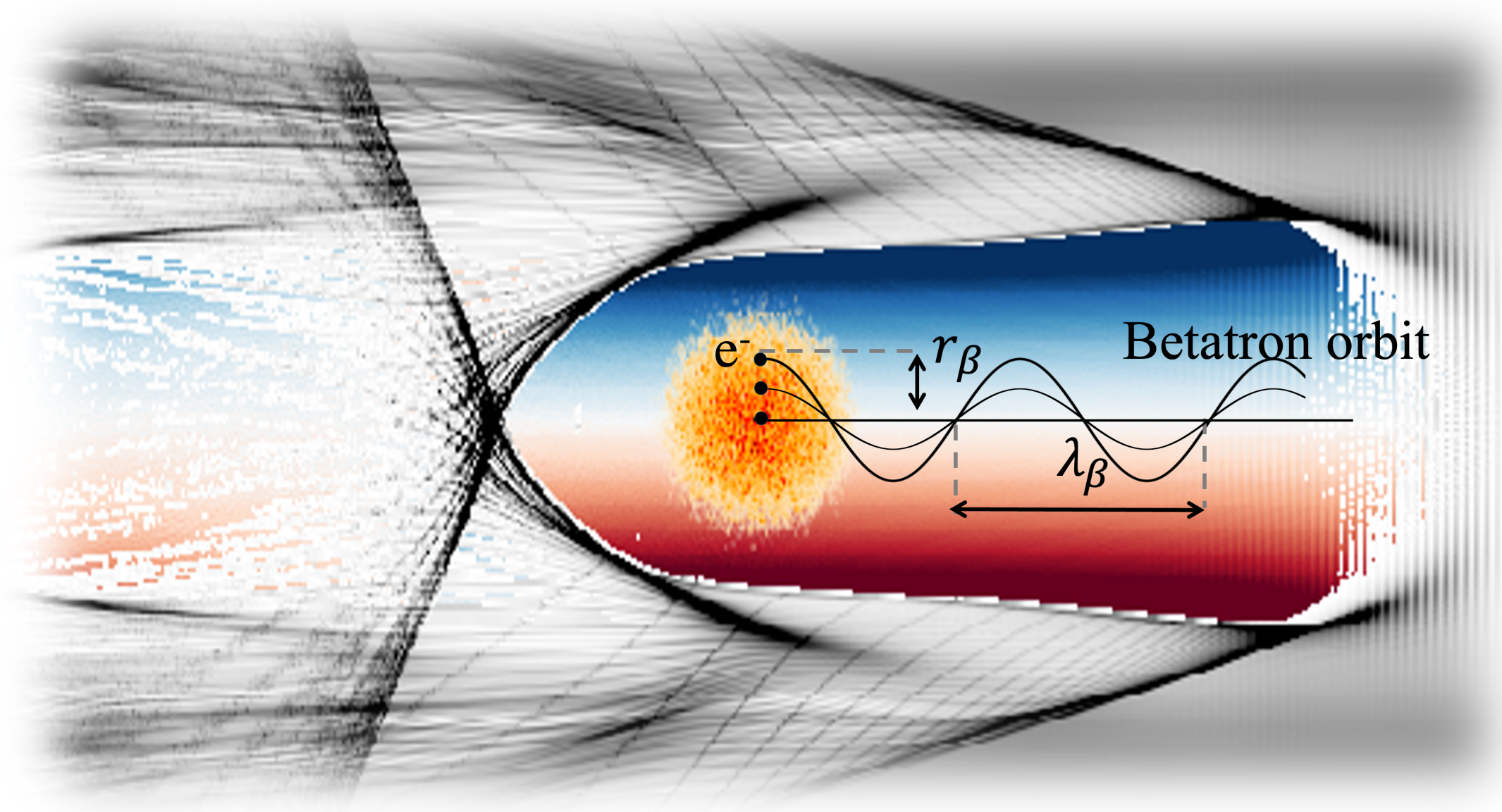
- Both Coulomb scattering and synchrotron radiation emission are shown to result in beam quality degradation [1].
- As beam energy get higher how about Quantum radiation effect?



Radiation effect on energy spread

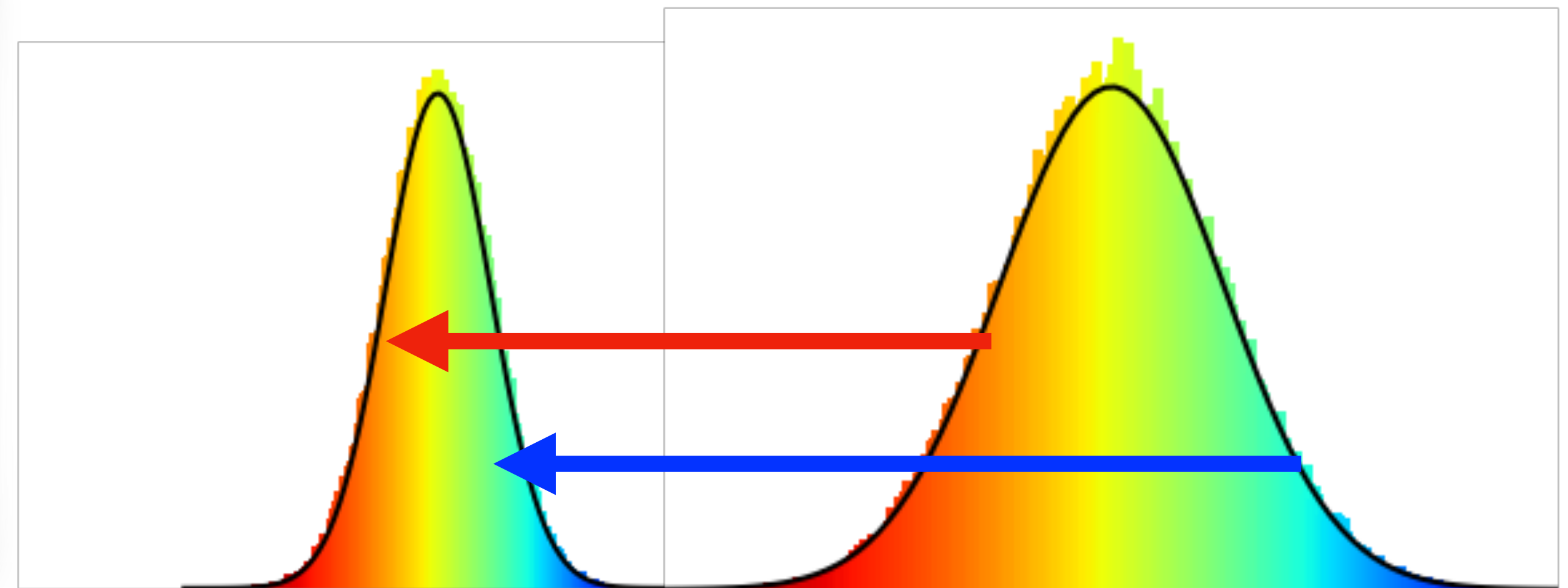


Radiation heating due to orbit effect



- Radiation reaction force inside wakefield scale with $F_{rad} \propto r_\beta^2$, r_β is the betatron amplitudes
- On-axis particles will not radiate, whereas off-axis particles radiate strongly.

Radiation cooling



- Radiation cooling due to high energy particle radiate more energy than those at lower energy

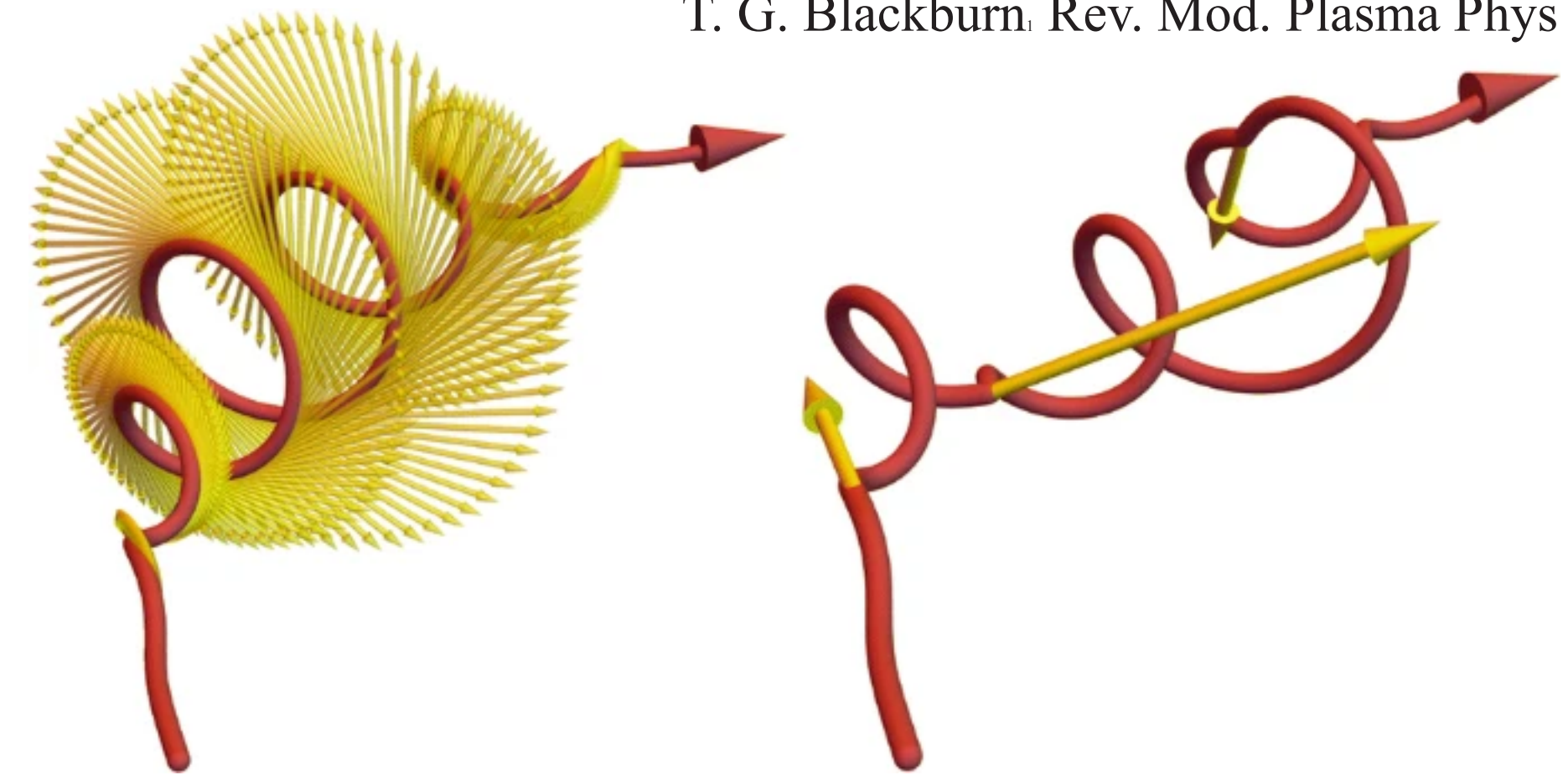
Quantum radiation effect on energy spread



T. G. Blackburn, Rev. Mod. Plasma Phys 2020

E_{cr} : characteristic field strength does $m_e c^2$ work over a (reduced) Compton length

$$eE_{cr}\bar{\lambda}_c > m_e c^2 \rightarrow E_{cr} \sim 10^{18} \text{ V} \cdot \text{m}^{-1}$$



Radiation reaction in classical picture (left) and quantum regime (right)

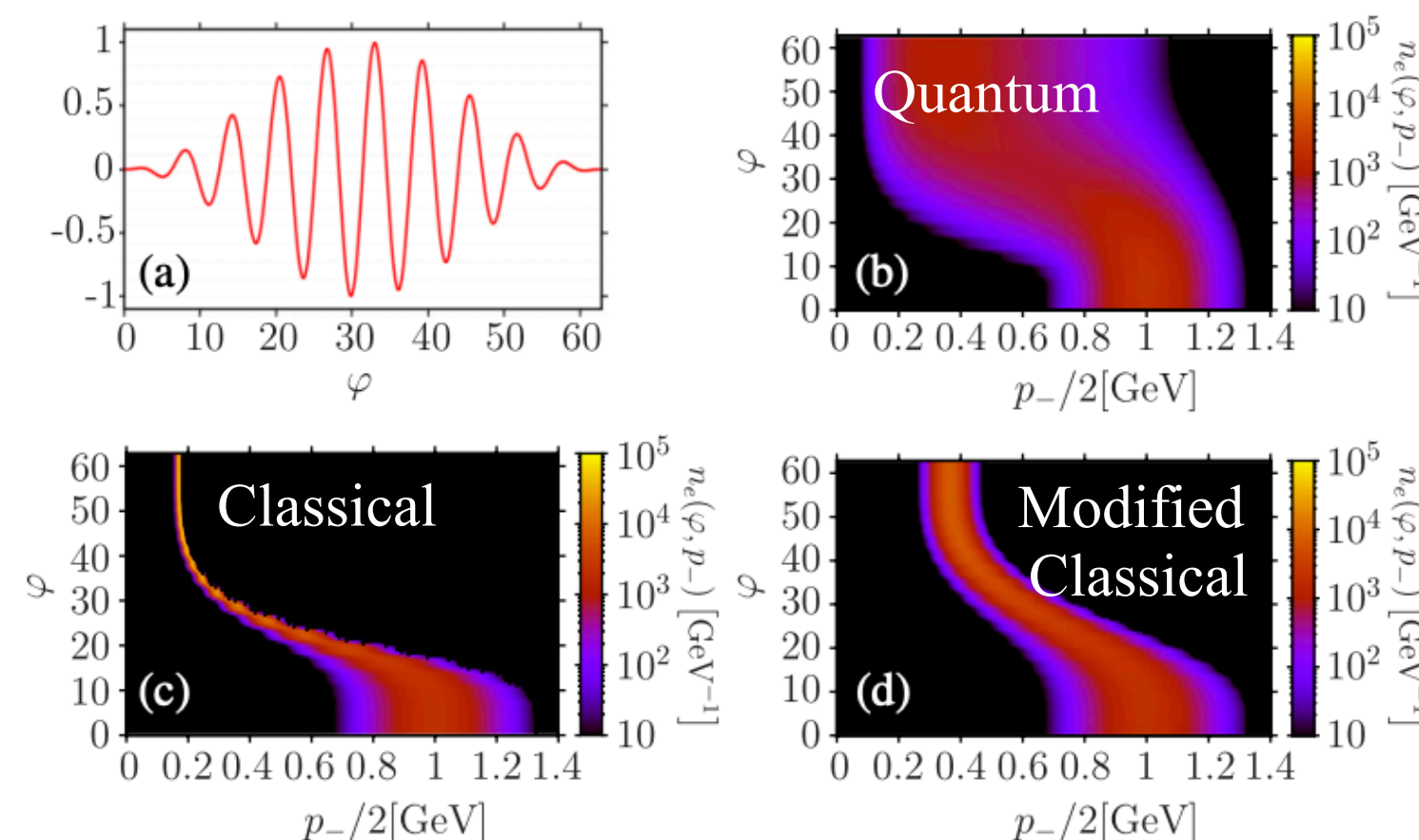
χ_p : Quantum strength parameter

$$\chi_p = \frac{||F_{\mu\nu}P_{(e,\gamma)}^\nu||}{m_e c E_{cr}} = \frac{E_{rest}}{E_{cr}} \approx \frac{\gamma E_{focus}}{E_{cr}}$$

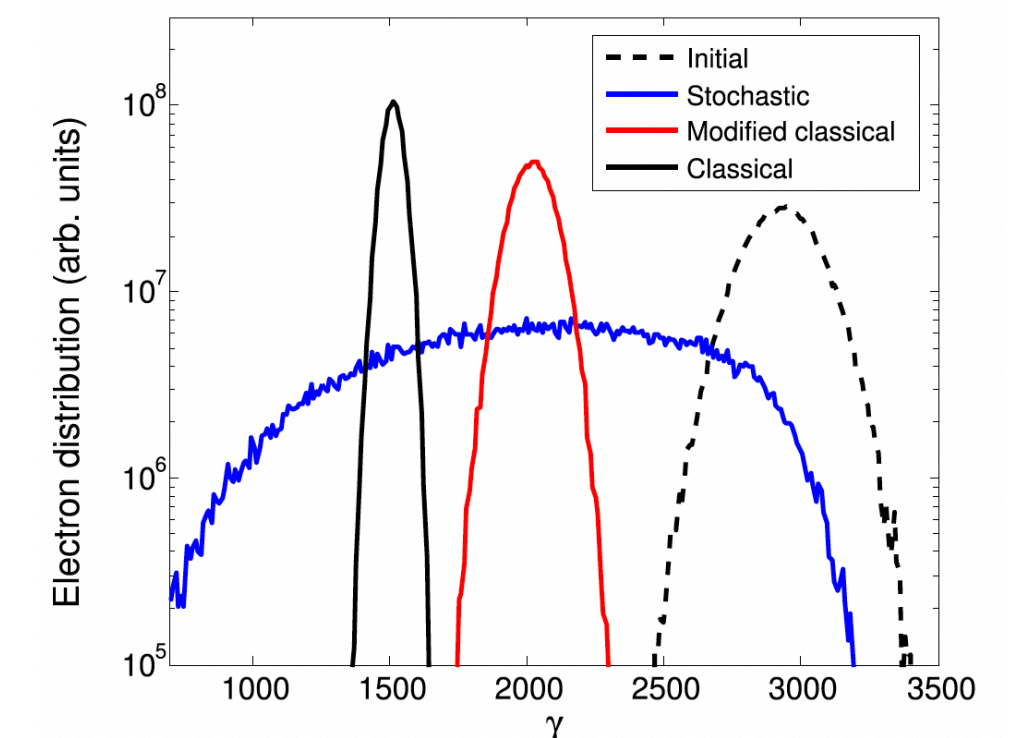
For TeV beam $\gamma = 10^6 - 10^7$

Stochastic broadening effect

N. Neitz, A. Di Piazza PRL 2013



C. Ridgers JPP 2017



Beam emittance control inside wakefield

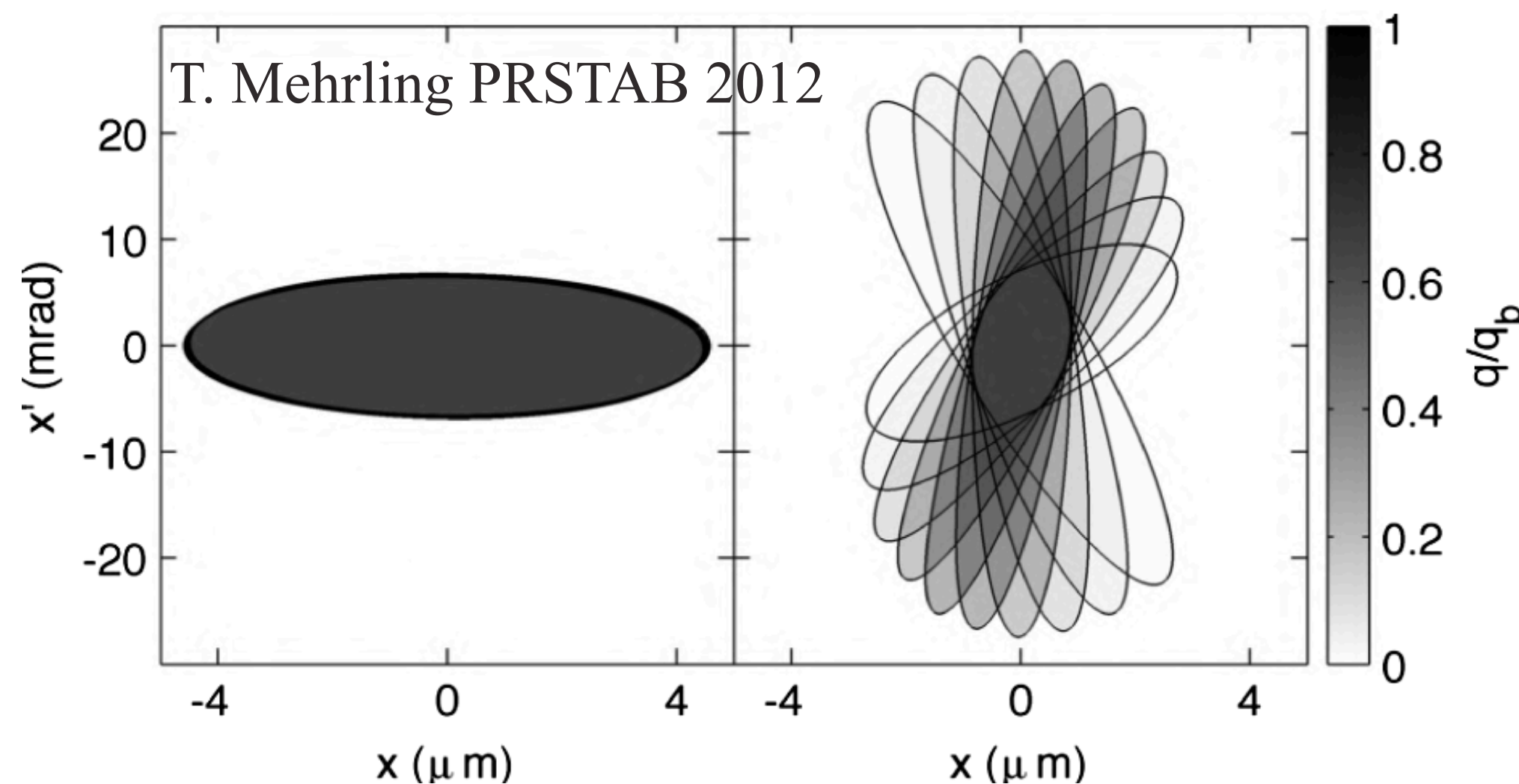


Ellipse equation:

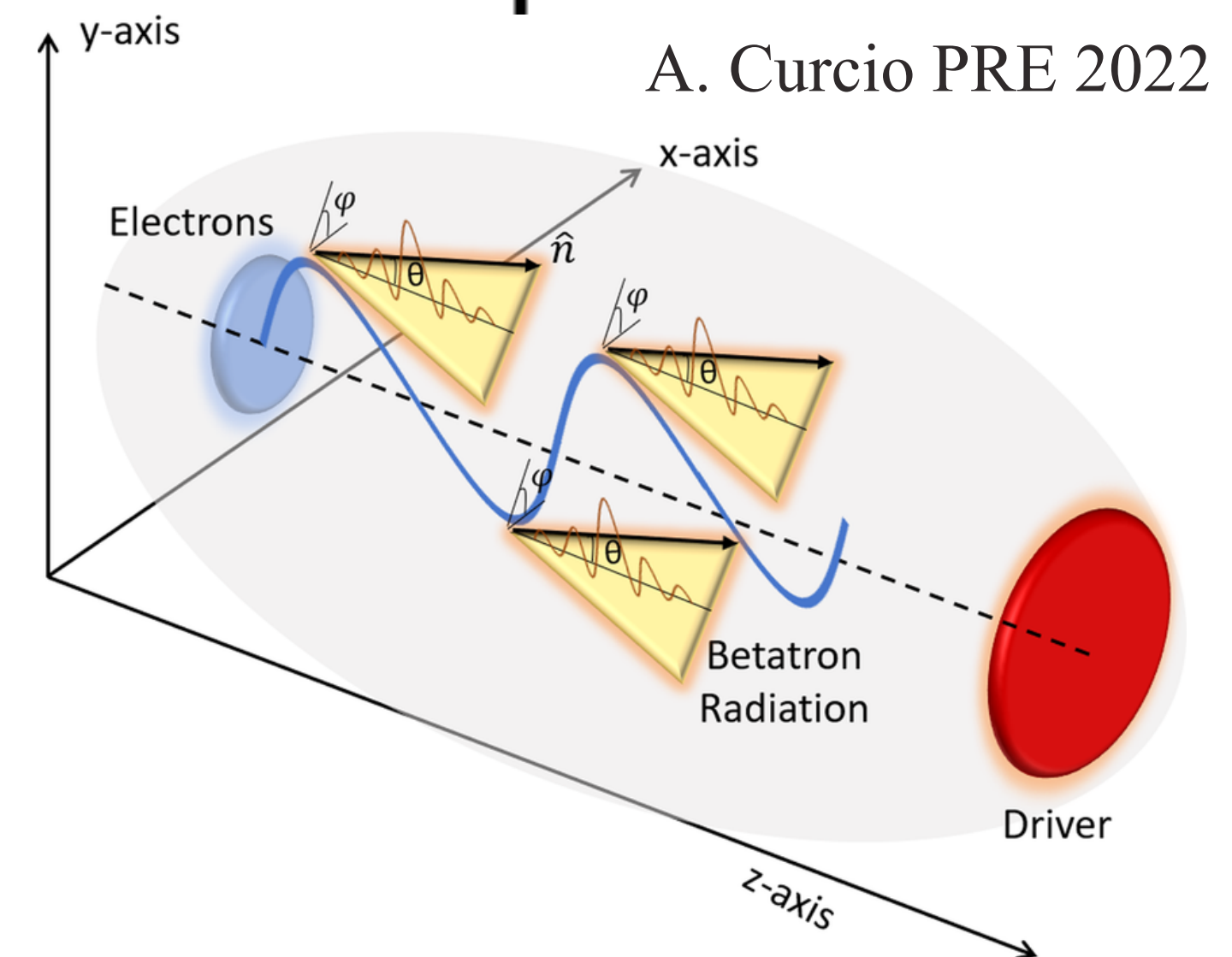
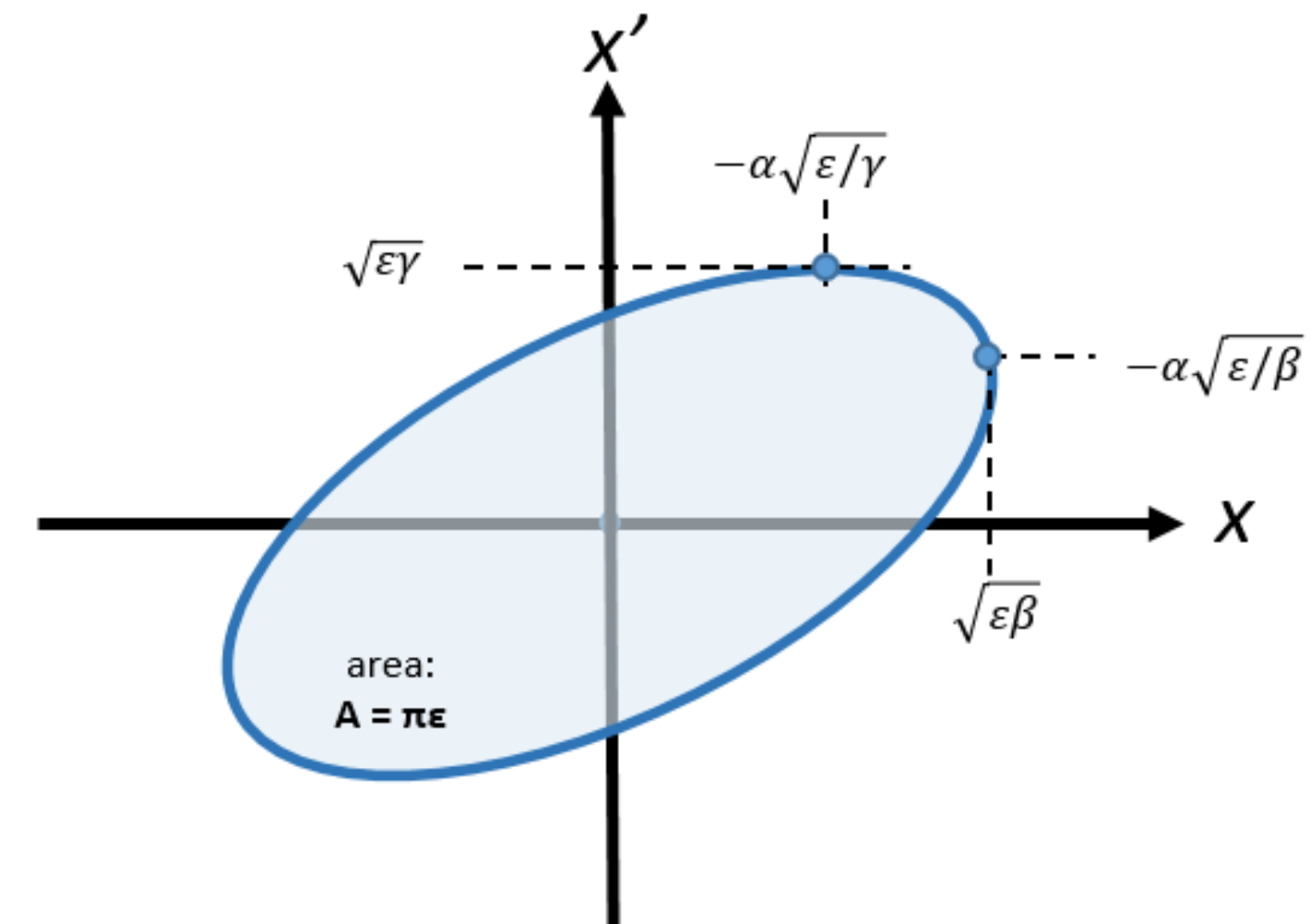
$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon$$

RMS normalized emittance:

$$\varepsilon_n = \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle x p_x \rangle^2}$$



Beam matching condition $r_{bm} = (\varepsilon_n / \gamma k_\beta)^{1/2}$



Can radiation reaction cool the emittance?

Spin polarized beam transportation in wakefield

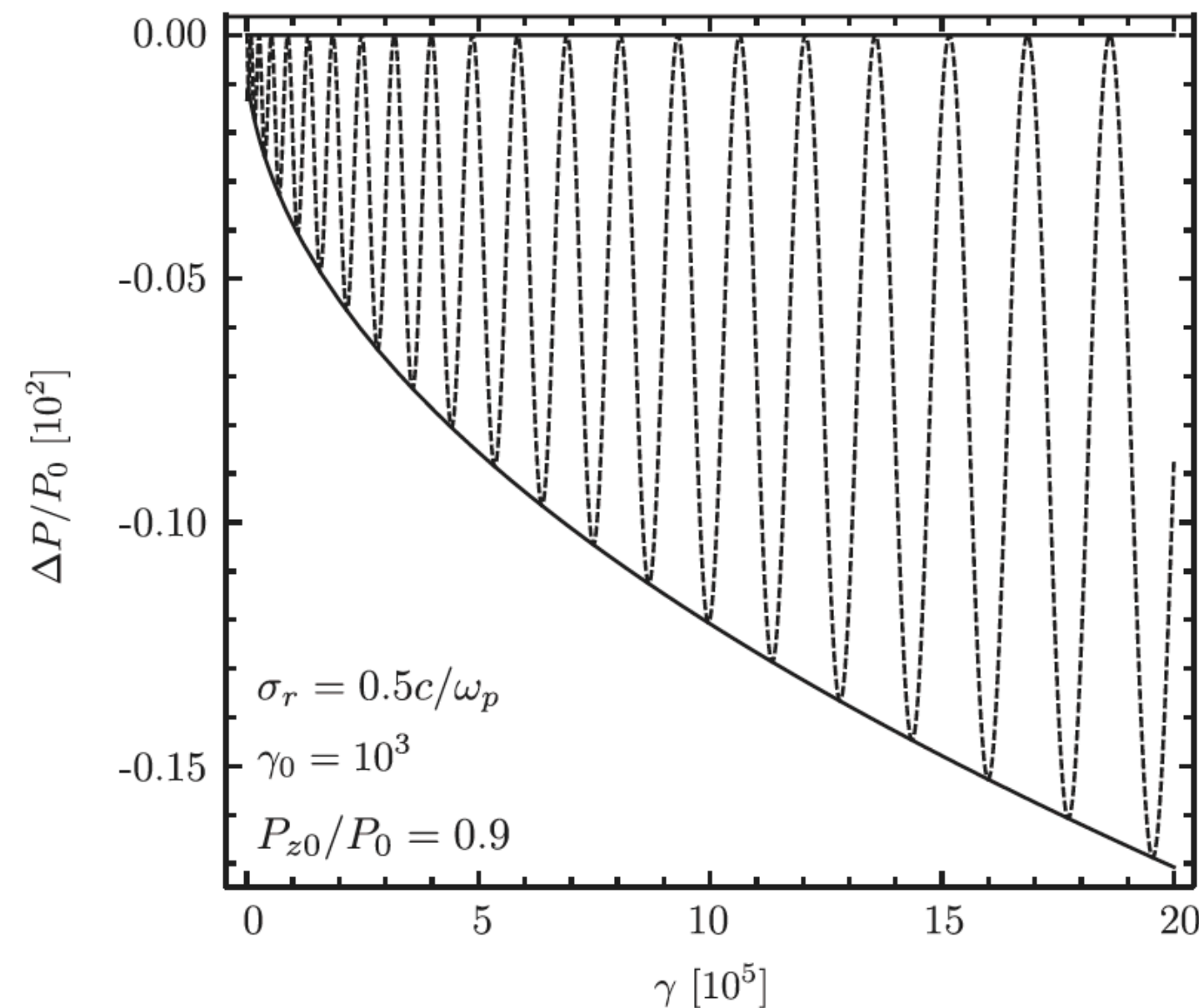


Polarized lepton beam is an important tool in HEP research



Proton spin partition
EIC Introduction bnl.gov/eic/

Classical spin precession induced depolarization can be controlled



J. Vieira PRSTAB 2011

How about the radiation induced spin change?

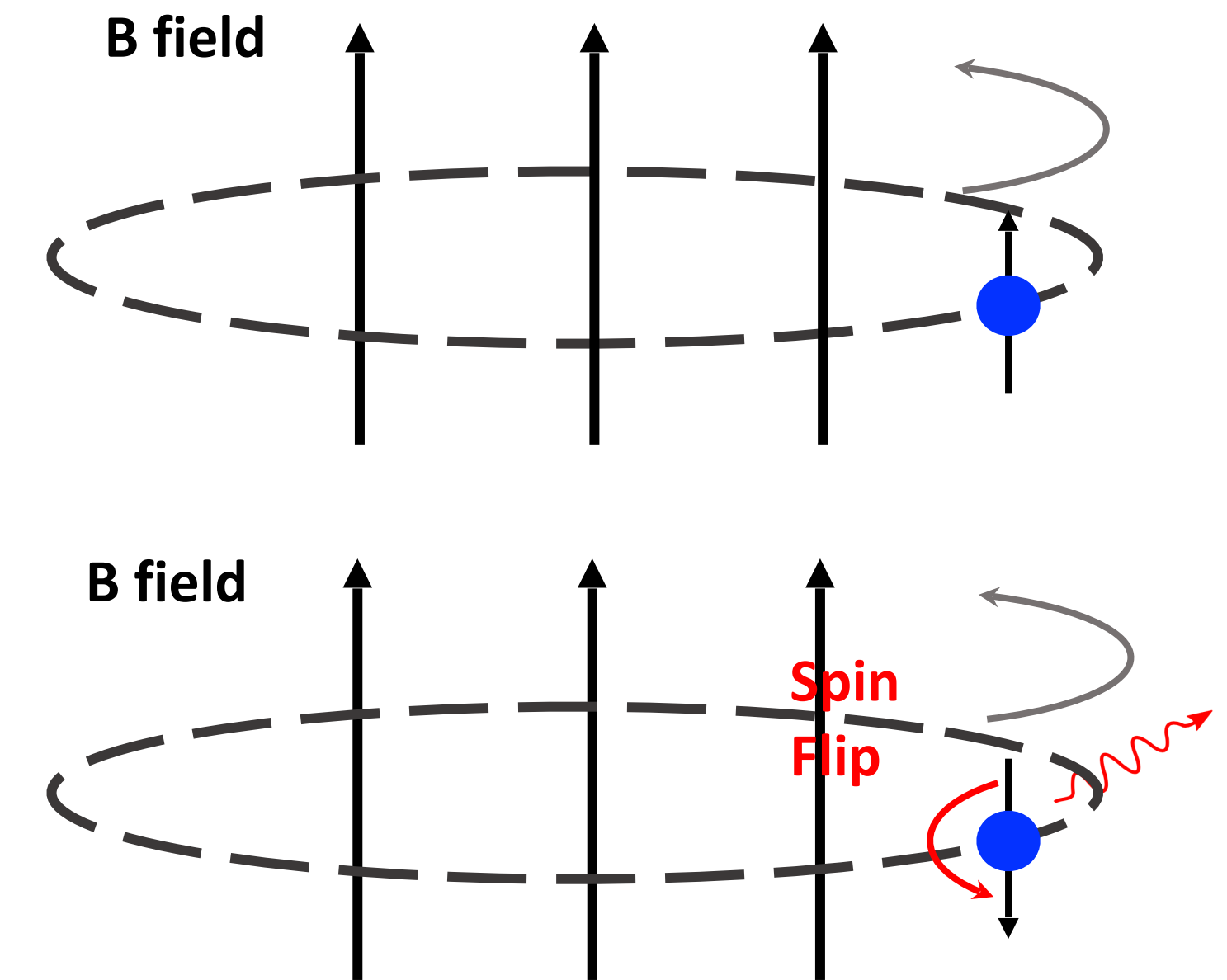


Illustration of the Sokolov-Ternov effect

Quantum parameter for TeV LWFA accelerator



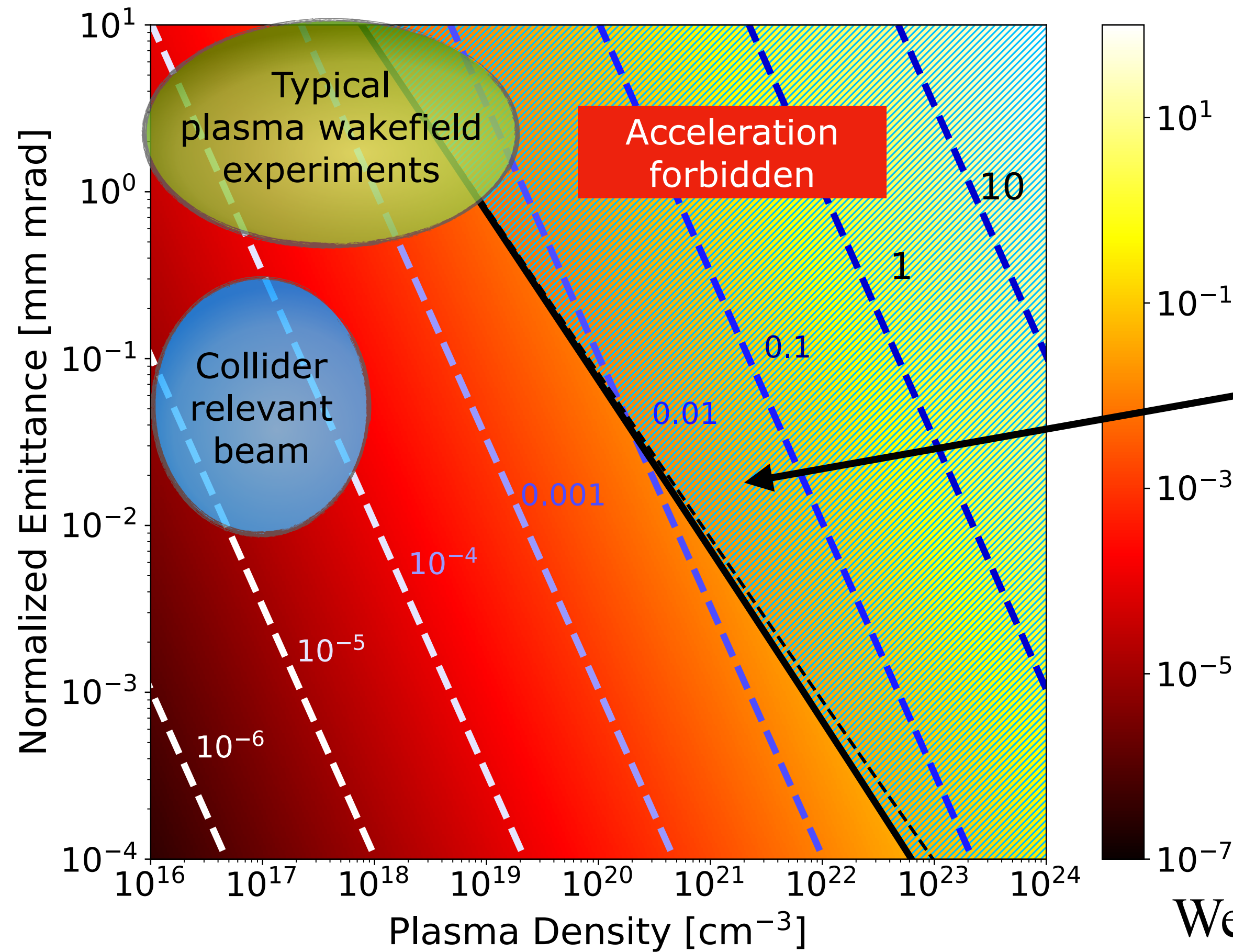
$$\chi_e \approx \gamma \frac{E_{focus}}{E_s}$$

$$E_{focus} \simeq \frac{en_p}{2\epsilon_0} \sigma_r$$

For matched beam

$$\chi_e \approx \frac{\lambda_c}{\lambda_p} \sqrt{\frac{2\pi\epsilon_n}{\lambda_p}} \left(\frac{\gamma}{2}\right)^{3/4}$$

χ_e for beam inside wakefield at 1TeV



$$F_{acc} = \pi \sqrt{a_0} m_e c^2 \left(\lambda_p\right)^{-1}$$

$$F_{rad} = \frac{4\pi\alpha_f}{3\lambda_c} m_e c^2 \chi_e^2$$

Radiation reaction force equals the acceleration force

$$\epsilon_n = \frac{3}{2\pi\alpha_f} \sqrt{\frac{a_0}{2}} \frac{\lambda_p^2}{\lambda_c} \gamma^{-3/2}$$

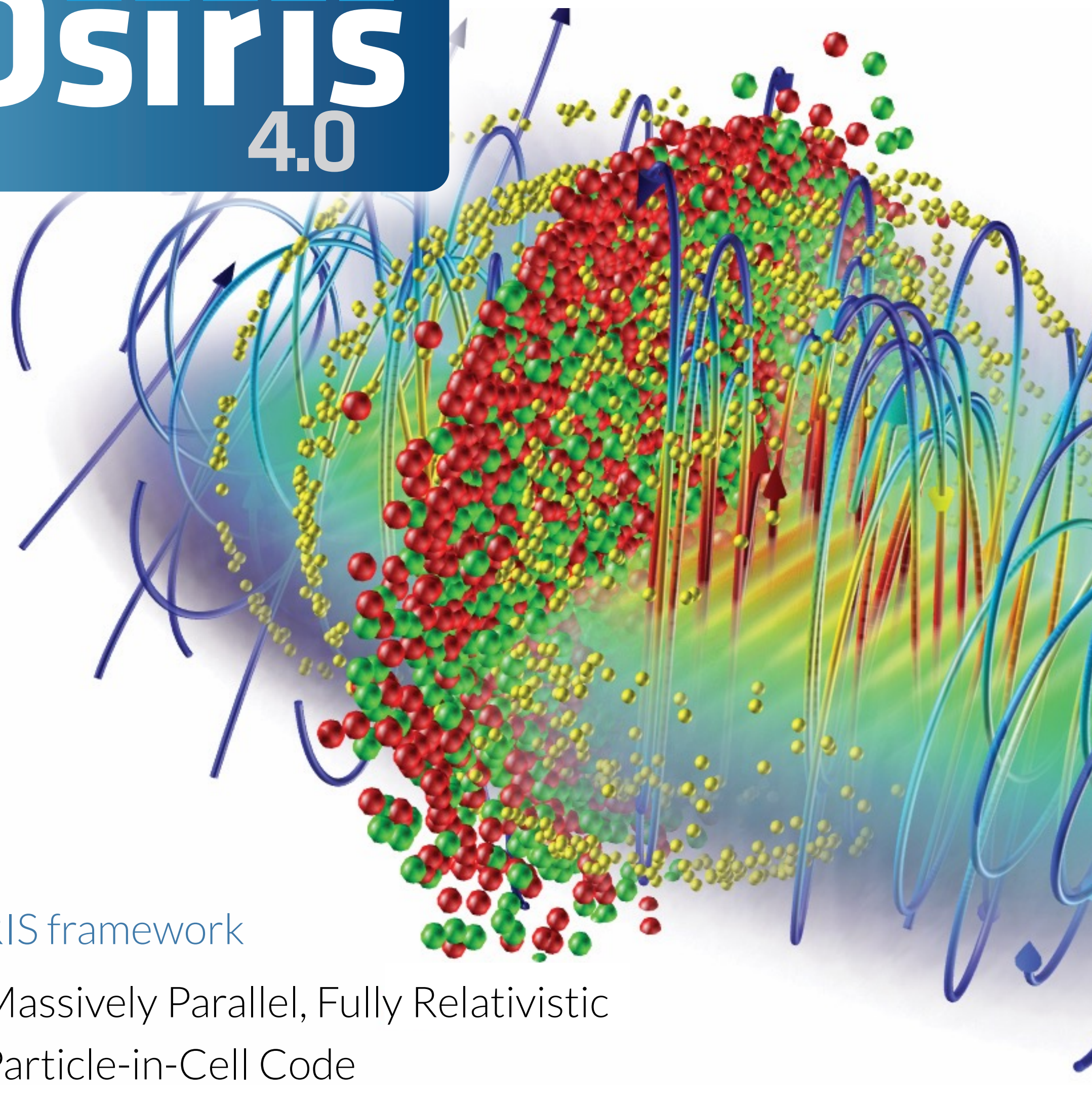
$$\chi_e \approx 10^{-3} \times \left(\frac{\mathcal{E}_{beam}}{1 \text{ [TeV]}}\right)^{3/4} \times \left(\frac{n_p}{10^{18} \text{ cm}^{-3}}\right)^{3/4} \times \sqrt{\frac{\epsilon_n}{1 \text{ [\mu m]}}}$$

We are looking at radiation effects on

- Energy spread growth/reduction
- Emittance growth/reduction
- Spin polarization detuning

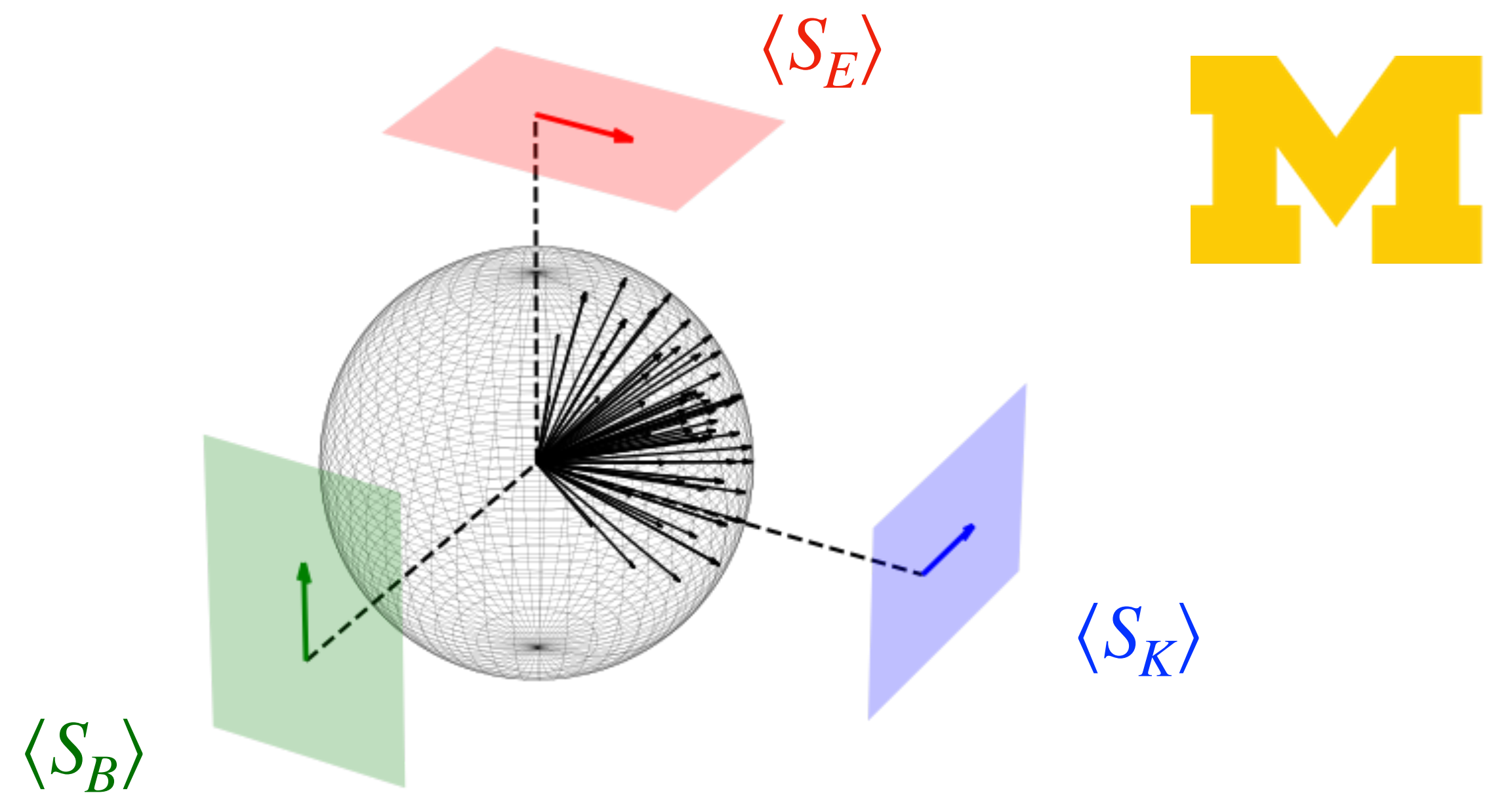
Osiris

4.0

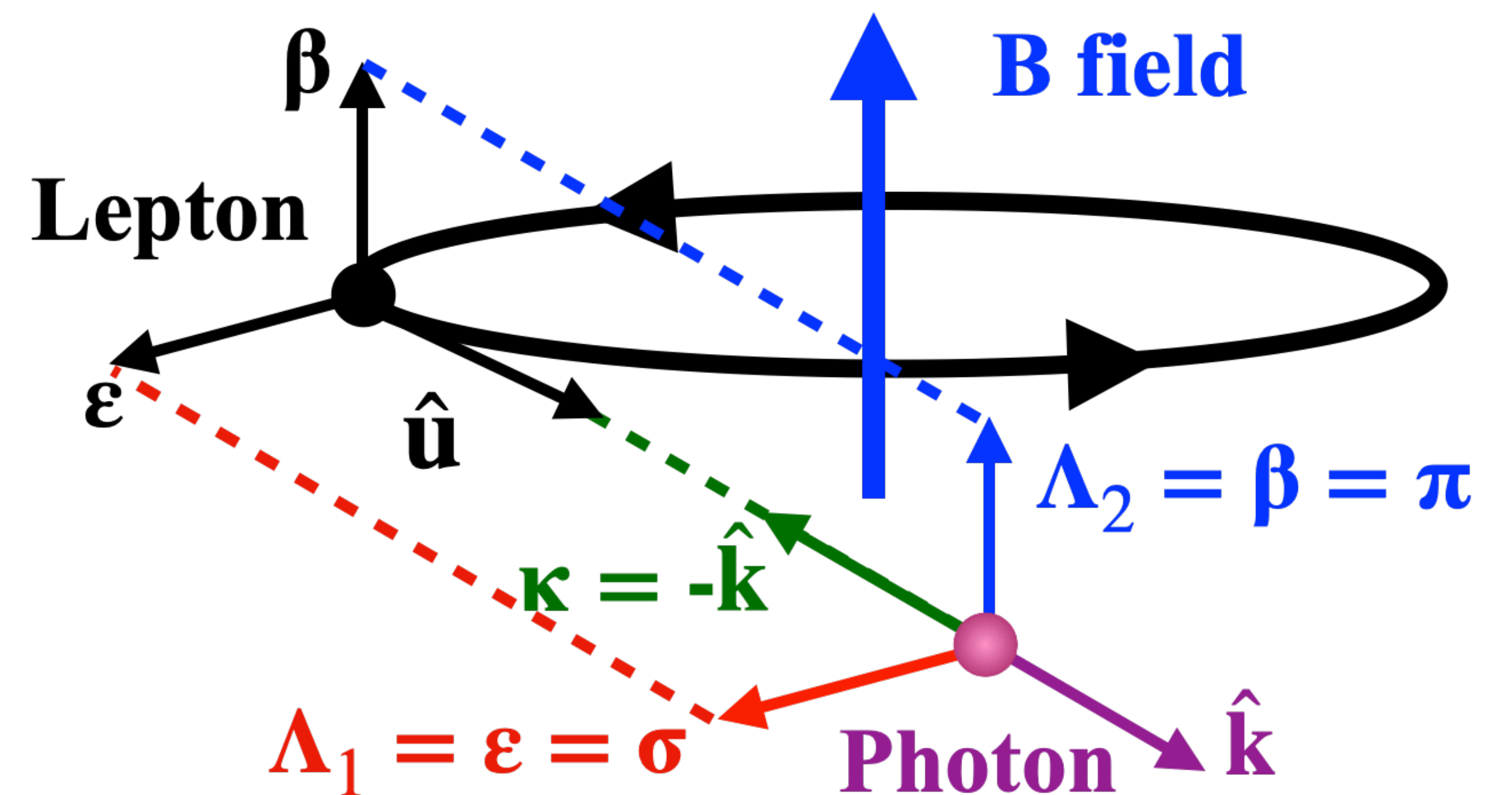


OSIRIS framework

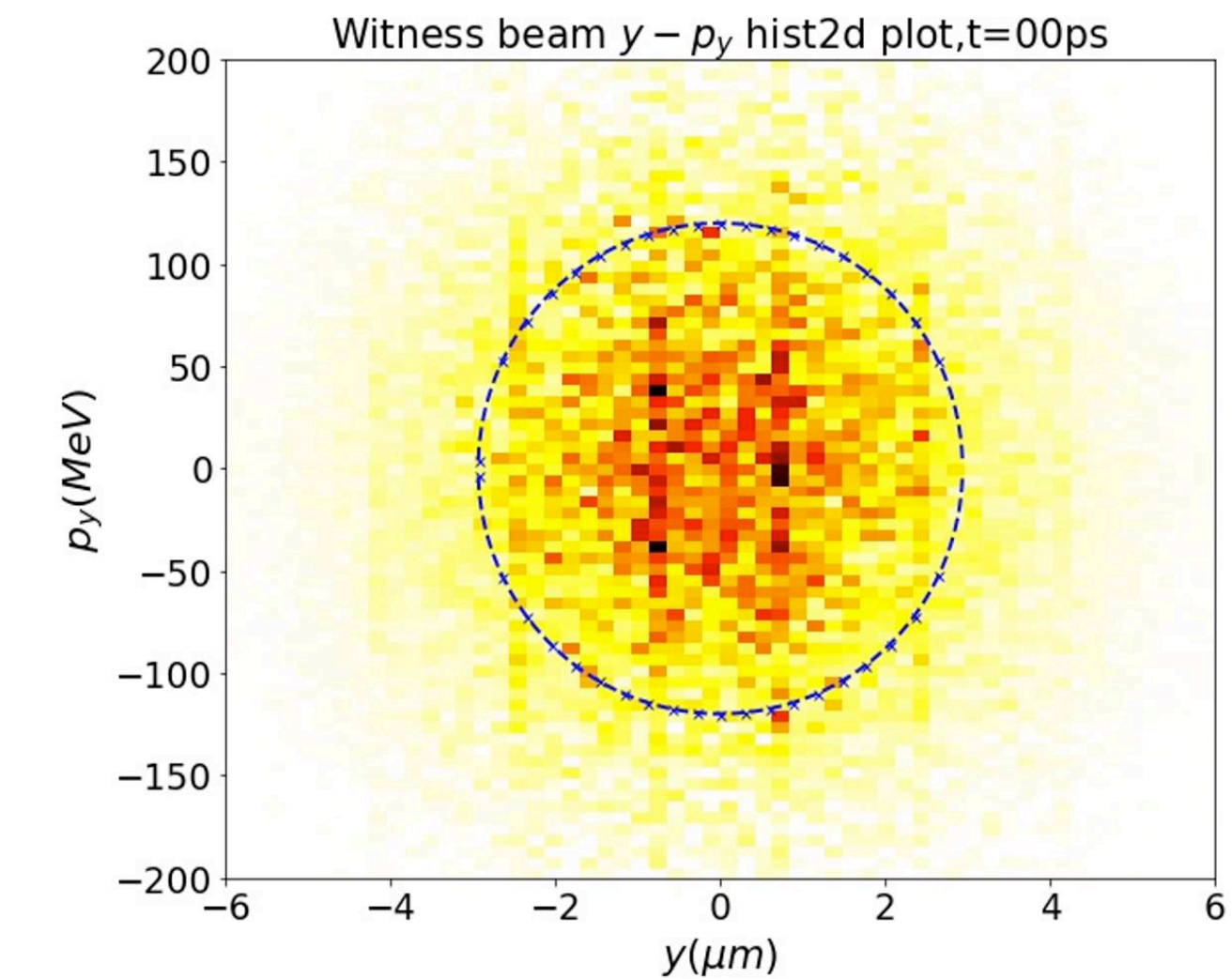
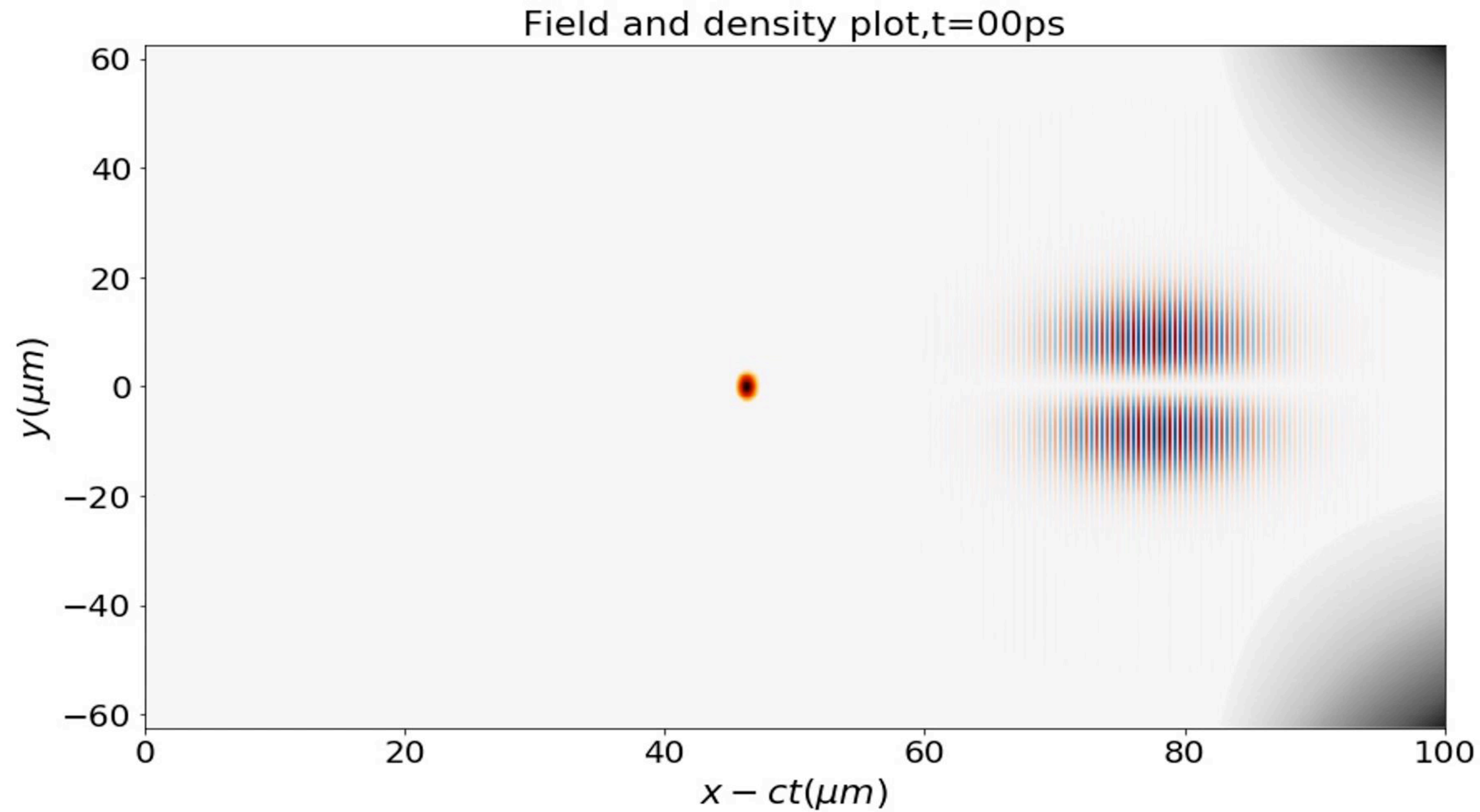
- Massively Parallel, Fully Relativistic Particle-in-Cell Code
- Support for advanced CPU / GPU architectures
- Extended physics/simulation models
- AI/ML surrogate models and data-driven discovery



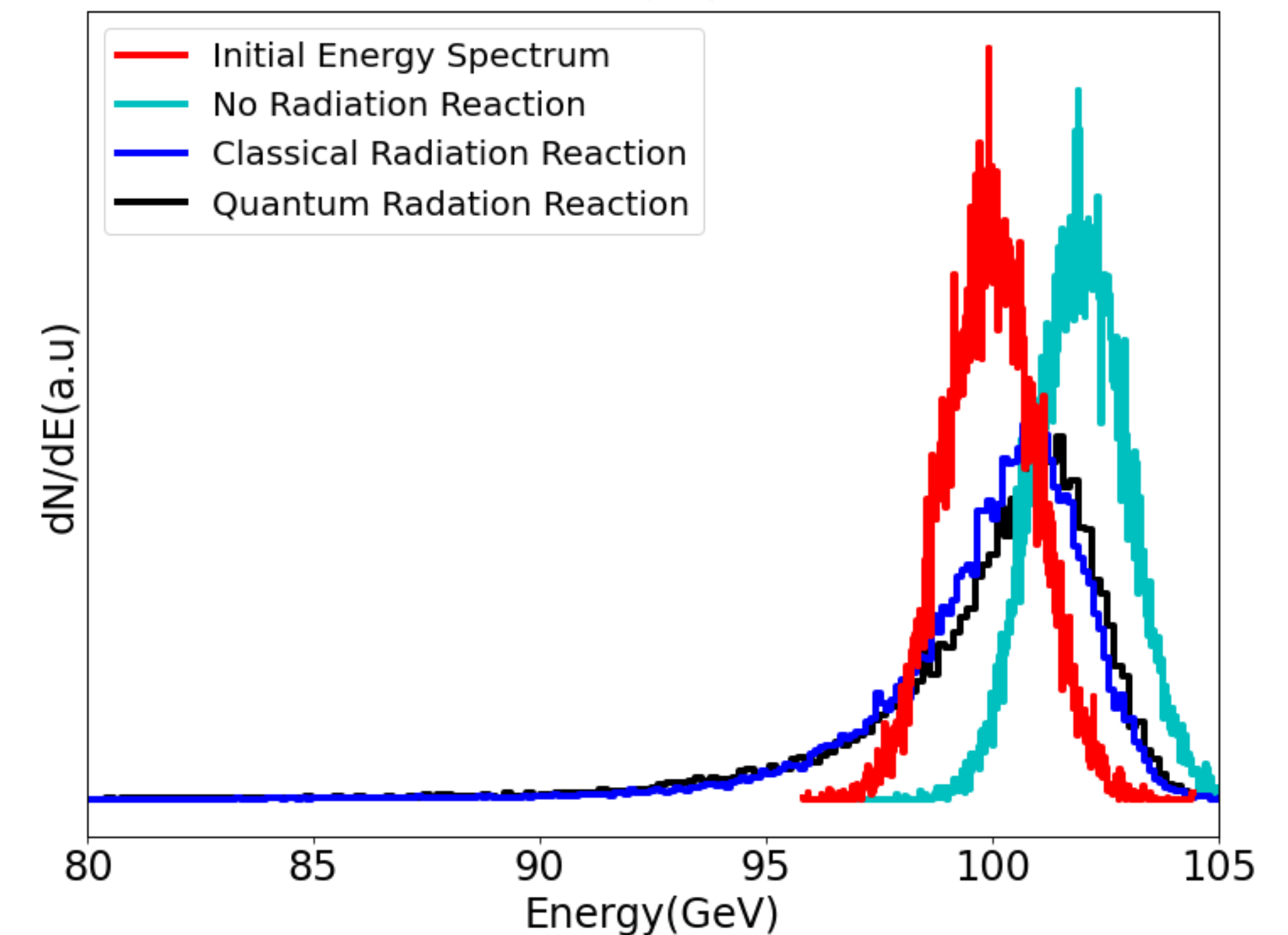
With spin and polarization resolve QED!



Radiation induced energy spread in LWFA



100 GeV beam inside LWFA, plasma density = 10^{18} cm^{-3}
Energy gain balances the radiation energy loss



An analytic model for predicting energy spread growth



Radiation effect on energy spread

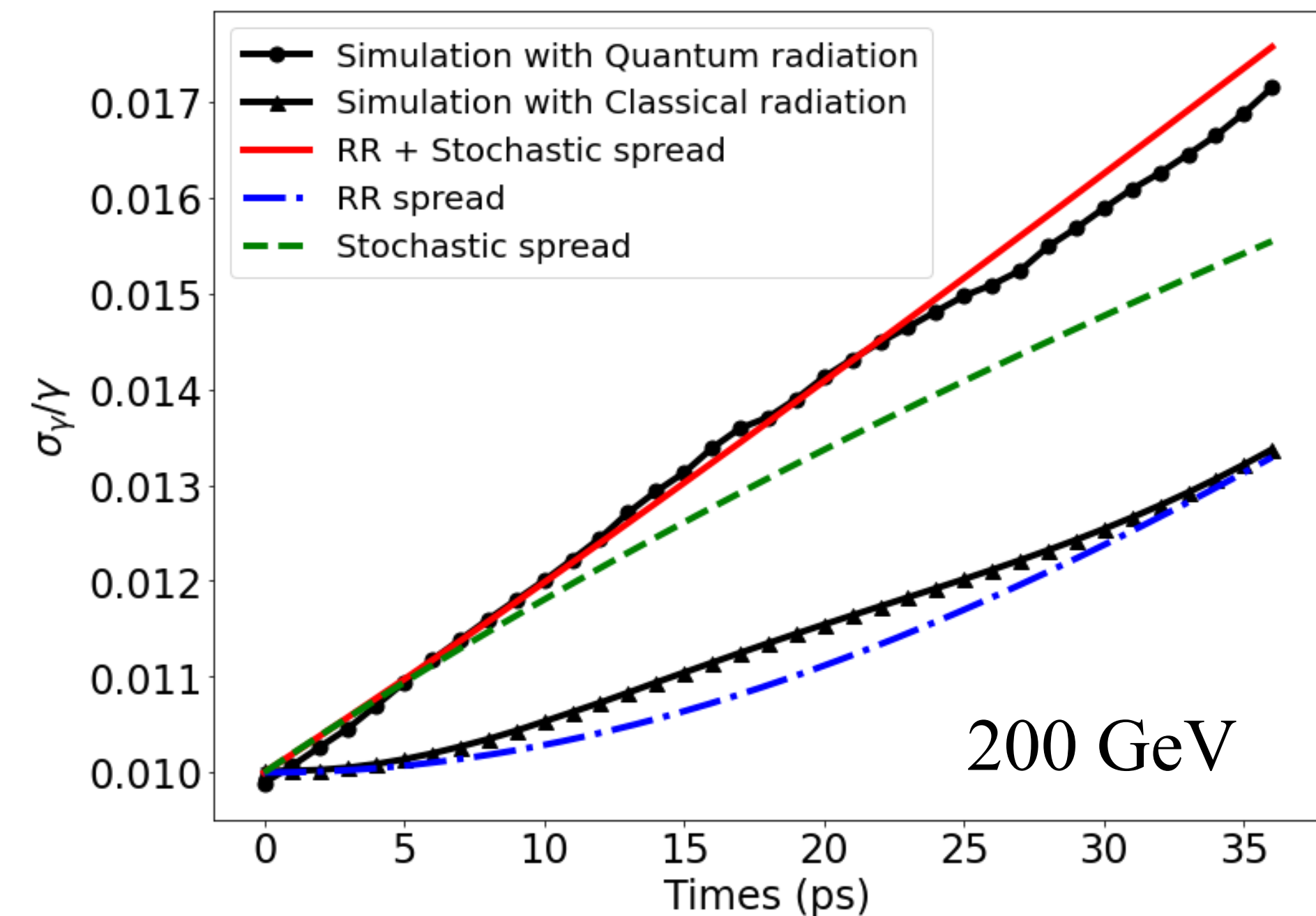
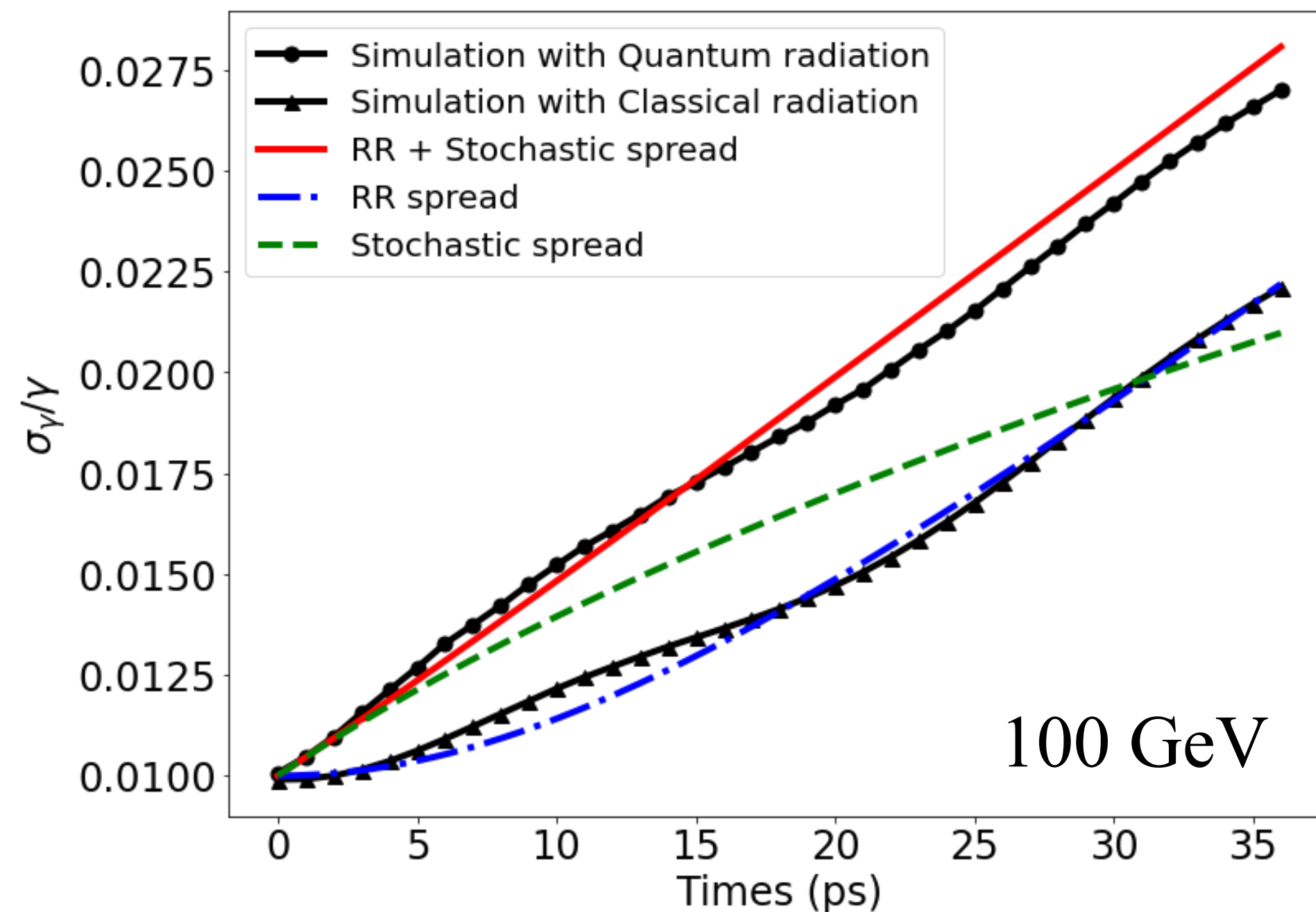
Stochastic broadening

P. Michel PRE 2006

N. Neitz, A. Di Piazza PRL 2013

$$\frac{d}{dt} \left(\sigma_\gamma \right)^2 = \frac{d}{dt} \left[\left(\sigma_\gamma \right)_{cool}^2 + \left(\sigma_\gamma \right)_{orb}^2 + \left(\sigma_\gamma \right)_{st}^2 \right]$$

$$\approx -C_{cool} \chi_e^2 \left(\sigma_\gamma \right)^2 + C_{orb} \chi_e^4 t + C_{st} \chi_e^3$$



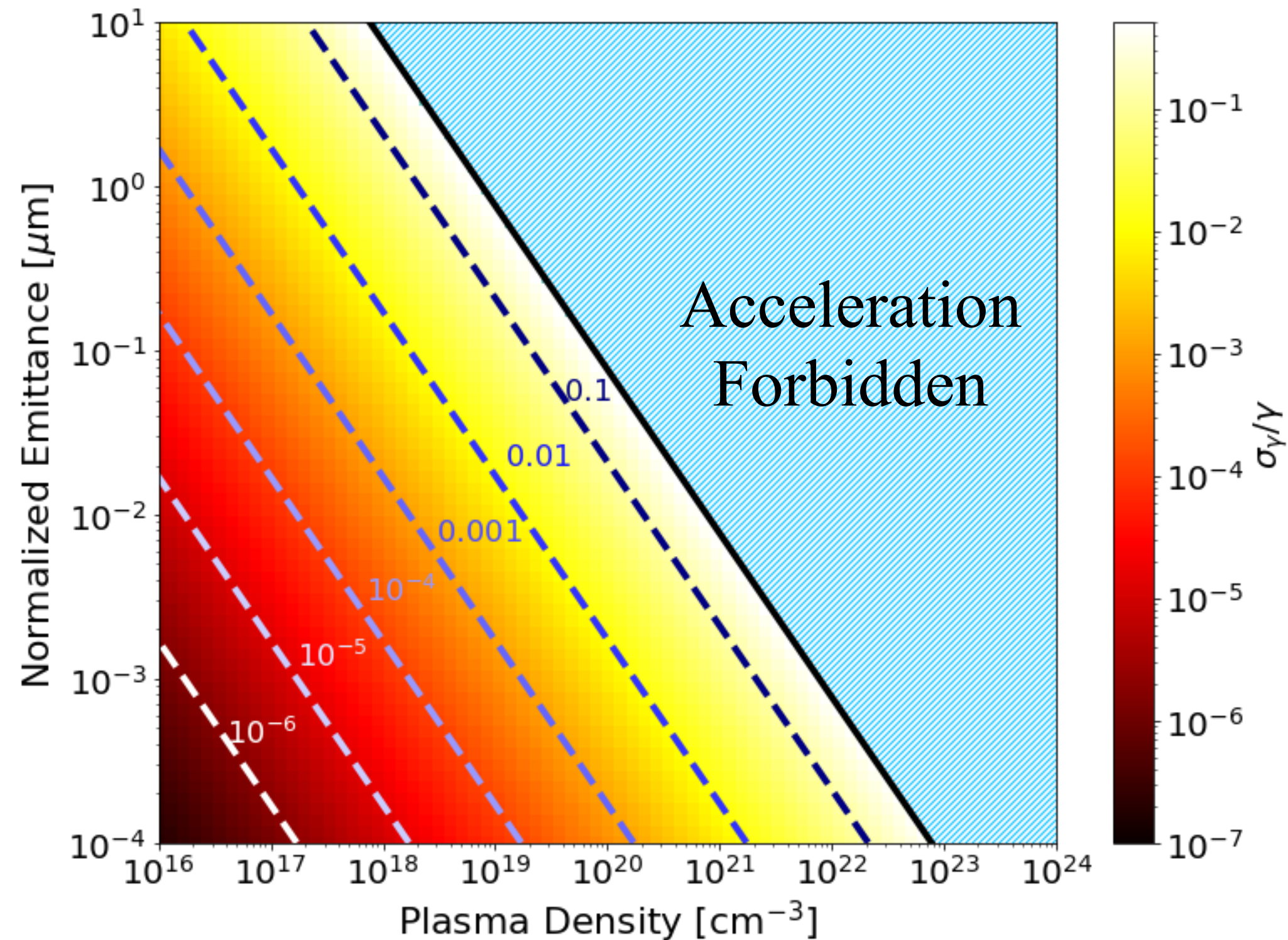
Radiation induced energy spread for TeV LWFA based accelerator



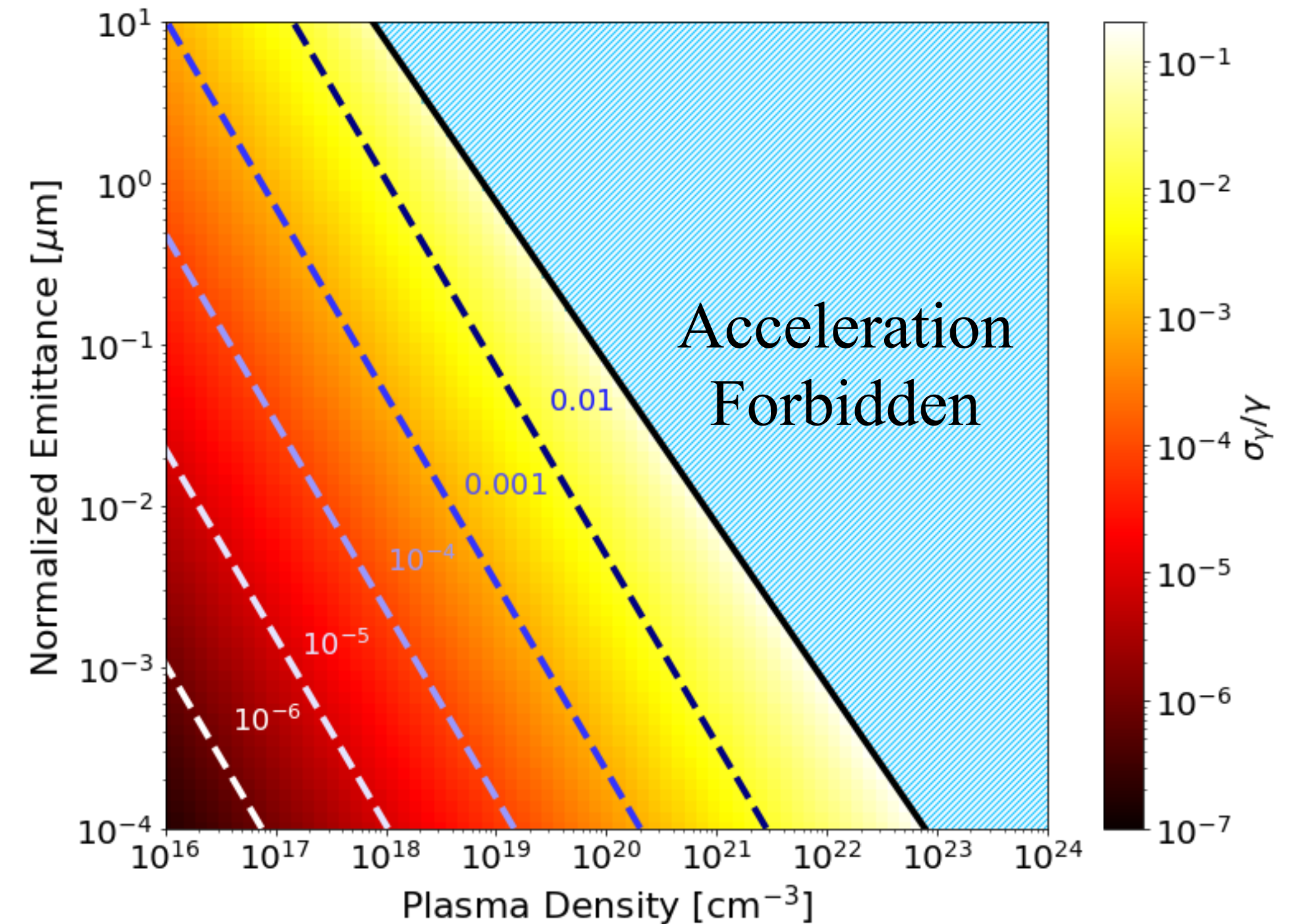
- Use the previous equations to estimate the energy spread change for beam accelerated to 1 TeV

$$\frac{d\gamma}{dct} = F_{acc} - F_{rad} \quad \frac{d}{dt} \left(\sigma_{\gamma} \right)^2 = \frac{d}{dt} \left[\left(\sigma_{\gamma} \right)_{cool}^2 + \left(\sigma_{\gamma} \right)_{orb}^2 + \left(\sigma_{\gamma} \right)_{st}^2 \right]$$

Total Energy spread



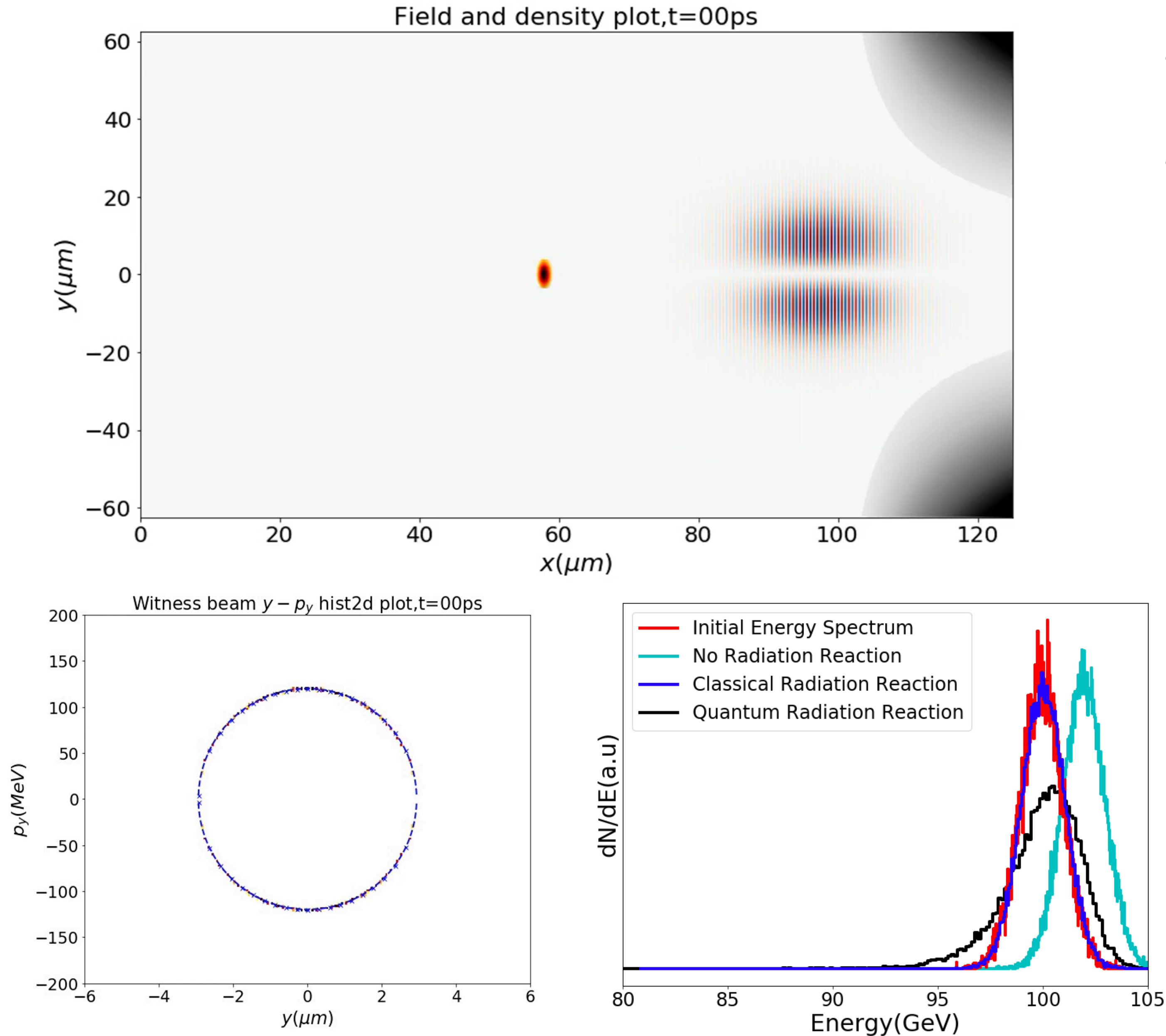
Energy spread from Stochastic broadening



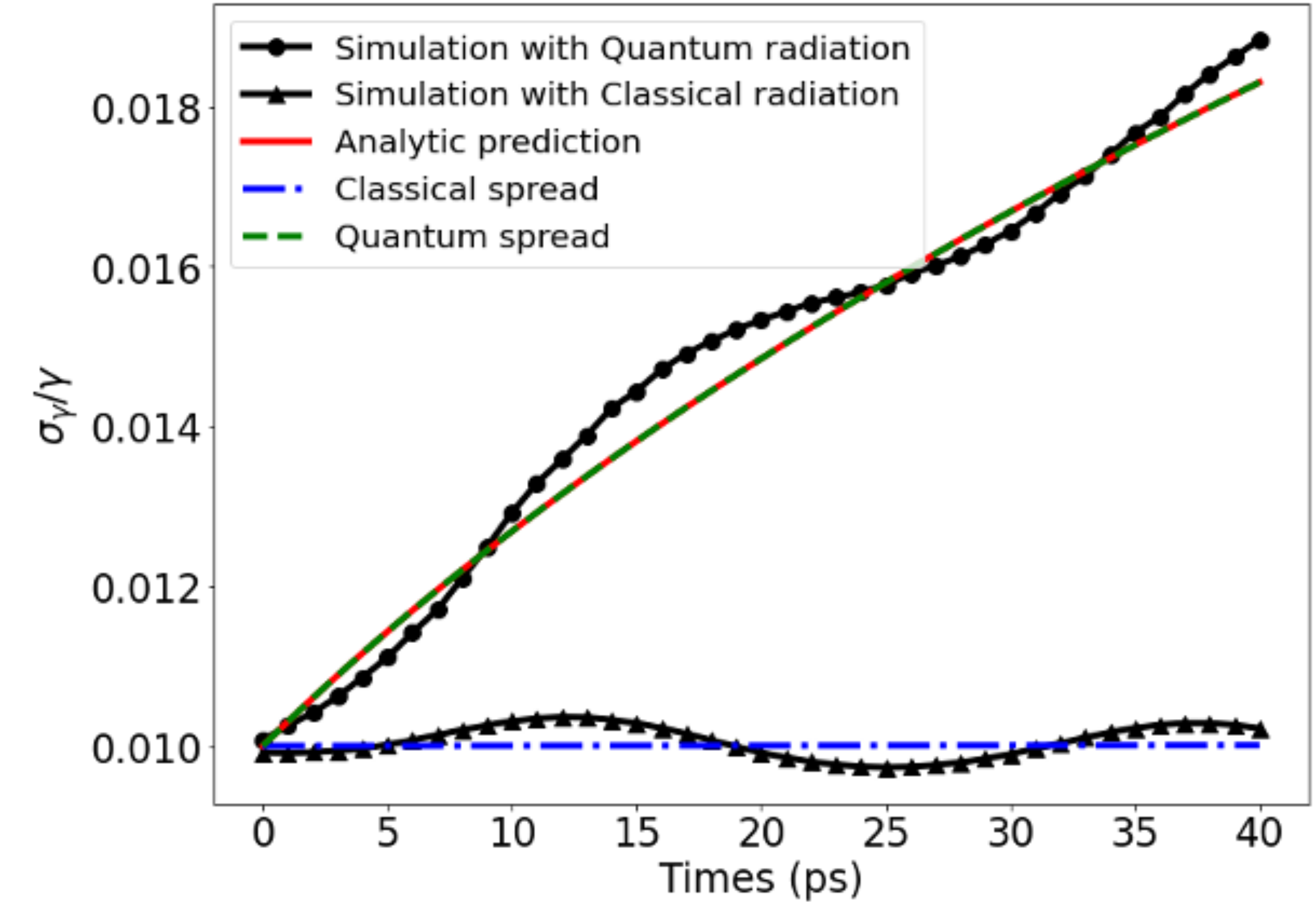
$$\left(\frac{\sigma_{\gamma}}{\gamma} \right)_{orb} = 5.36\% \times a_0^{-0.5} (n_0 [10^{18} \text{ cm}^{-3}]) (\varepsilon_n [\text{mm mrad}])$$

$$\left(\frac{\sigma_{\gamma}}{\gamma} \right)_{st} = 0.96\% \times a_0^{-0.25} (n_0 [10^{18} \text{ cm}^{-3}])^{0.875} (\varepsilon_n [\text{mm mrad}])^{0.75}$$

Ring distribution to reduce orbit effect induced energy spread



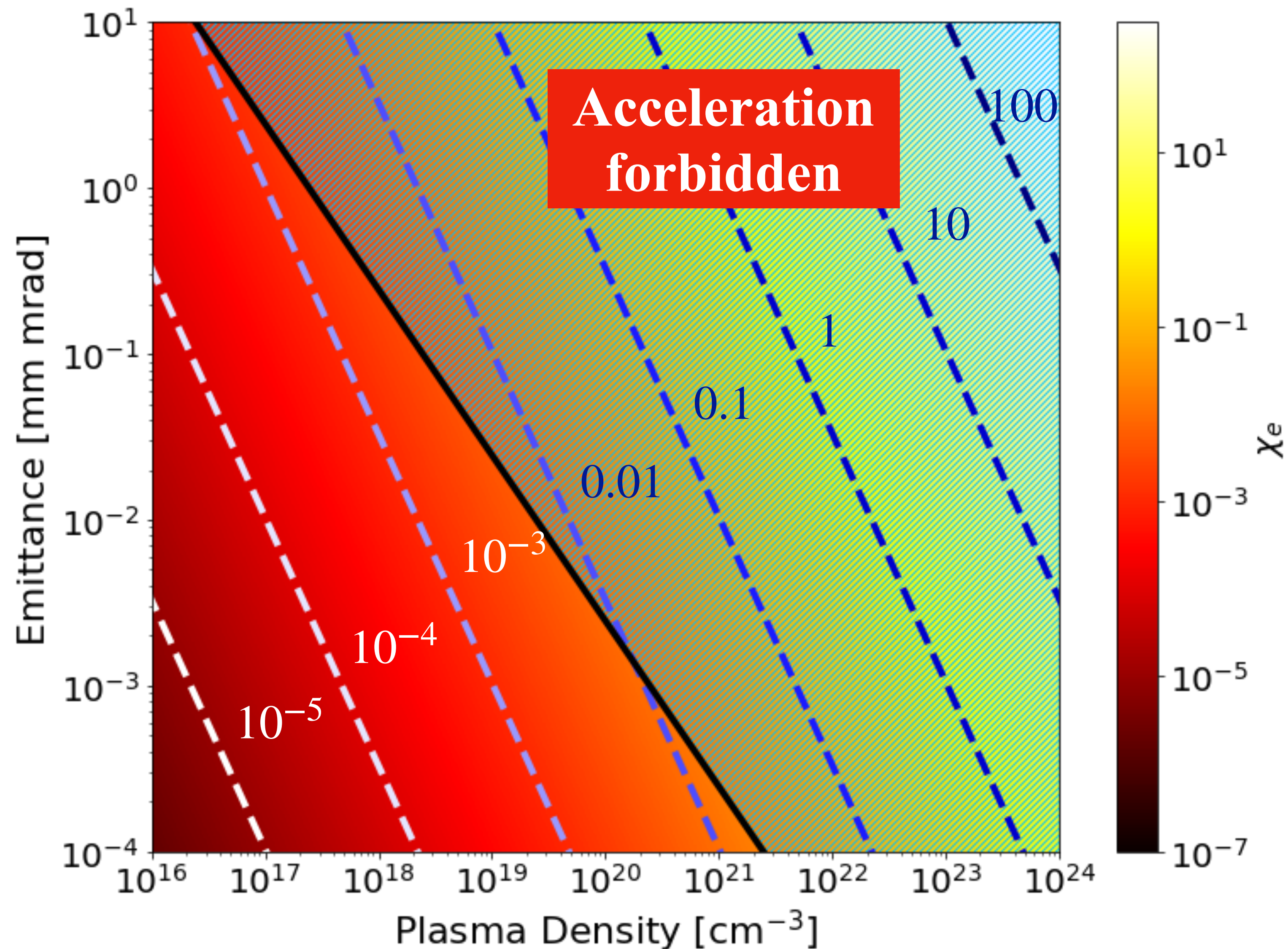
- Using ring shape distribution to reduce the orbit effect induced energy spread
- The energy spread is now mainly due to the quantum radiation effect



Quantum parameter for 10 TeV LWFA accelerator



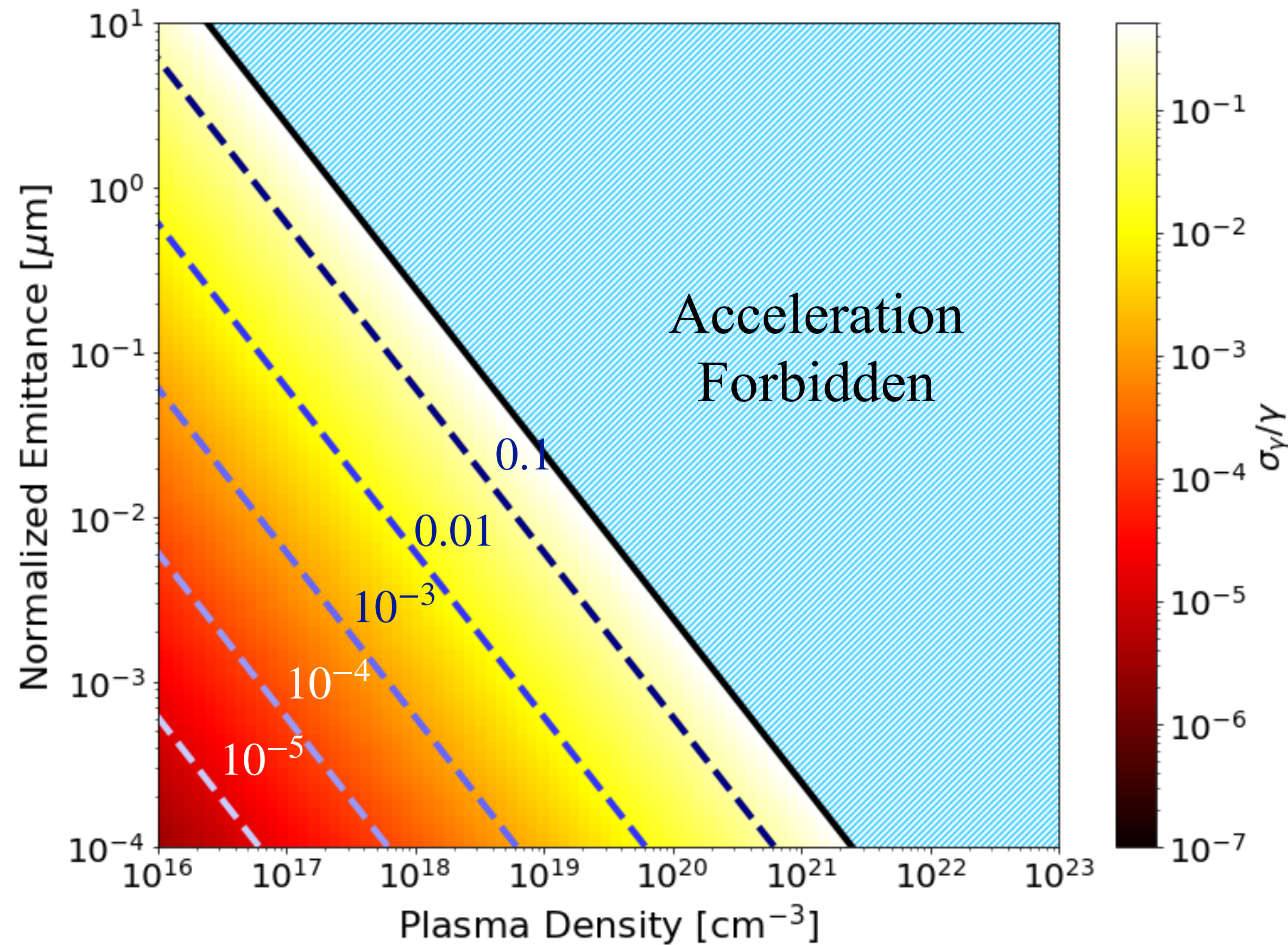
χ_e for beam inside wakefield at 10TeV



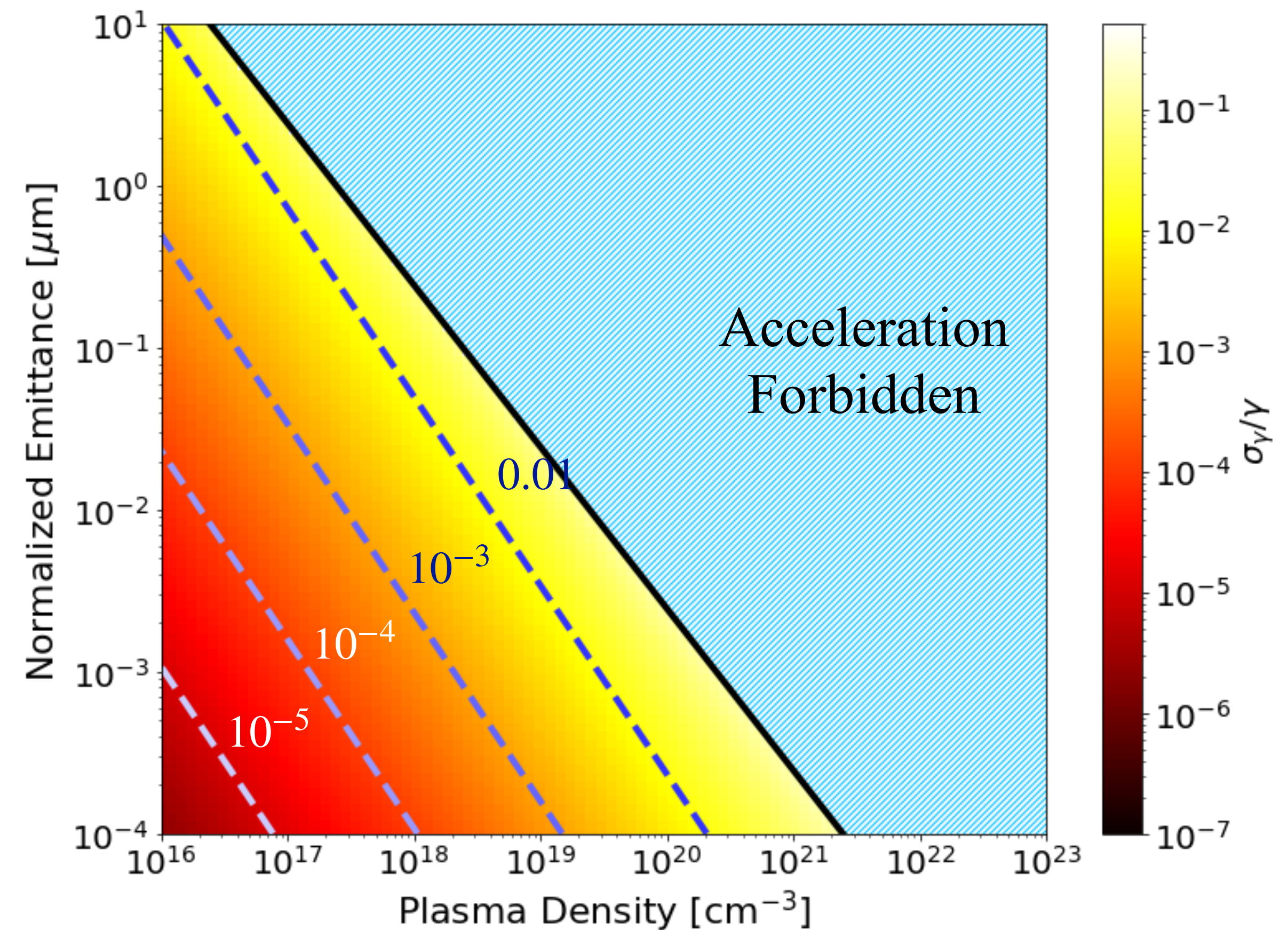
Radiation induced energy spread for 10 TeV LWFA based accelerator



Total Energy spread



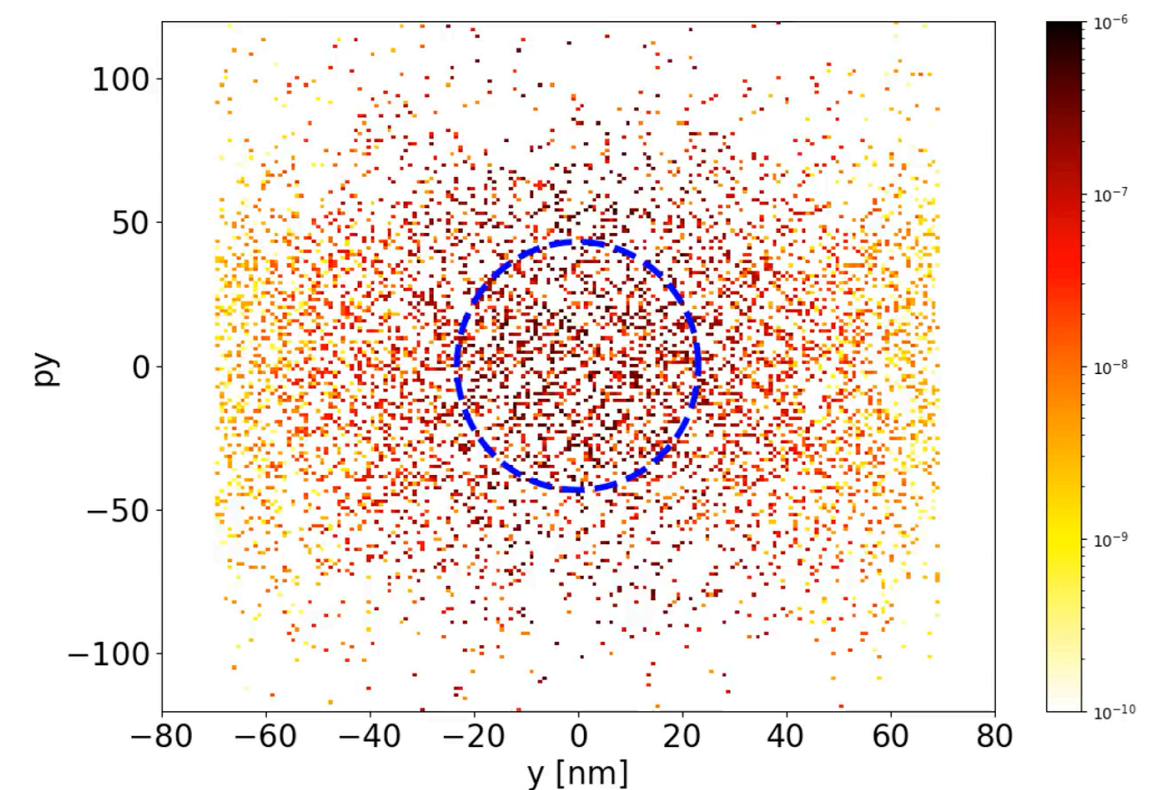
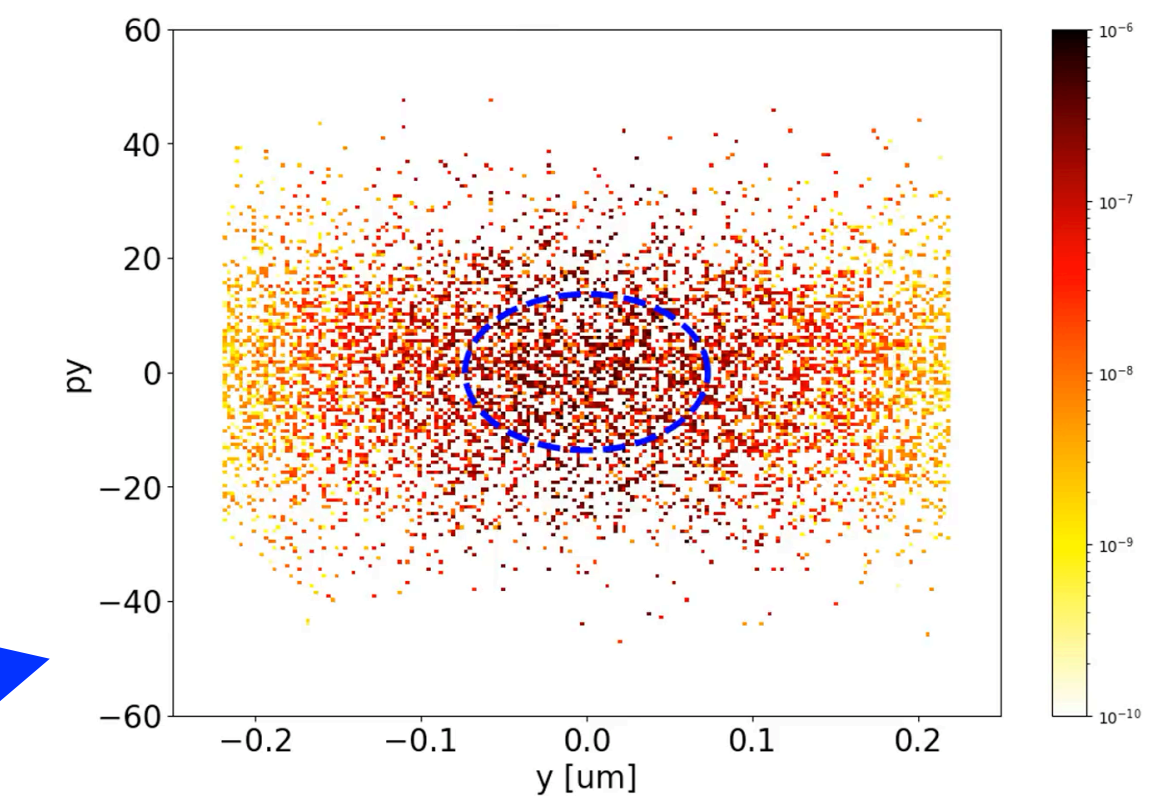
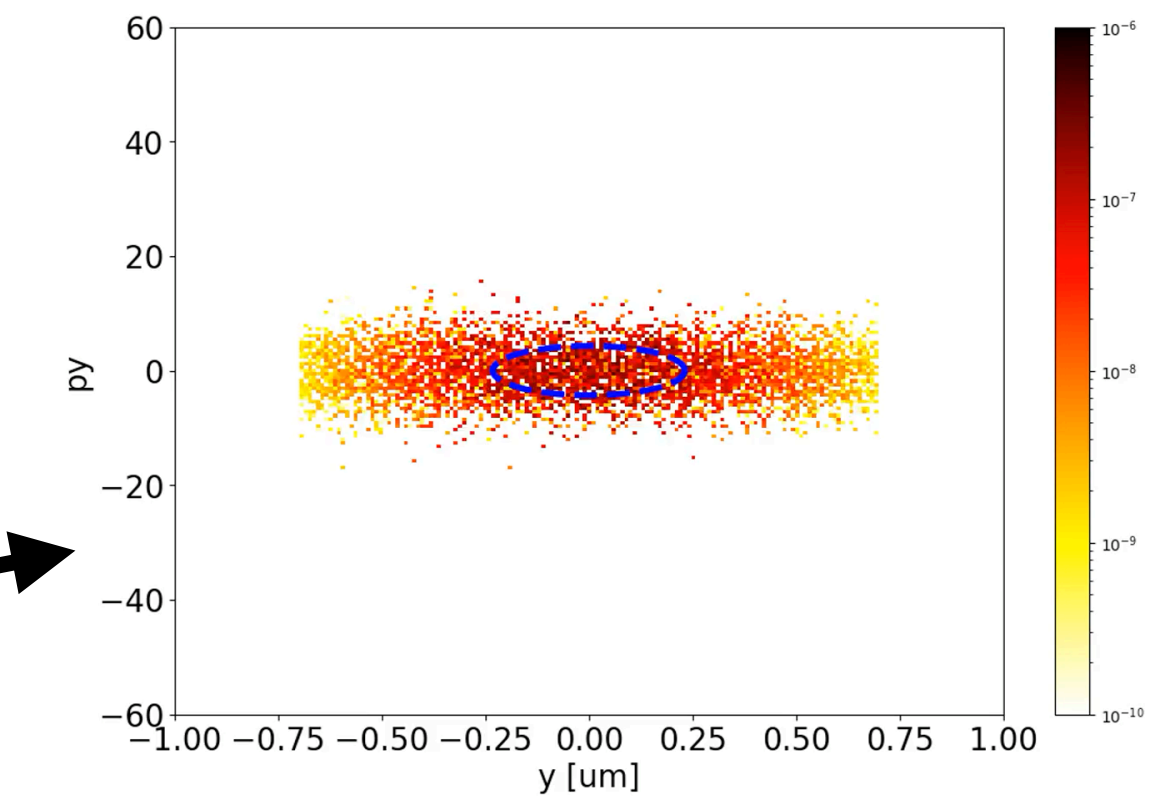
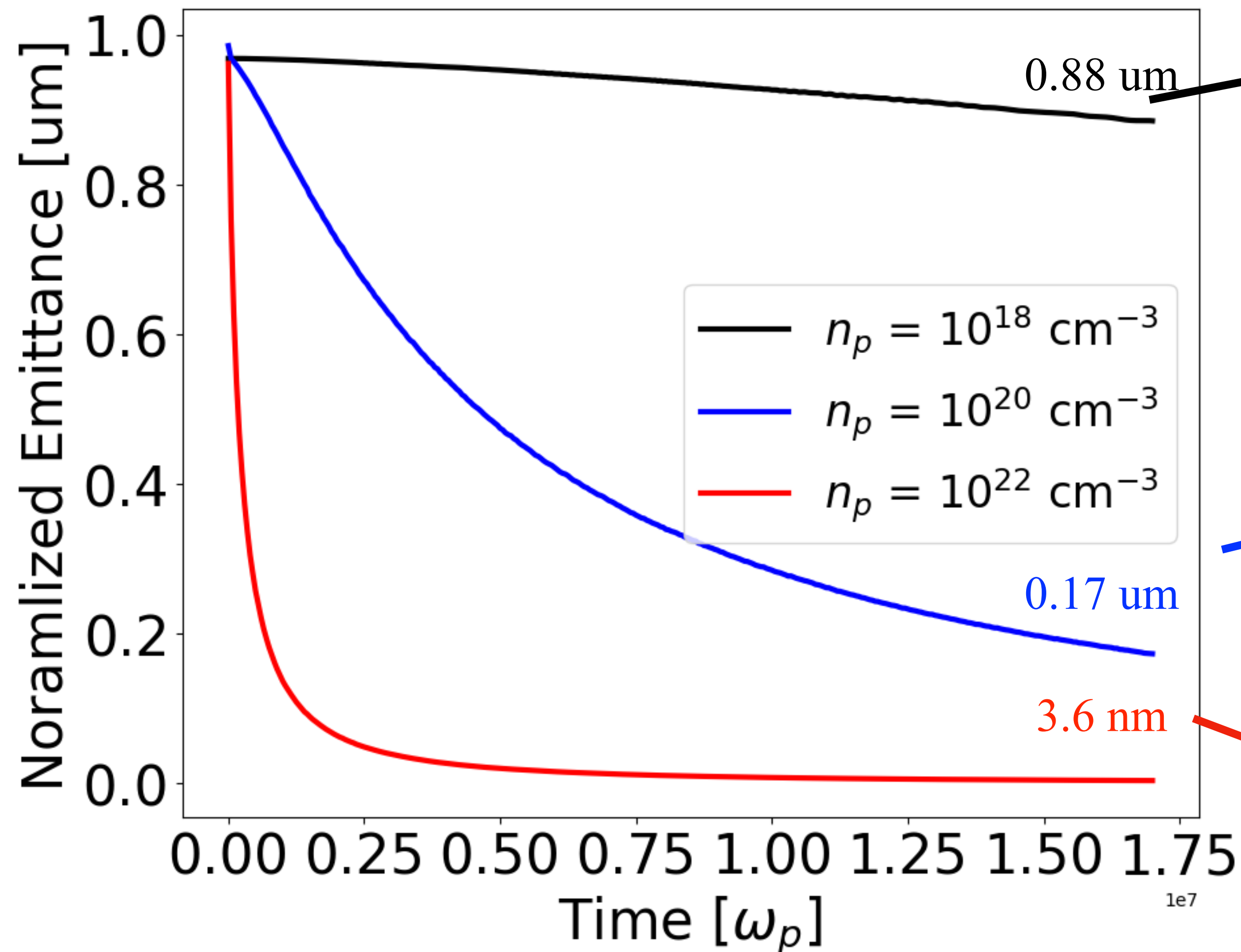
Energy spread from Stochastic broadening



Radiation induced Emittance cooling



Osiris 2D simulation: 10 GeV beam with a normalized emittance of 1 μm propagating inside an infinitely long ion channel with a constant acceleration field: $F_{acc} = \pi\sqrt{a_0}m_e c^2 (\lambda_p)^{-1}$

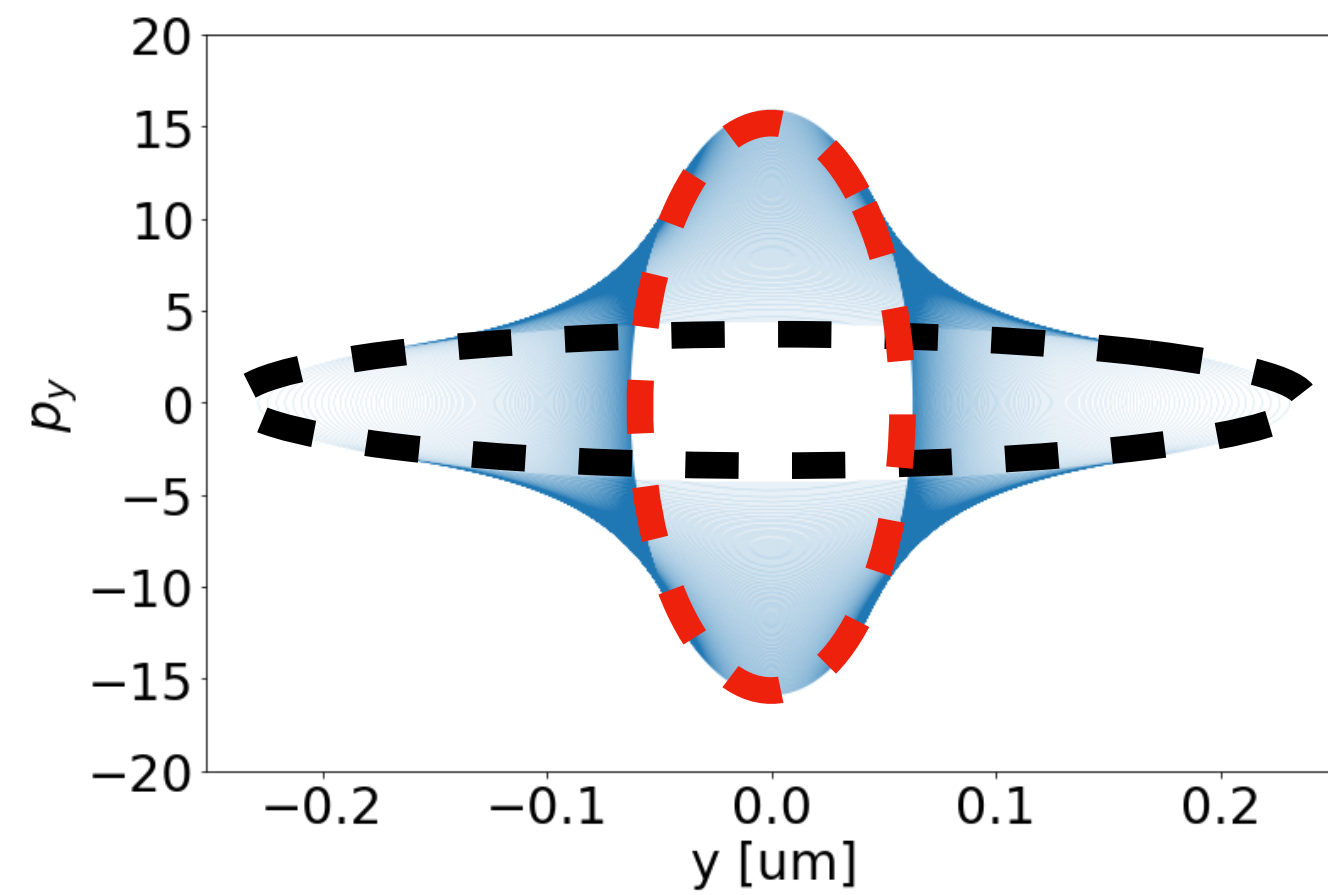


Radiation induced Emittance cooling

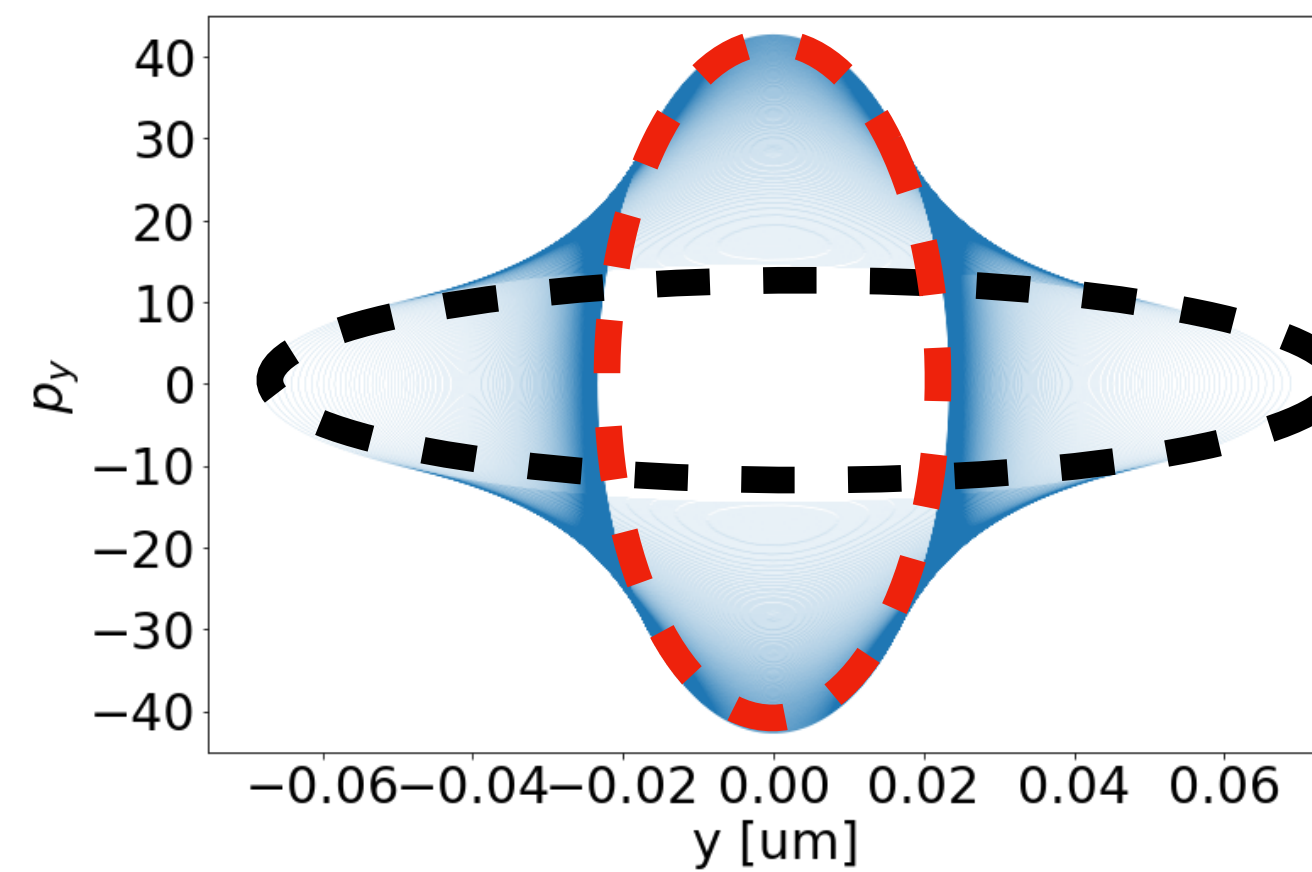


$$n_p = 10^{18} \text{ cm}^{-3}$$

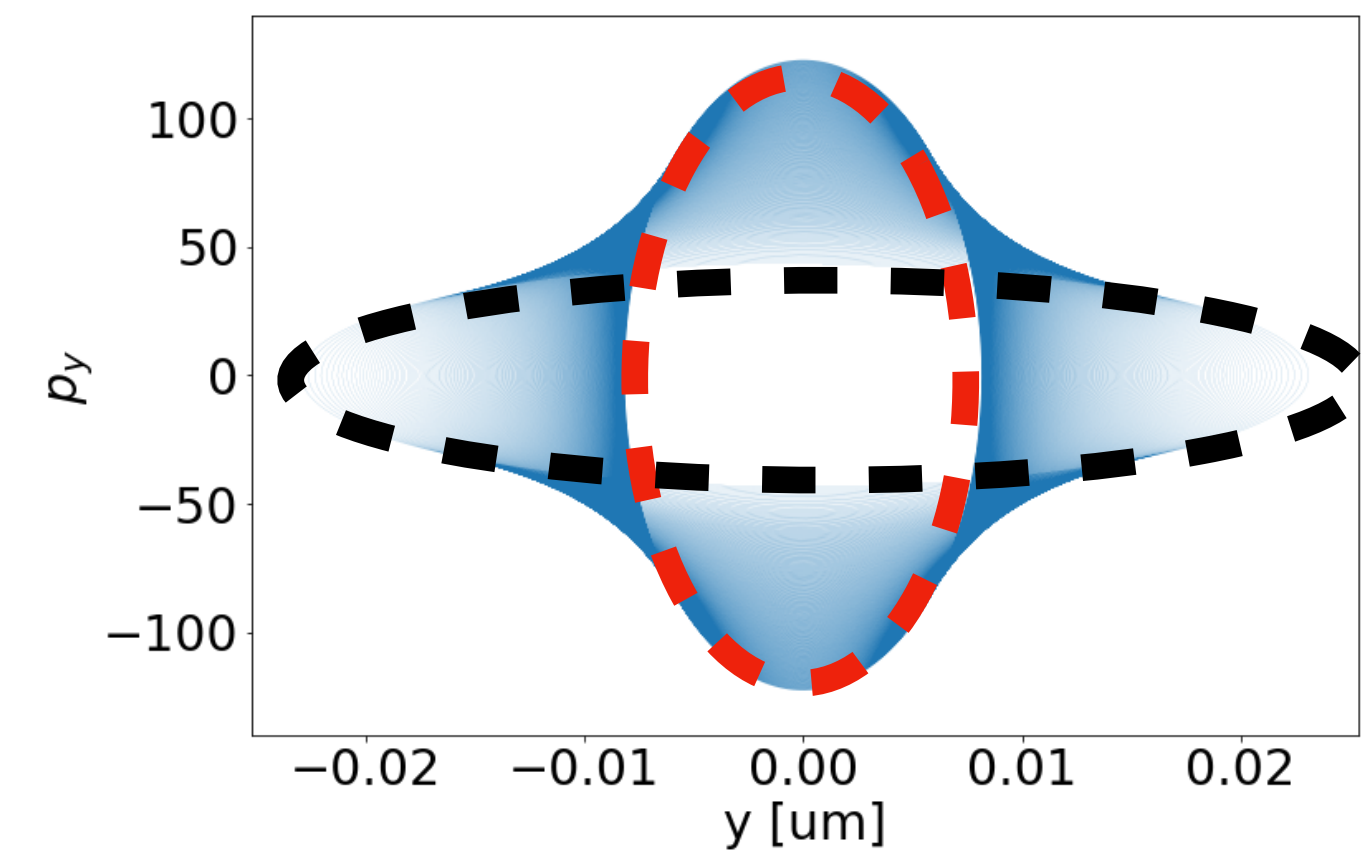
No radiation reaction



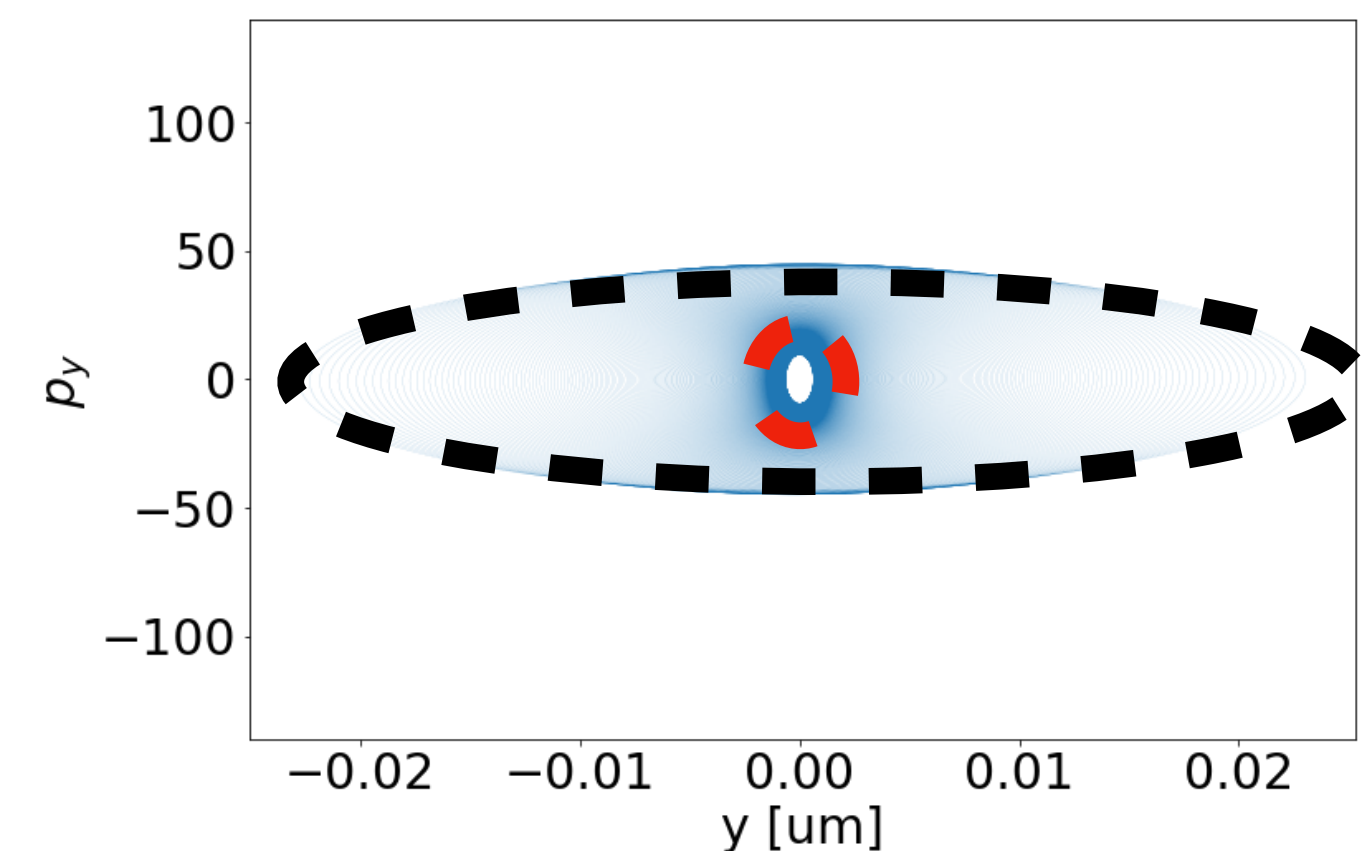
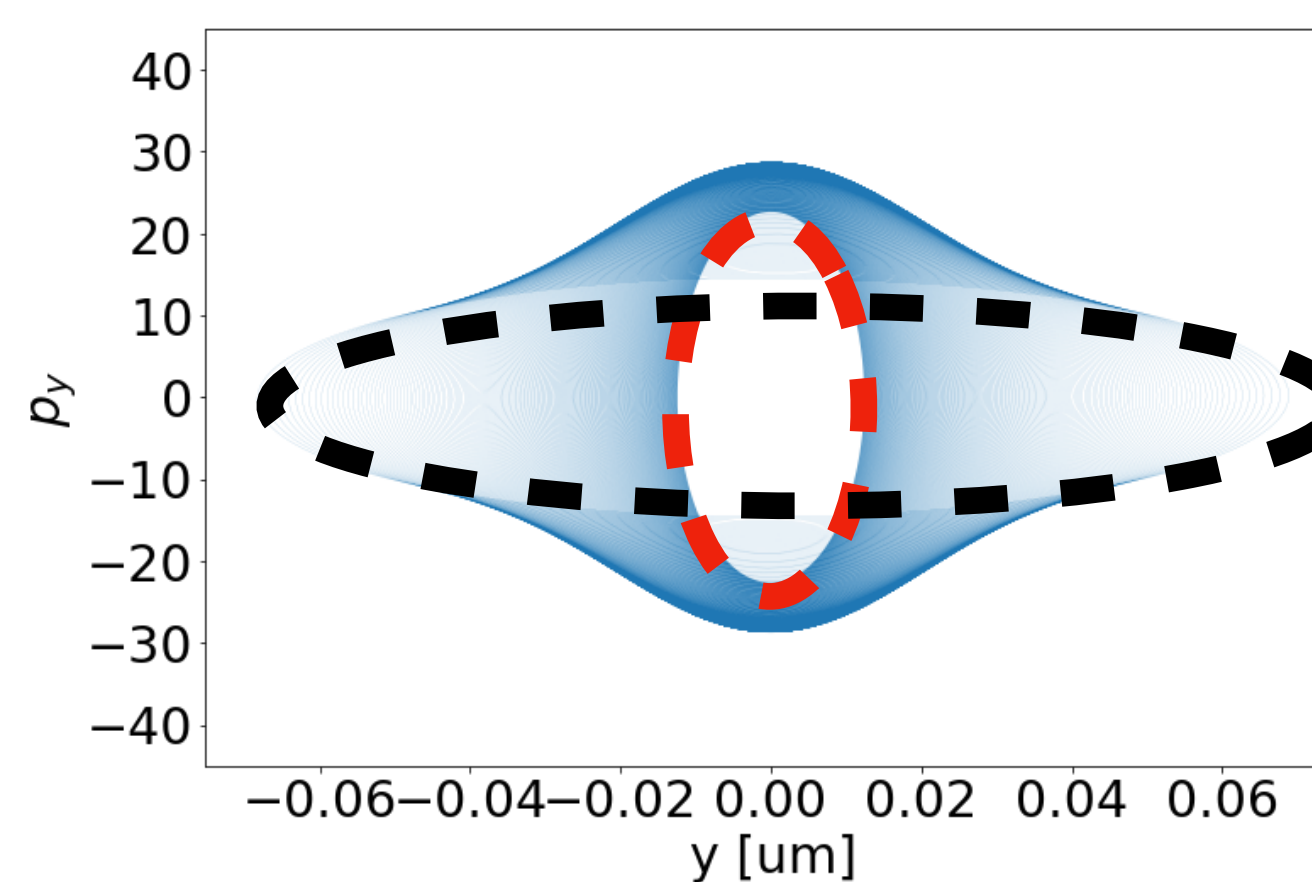
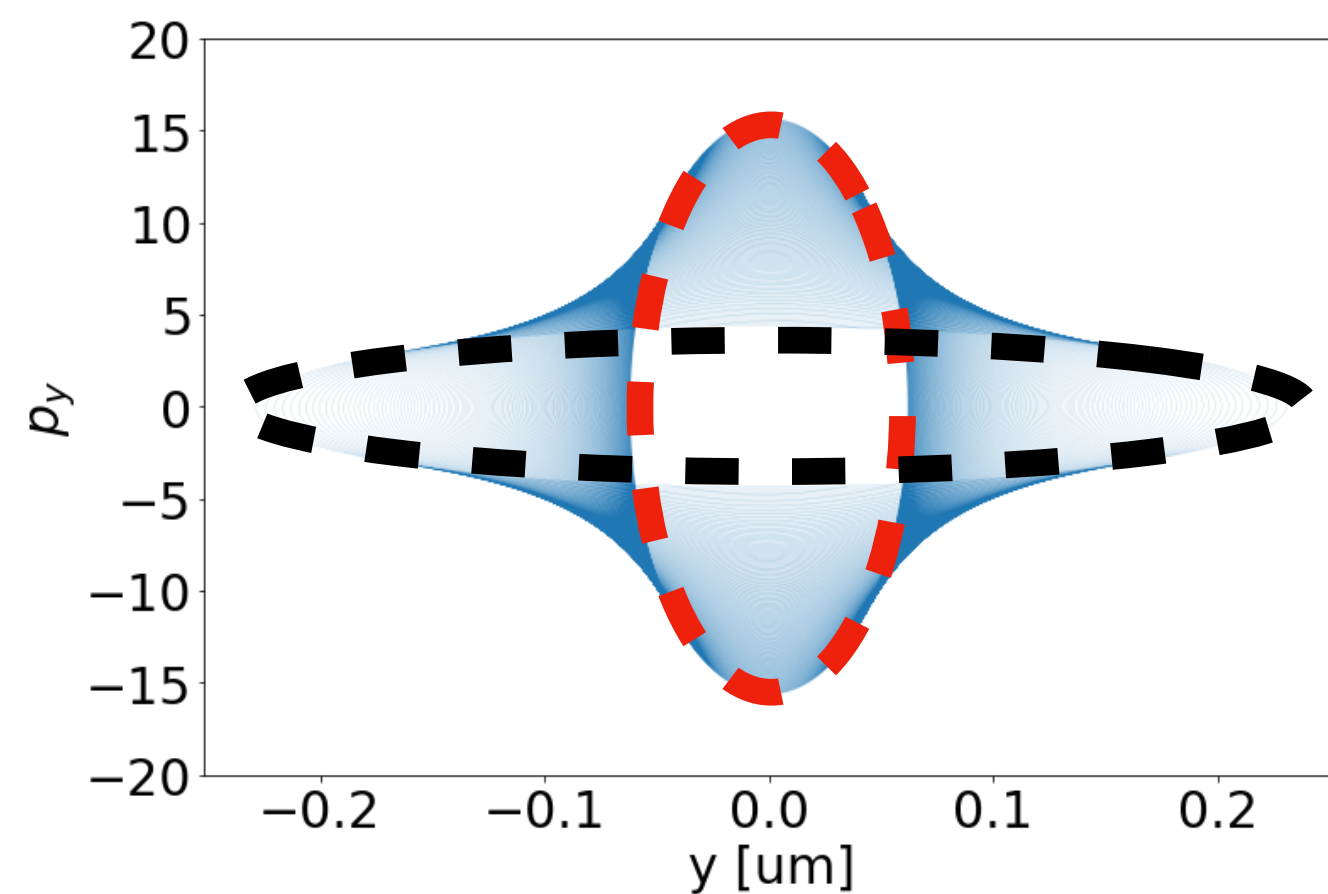
$$n_p = 10^{20} \text{ cm}^{-3}$$



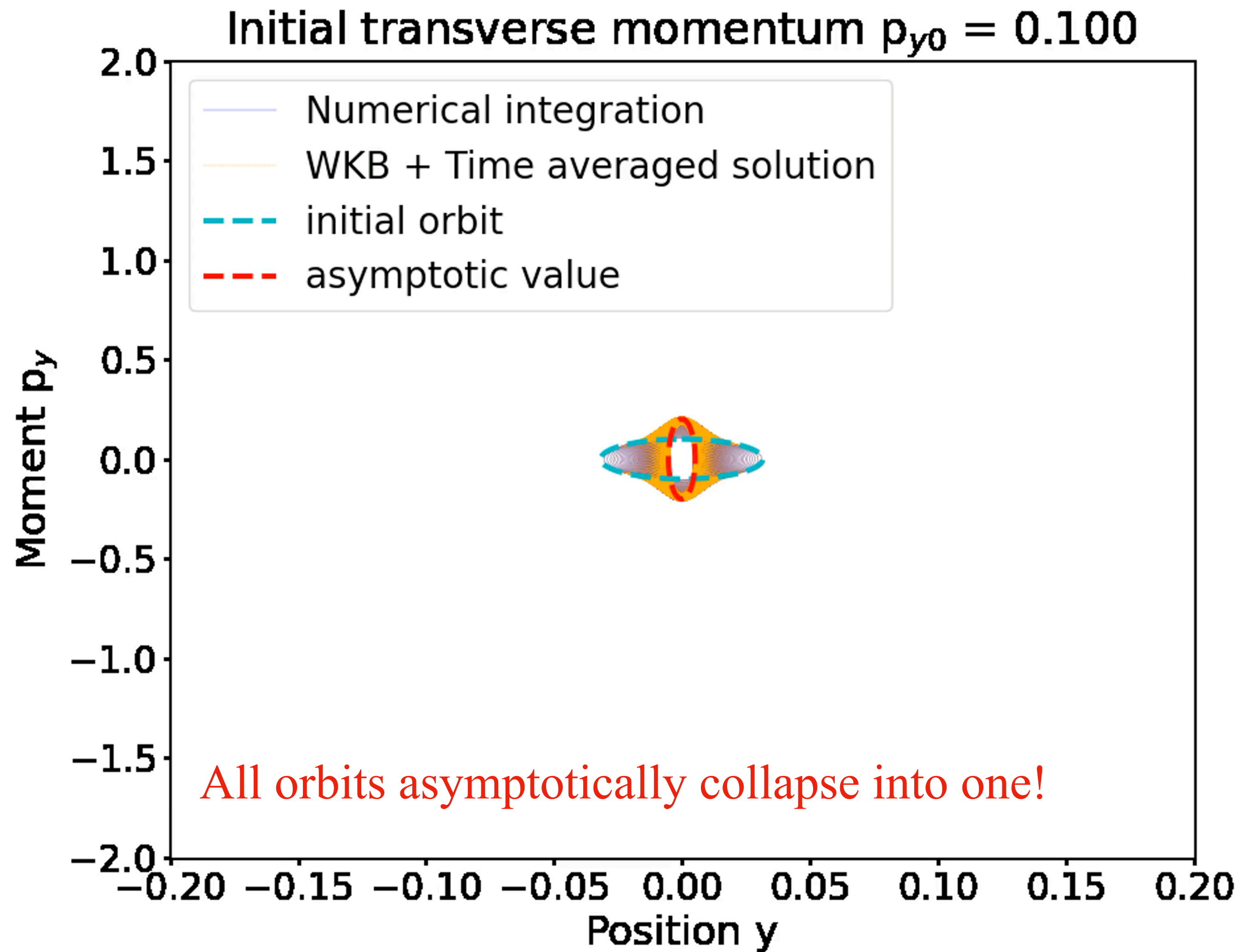
$$n_p = 10^{22} \text{ cm}^{-3}$$



With radiation reaction



Ring shape forming mechanism



$$\frac{d\gamma}{d\tau} = \gamma\kappa - \beta\gamma^3 y^2 \quad \frac{dp_y}{d\tau} = -\alpha\gamma y - \beta\gamma^2 y^2 p_y$$

κ is the linear acceleration, $\kappa \propto \sqrt{n_p}$

α is the focusing strength, $\alpha \propto \sqrt{n_p}$

β is the radiation reaction, $\beta \propto n_p$

Assume radiation reaction is small. Also energy change is slower than betatron oscillation

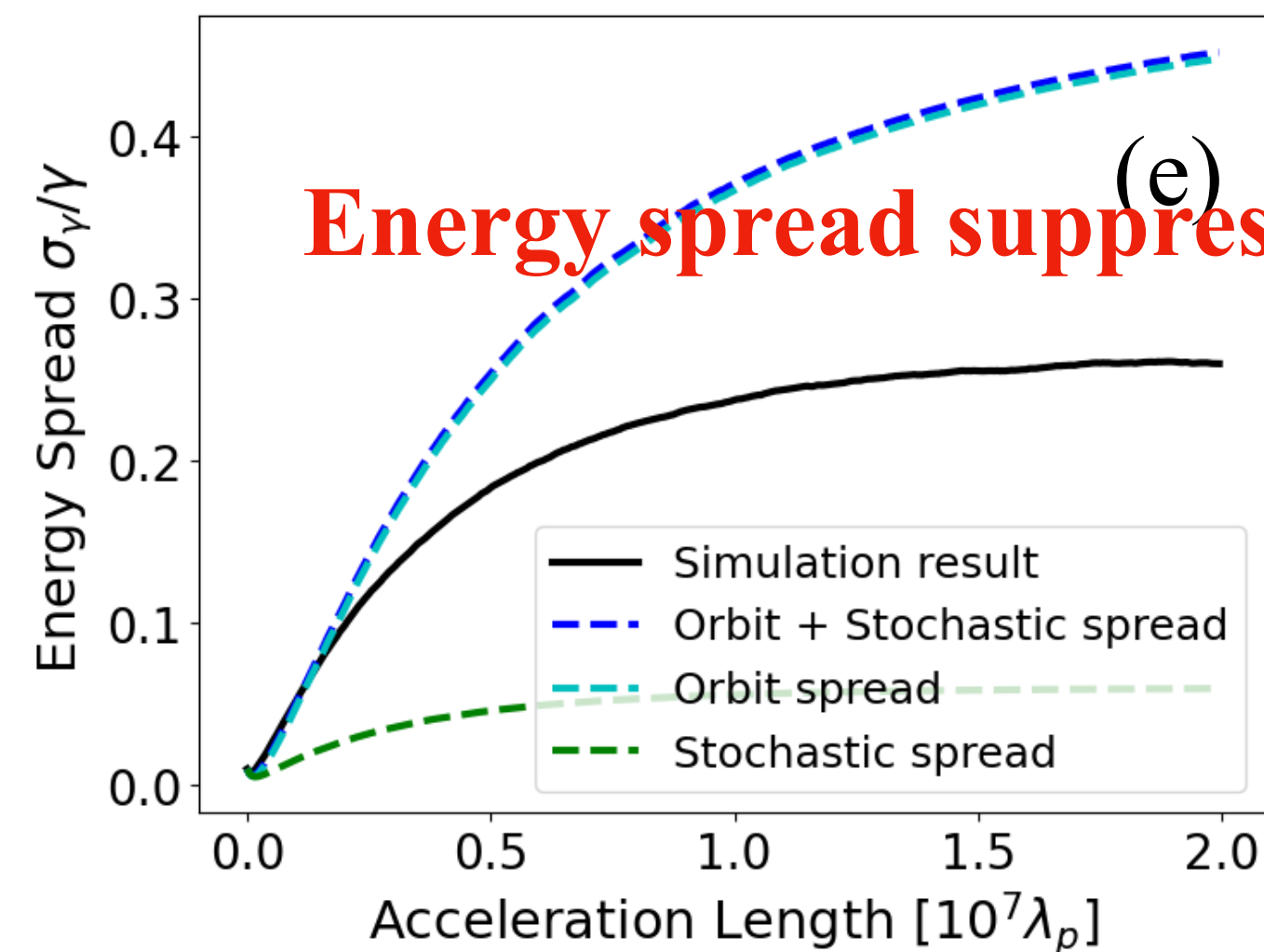
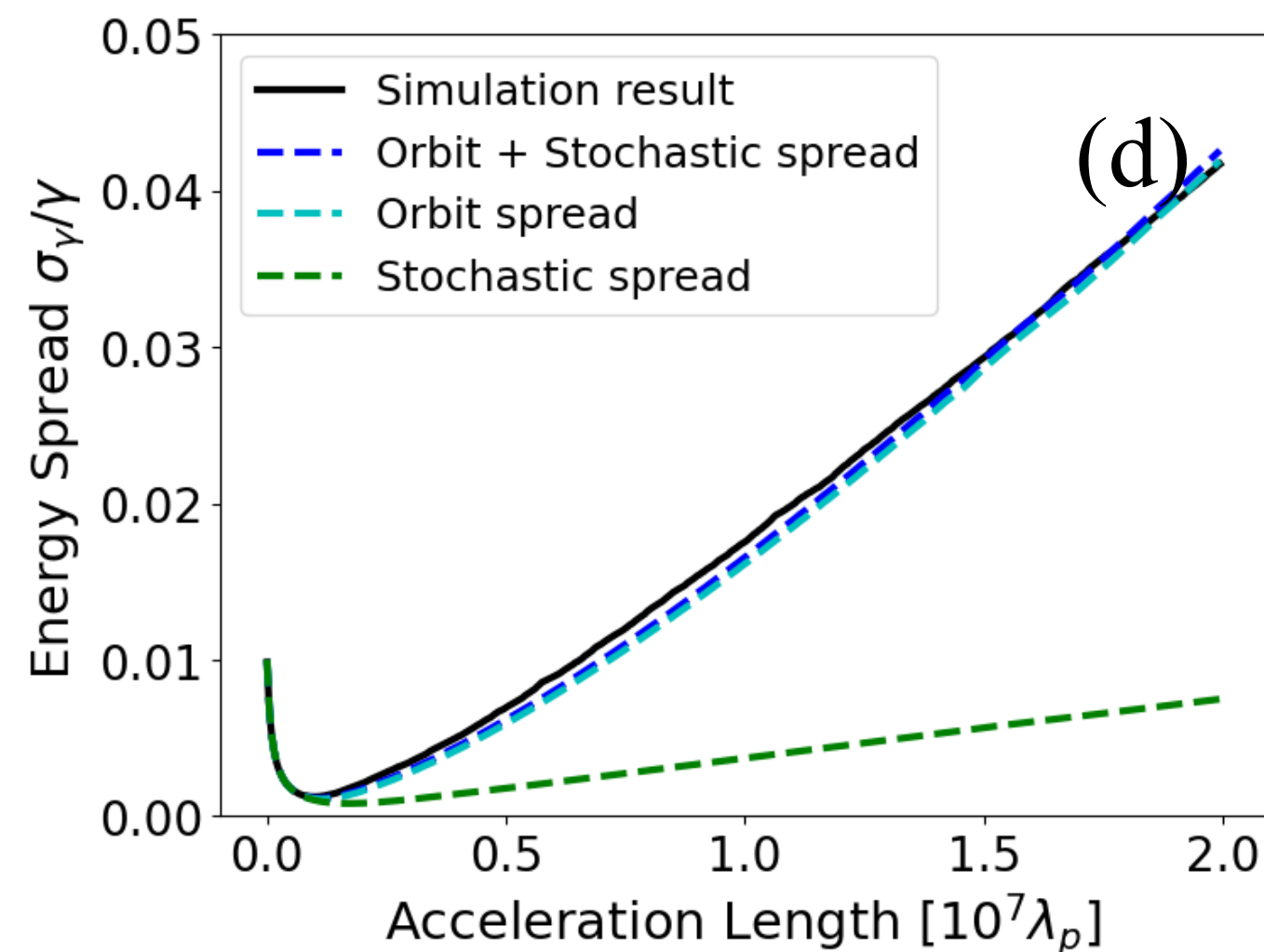
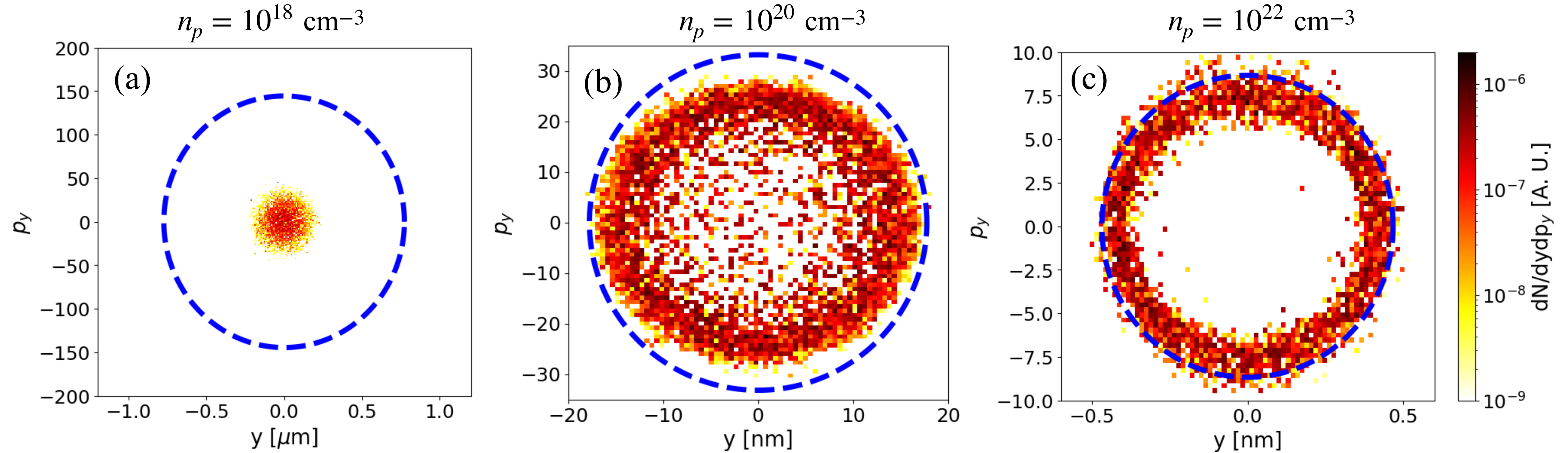
The evolution of $J = \frac{p_y^2}{2\sqrt{\alpha\gamma}} + \frac{y^2\sqrt{\alpha\gamma}}{2}$:

$$J \approx \frac{J_0}{1 + \frac{\beta\gamma_0^{3/2}}{3\sqrt{\alpha\kappa}} J_0 \left(\exp\left(\frac{3\kappa\tau}{2}\right) - 1 \right)}$$

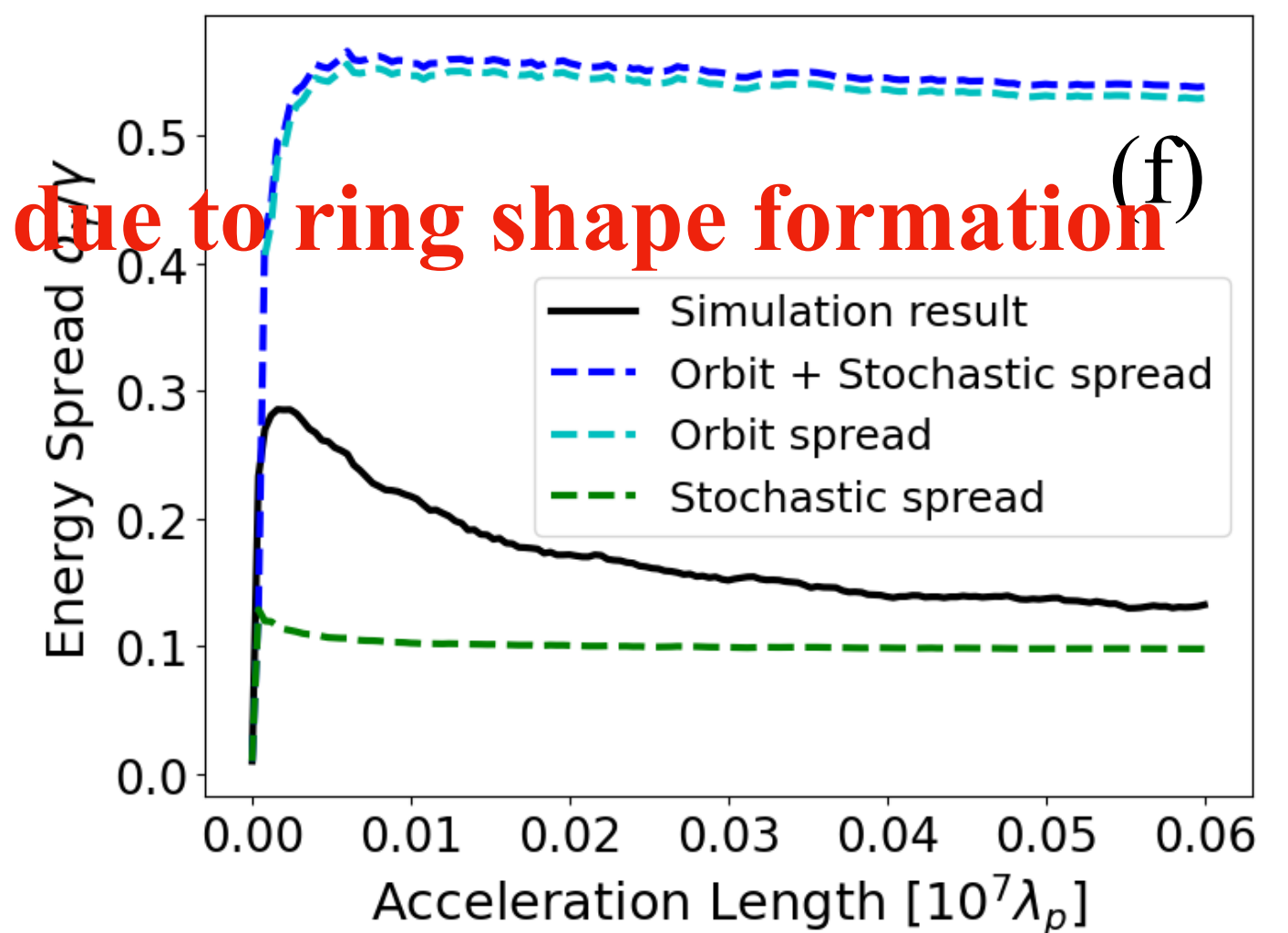
The asymptotic limit of J as τ is large is: $J_{\tau \rightarrow \infty} = \frac{3\sqrt{\alpha\kappa}}{\beta\gamma^{3/2}}$

$$|r|_{\tau \rightarrow \infty} = \frac{1}{\gamma} \sqrt{\frac{6\kappa}{\beta}} \quad |p_r|_{\tau \rightarrow \infty} = \sqrt{\frac{6\alpha\kappa}{\gamma\beta}} \quad \text{Independent on initial condition!}$$

Ring shape forming mechanism



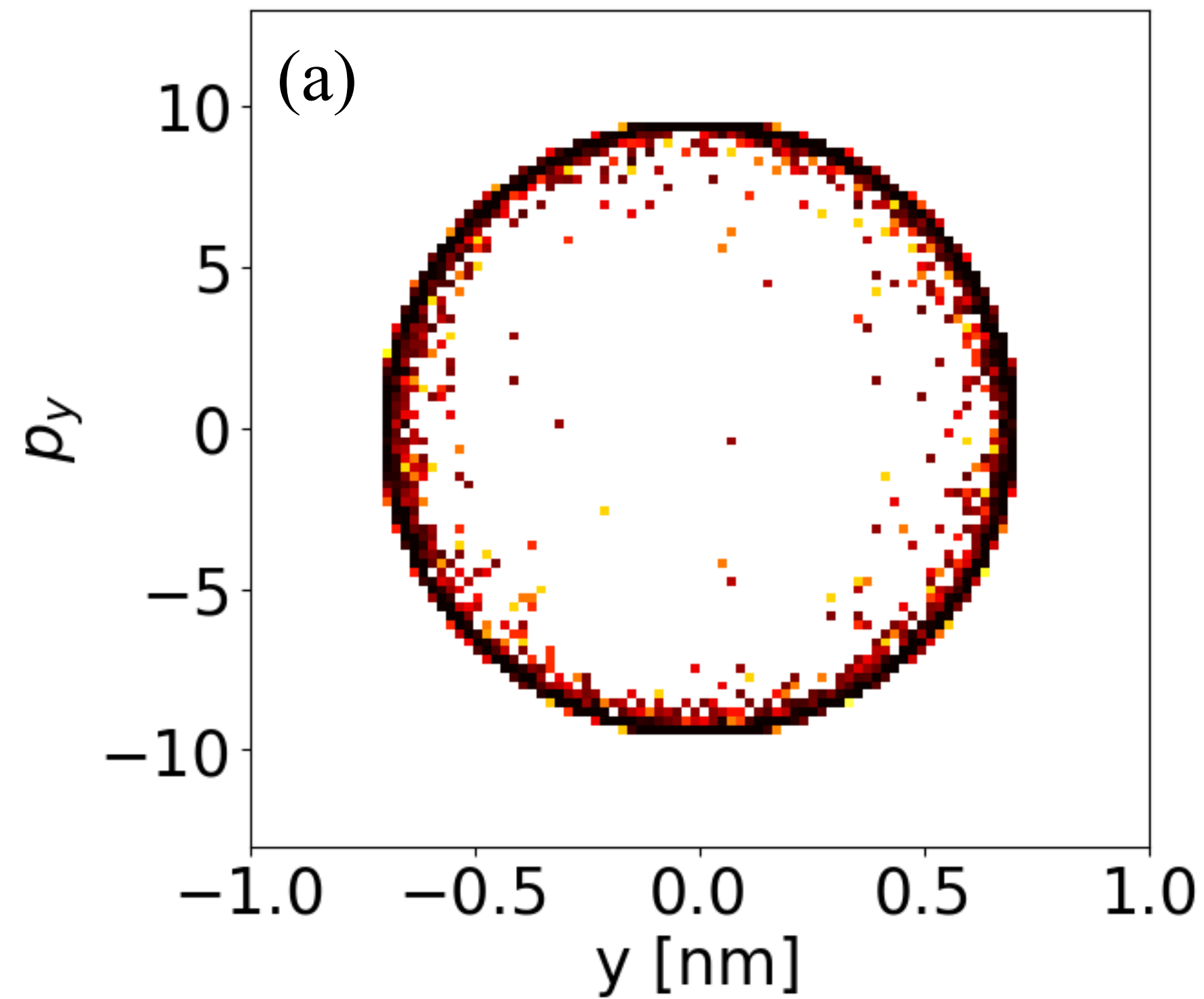
Energy spread suppression due to ring shape formation



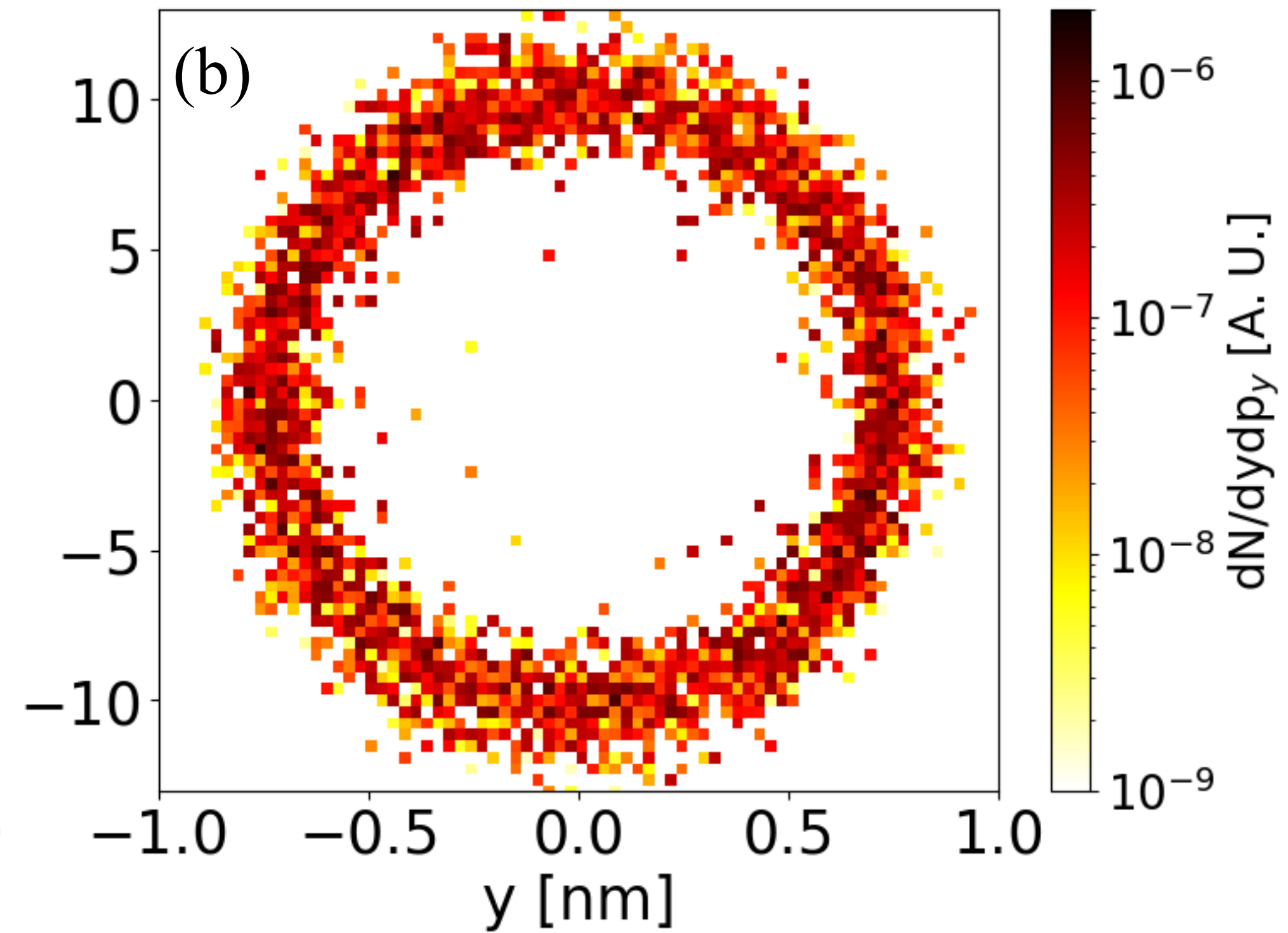
Ring shape forming mechanism



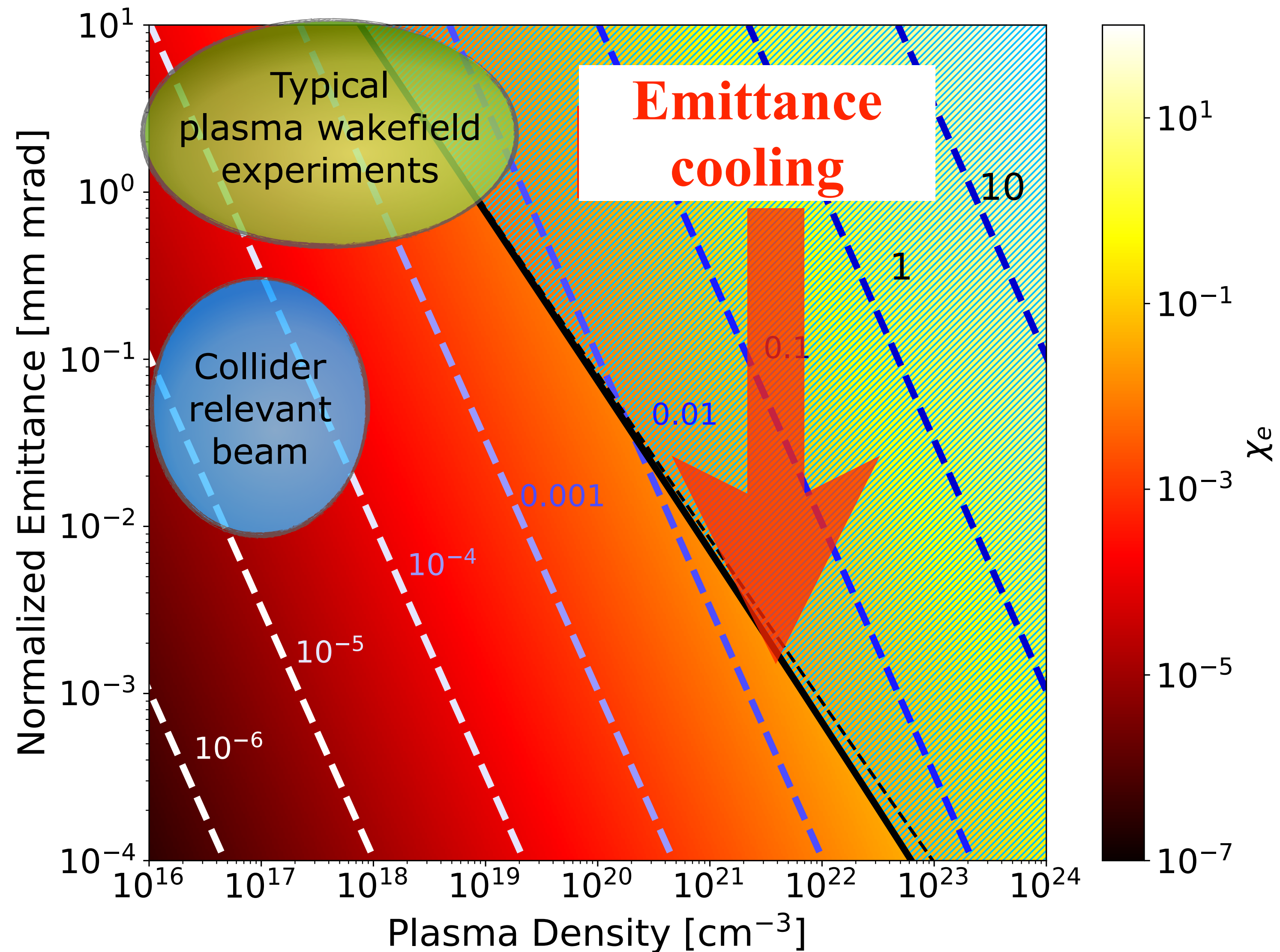
Classical Radiation reaction



Quantum Radiation reaction



Radiation induced Emittance cooling

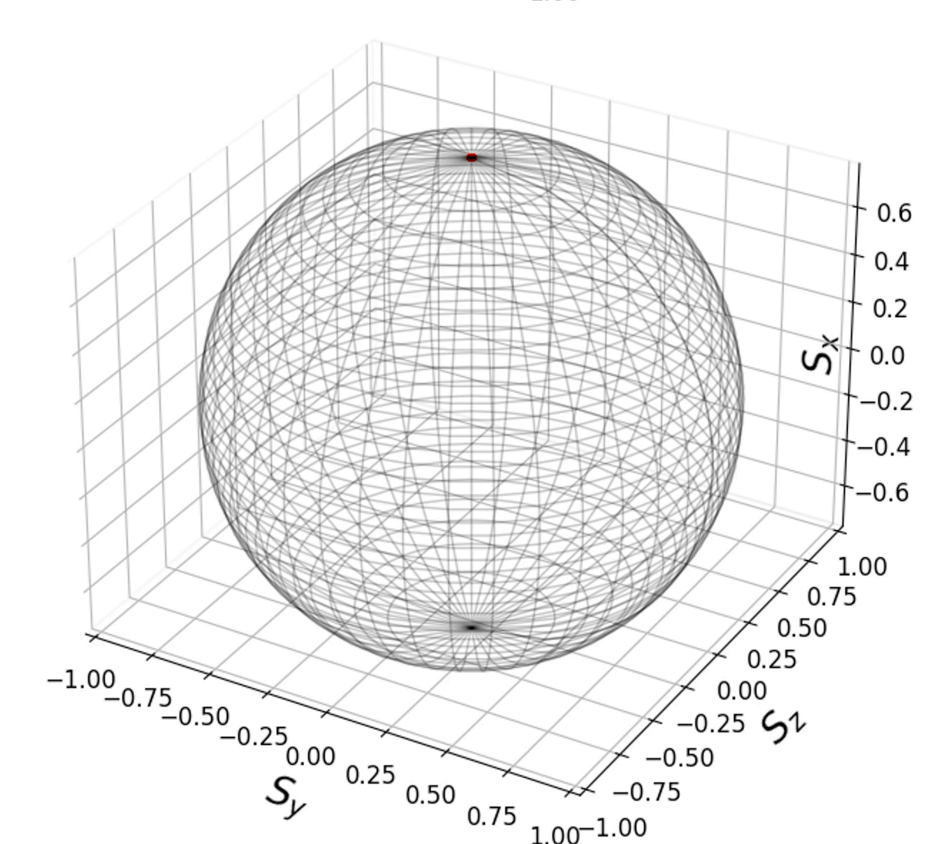
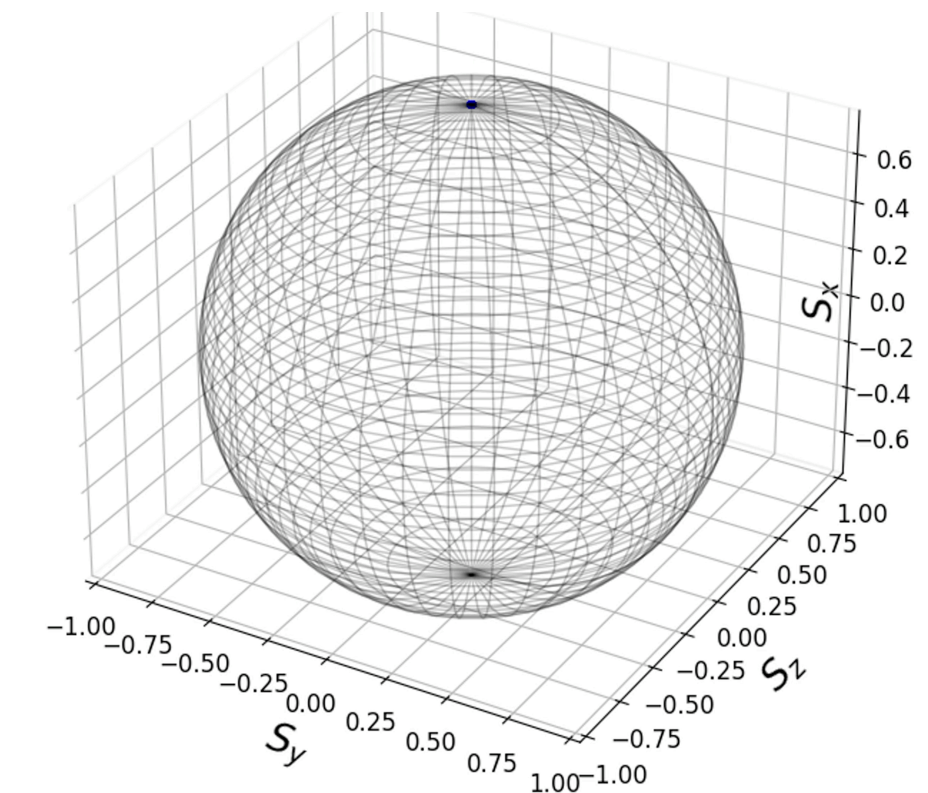
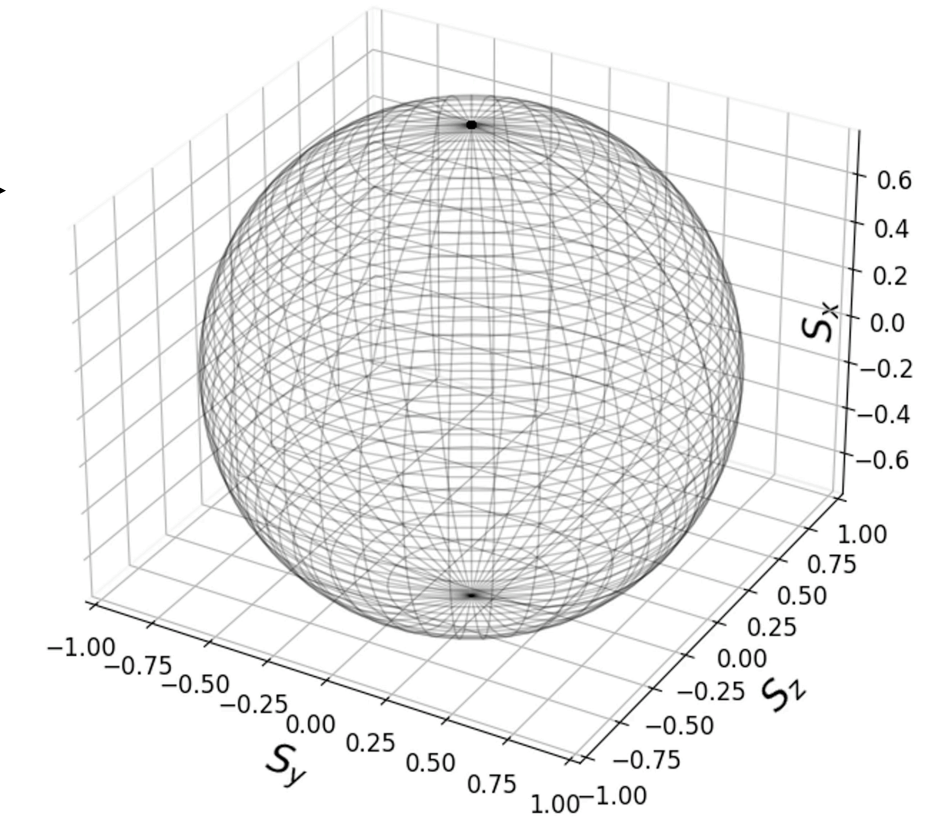
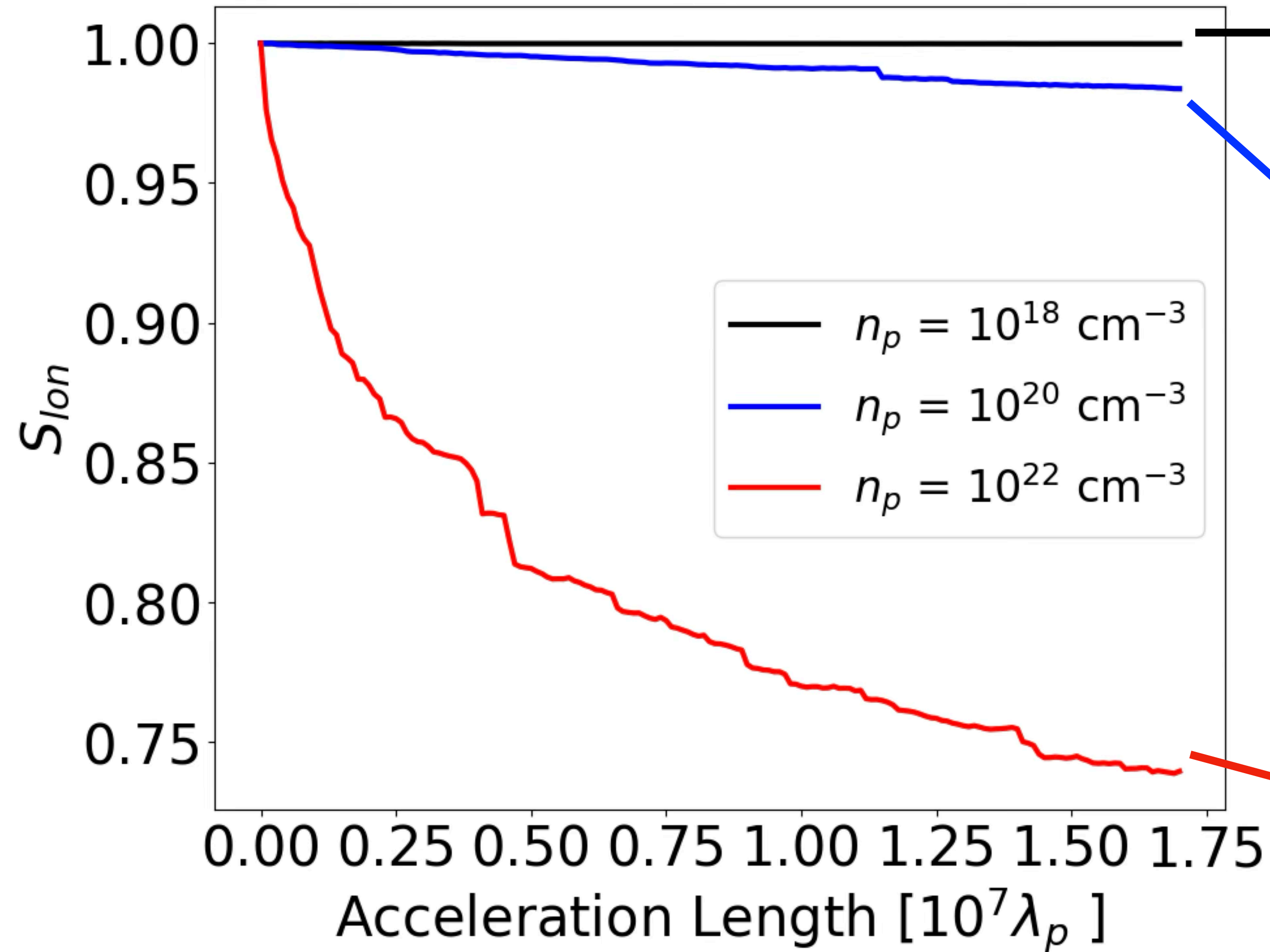


Due to the radiative emittance cooling, it is possible to use high density LWFA to accelerate beam to TeV

QED effect on spin transportation:



Begin with a longitudinally polarized beam



$$S \approx \frac{10^{-4}}{\sqrt{a_0}} (\mathcal{E}_{beam} [\text{TeV}])^{5/4} \times (n_p [10^{18} \text{ cm}^{-3}])^{7/4} \times (\epsilon_n [\mu\text{m}])^{3/2}$$

- The quantum parameter for a 1 TeV electron beam inside the wakefield could be about 10^{-3} (For 10 TeV is 5.6×10^{-3}). Although small, quantum effects on the beam energy spread can accumulate and eventually become unignorable.
- Radiative emittance cooling acting together with the evolution of the matched beam leads to a ring-shaped distribution in transverse phase space, which reduces radiation induced energy spread.
- Generally, the radiative effect does not influence the polarized beam transportation inside the laser wakefield for lower density.
- The emittance cooling effect may be a consideration for laser produced positron beams, which have much higher emittance

A wide-angle photograph of a university campus during autumn. In the foreground, there are white daisy-like flowers. A large, rectangular pond with a central fountain is the middle ground. A paved walkway with benches and black lampposts runs along the pond. In the background, a large, multi-story brick building with a green roof and glass sections is visible. The sky is blue with scattered white clouds. The word "Questions" is written in large white letters across the pond.

Questions

Backup

An analytic model for predicting energy spread growth



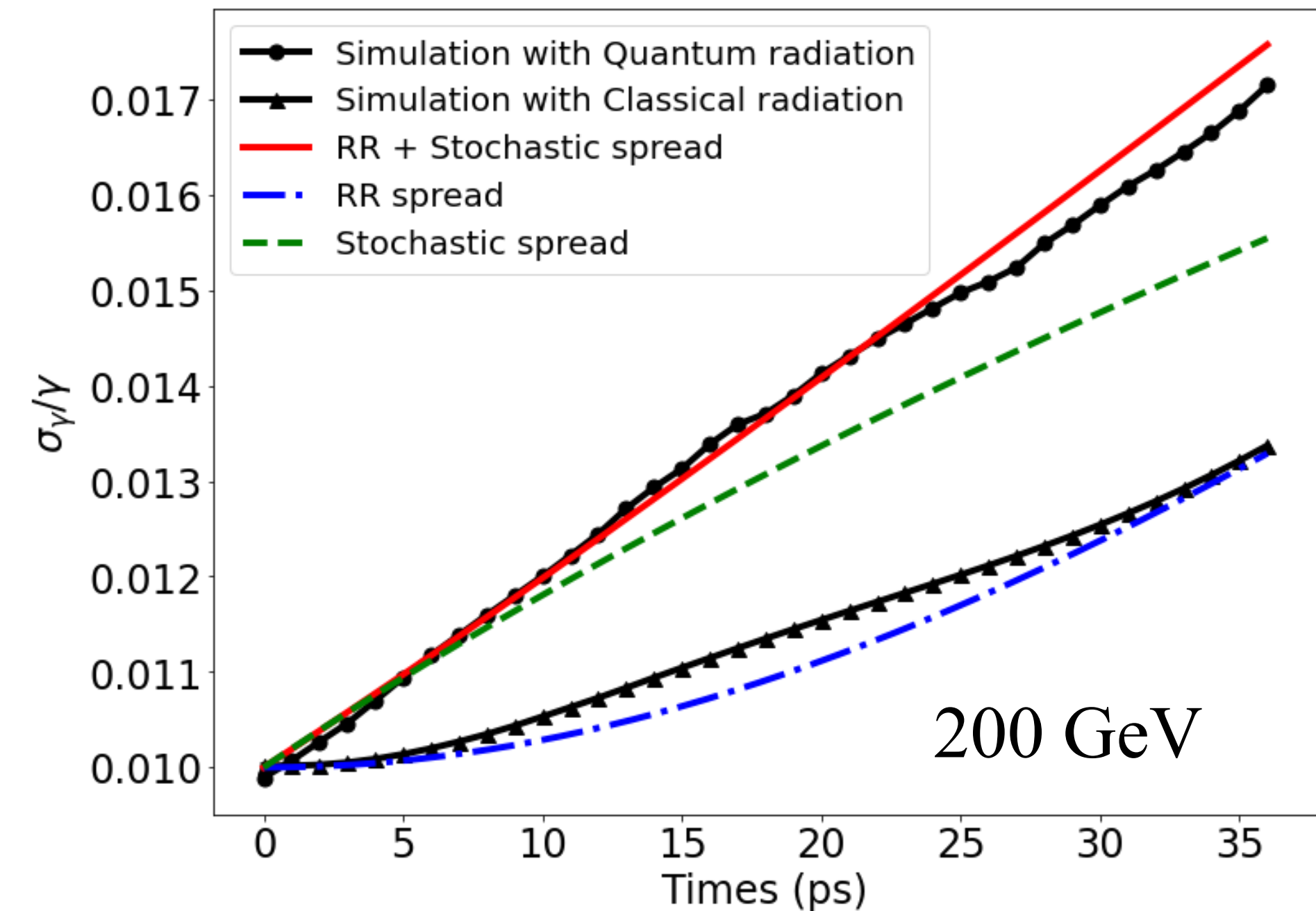
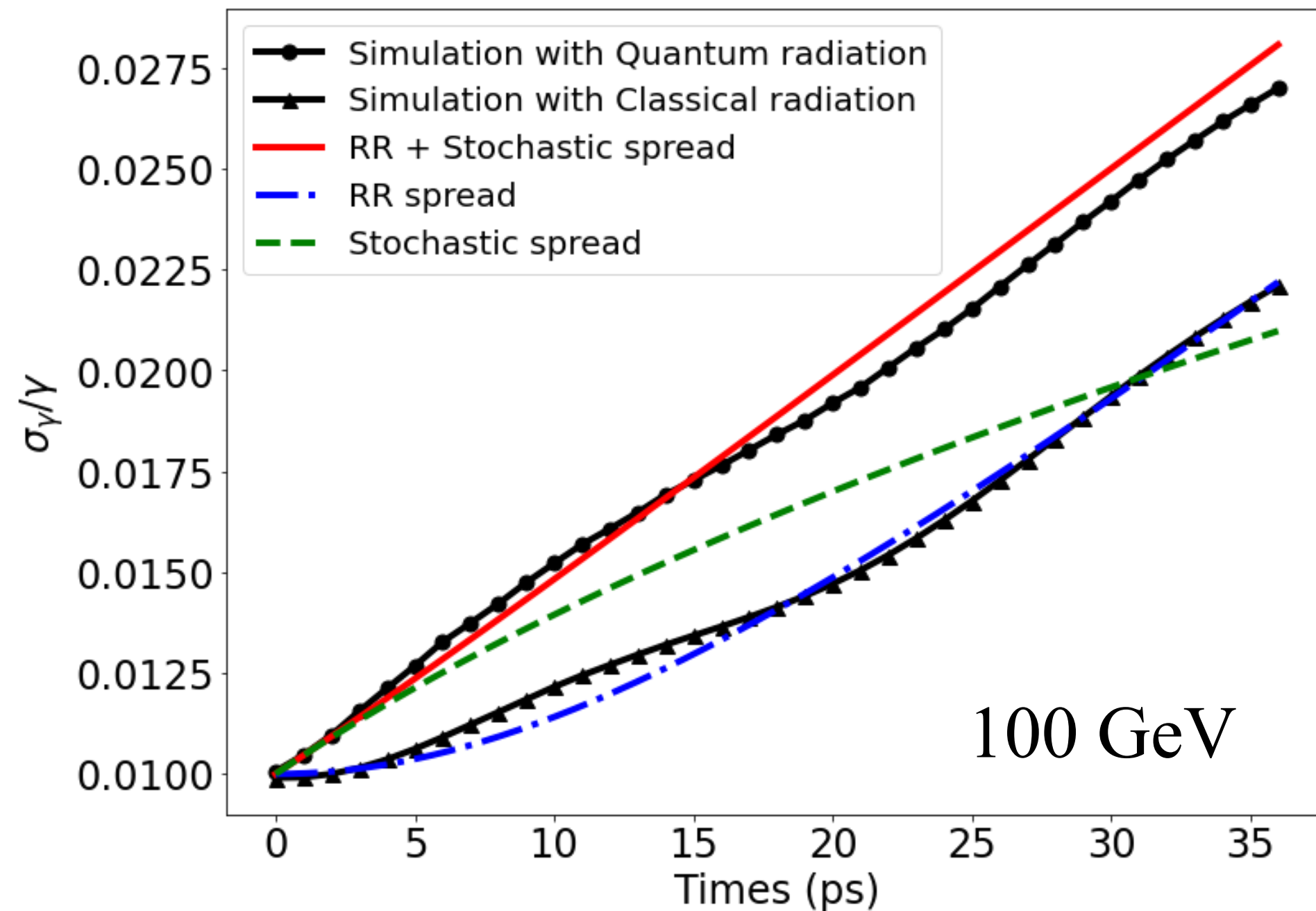
$$\frac{d}{dt} \left(\sigma_\gamma \right)_{orb}^2 = \frac{d}{dt} \left(\frac{F_{rad}(\varepsilon_n)t}{m_e c} \right)^2 = \frac{32\pi^2}{9} \left(\frac{\alpha_f c}{\lambda_c} \right)^2 \chi_e^4 g_1^2(\chi_e) t$$

$$\frac{d}{dt} \left(\sigma_\gamma \right)_{st}^2 = \frac{55\pi\alpha_f c}{12\sqrt{3}\lambda_c} \gamma \chi_e^3 g_2(\chi_e)$$

$$\frac{d}{dt} \left(\sigma_\gamma \right)_{cool}^2 = \frac{d}{dt} (2\Delta\gamma F_{rad}) = -\frac{16\pi\alpha_f c}{3\lambda_c \gamma} \chi_e^2 \sigma_\gamma^2 g_1(\chi_e)$$

$$\frac{d}{dt} \left(\sigma_\gamma \right)^2 = \frac{d}{dt} \left[\left(\sigma_\gamma \right)_{cool}^2 + \left(\sigma_\gamma \right)_{orb}^2 + \left(\sigma_\gamma \right)_{st}^2 \right]$$

$$\approx -C_{cool} \chi_e^2 \left(\sigma_\gamma \right)^2 + C_{orb} \chi_e^4 t + C_{st} \chi_e^3$$

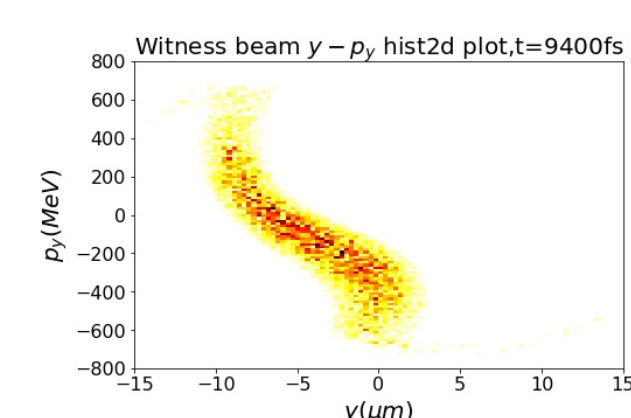
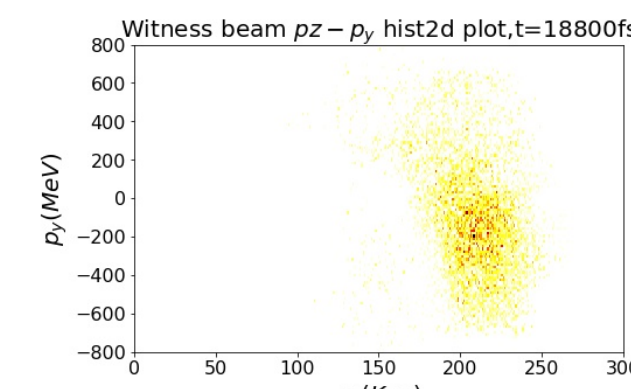
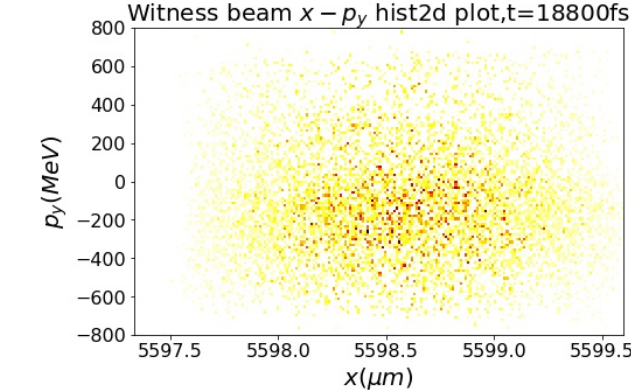
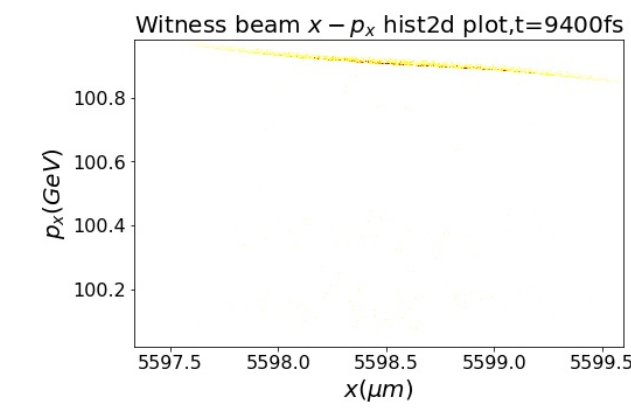
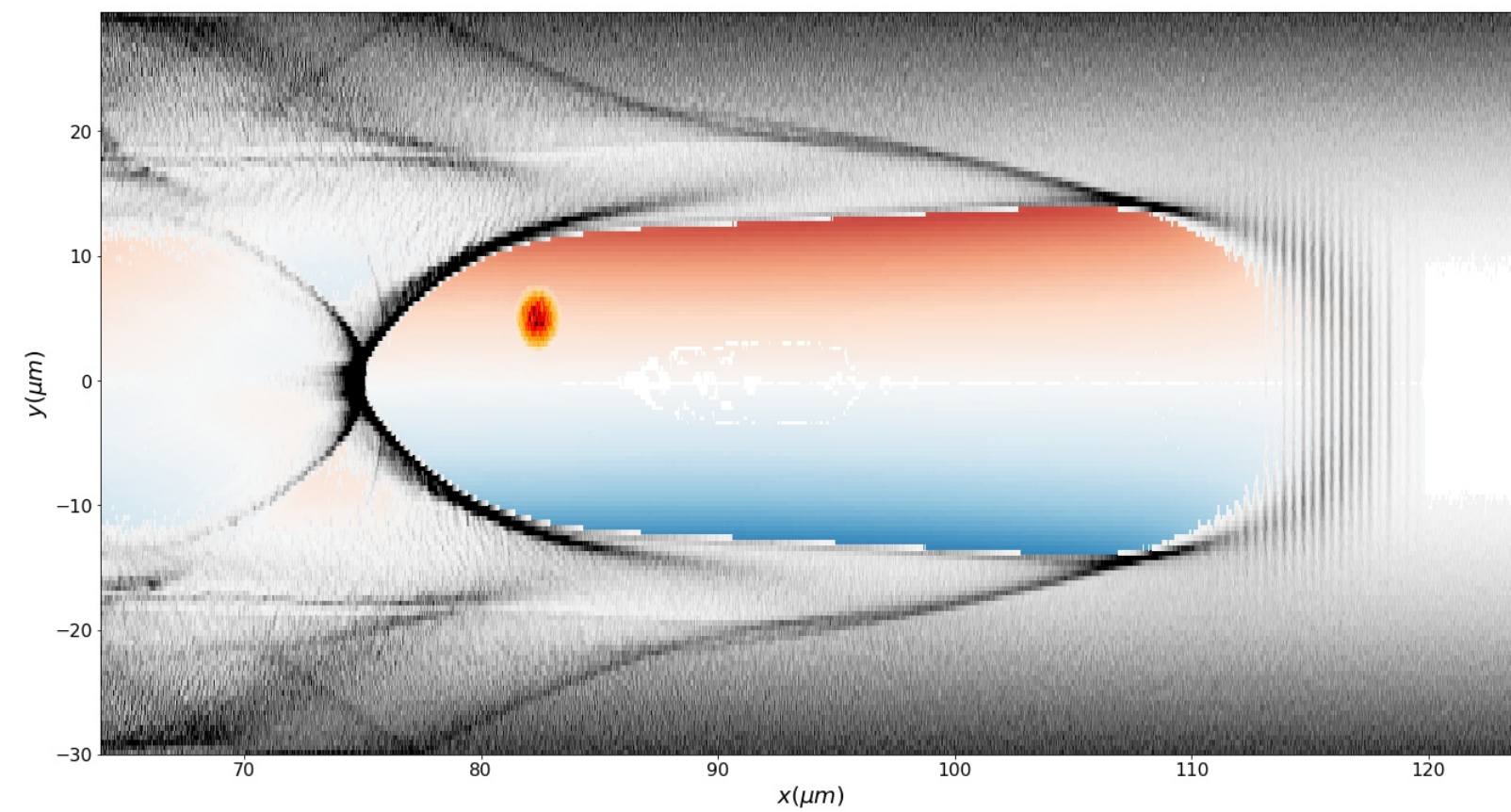
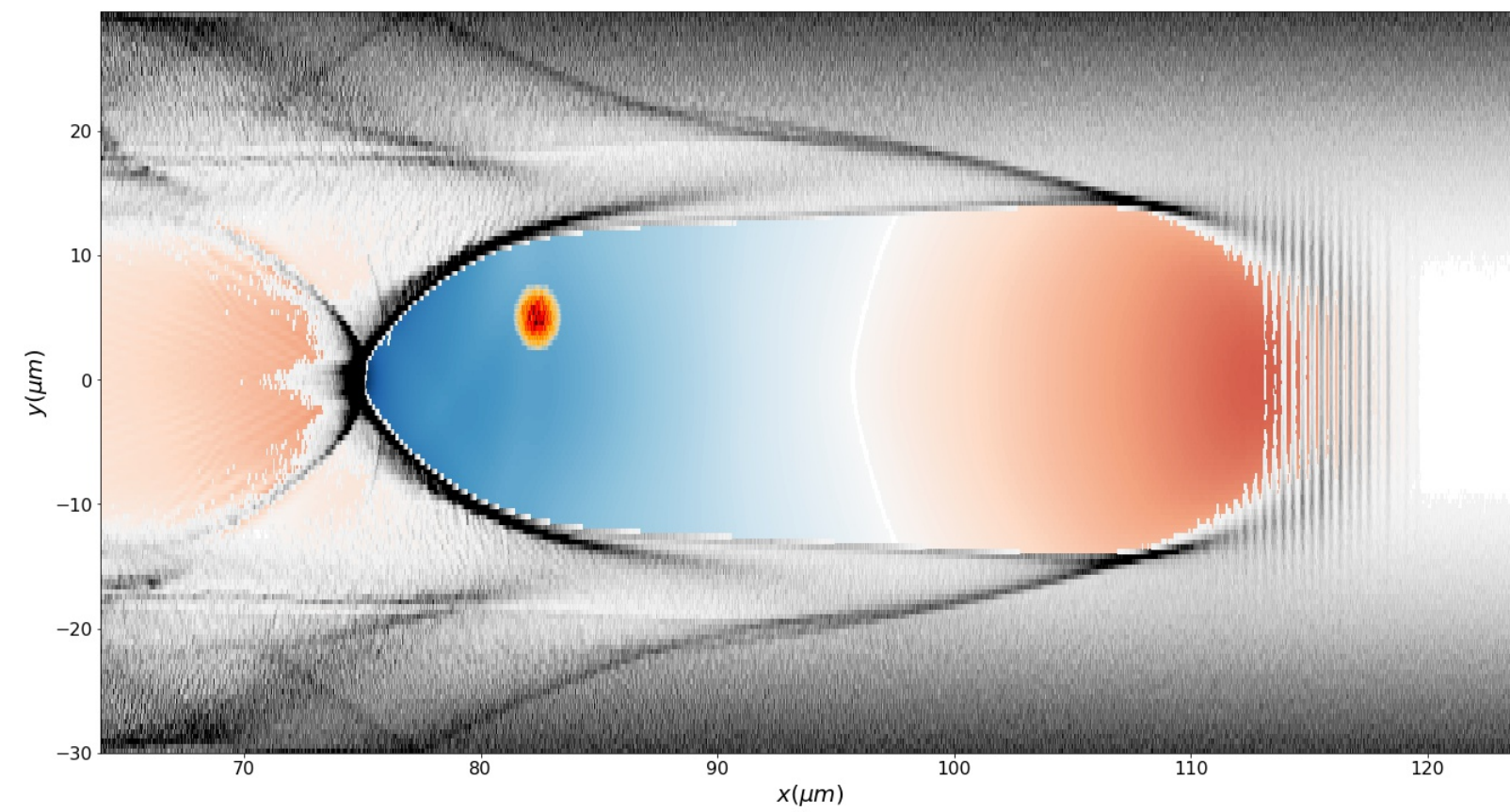
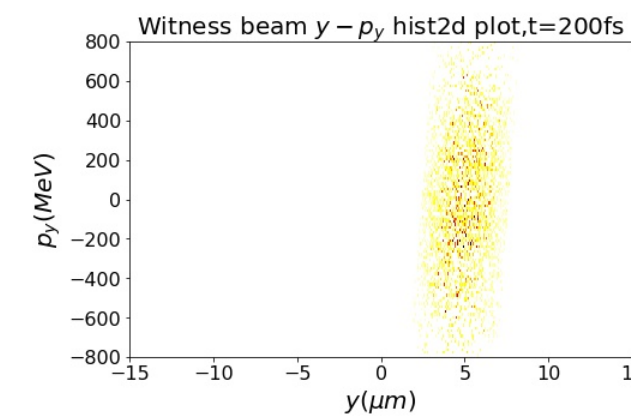
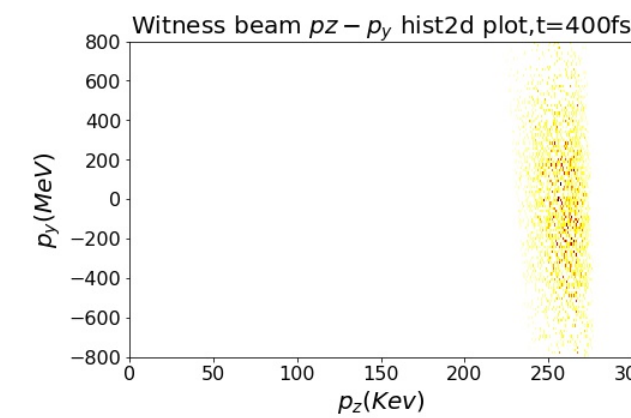
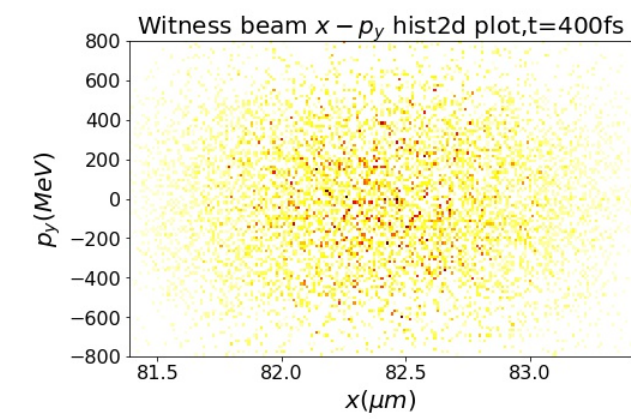
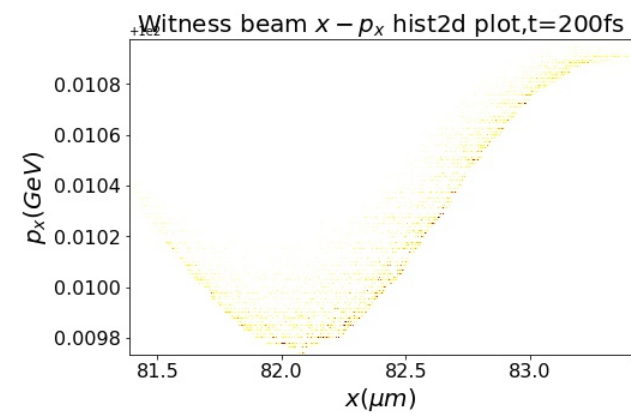


Beam off axis

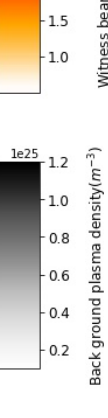
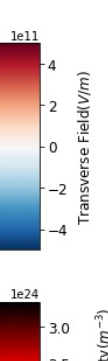
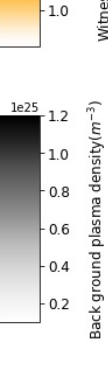
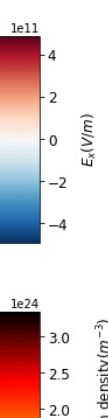
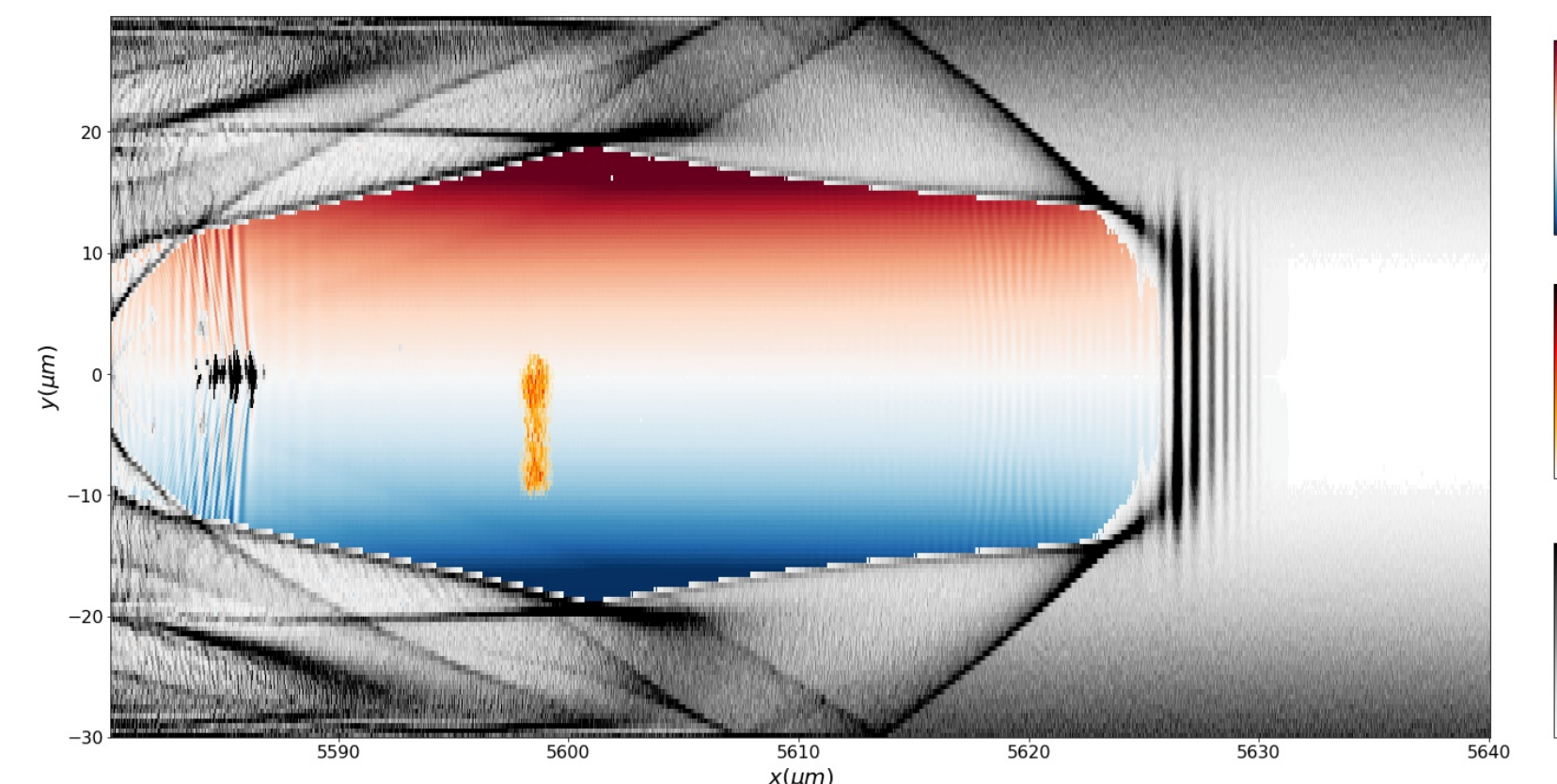
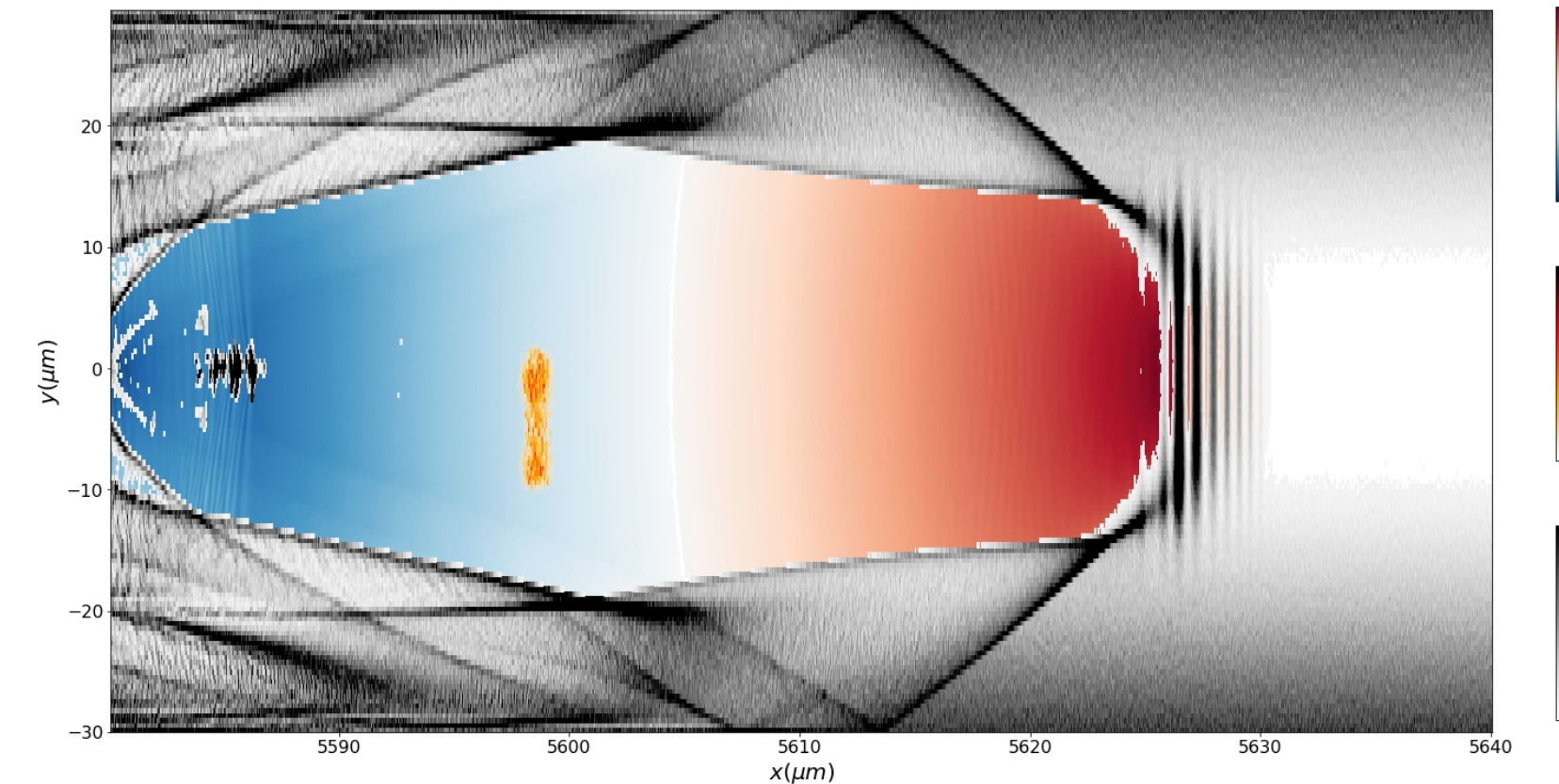


100 GeV beam inside LWFA,
plasma density = $5 \times 10^{18} \text{ cm}^{-3}$
Beam off axis

$T = 400 \text{ fs}$, $n_w = 5 \times 10^{24} (\text{m}^{-3})$, $n_p = 10^{24} (\text{m}^{-3})$, $p_{x0} = 100 (\text{GeV})$



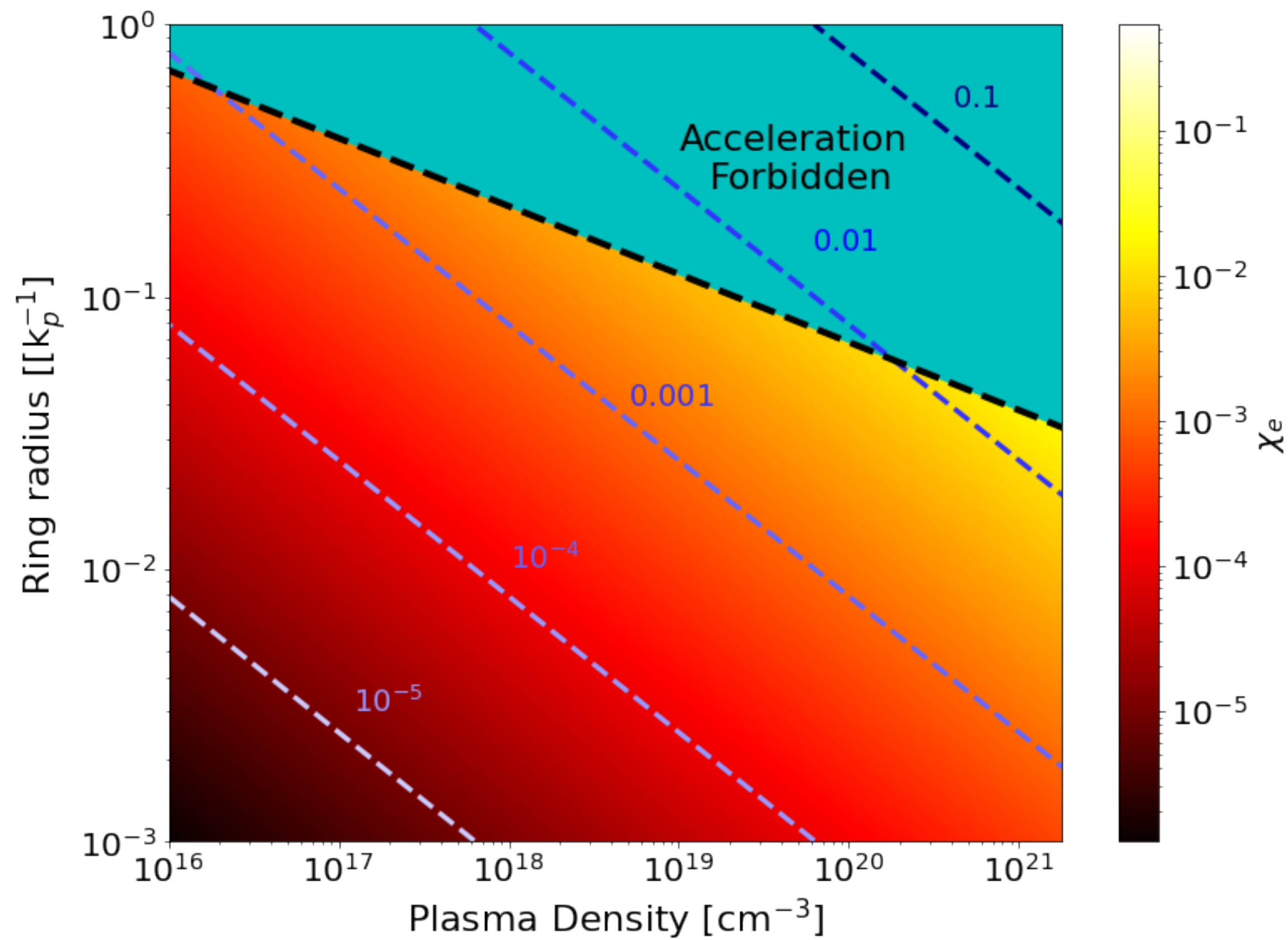
$T = 18800 \text{ fs}$, $n_w = 5 \times 10^{24} (\text{m}^{-3})$, $n_p = 10^{24} (\text{m}^{-3})$, $p_{x0} = 100 (\text{GeV})$



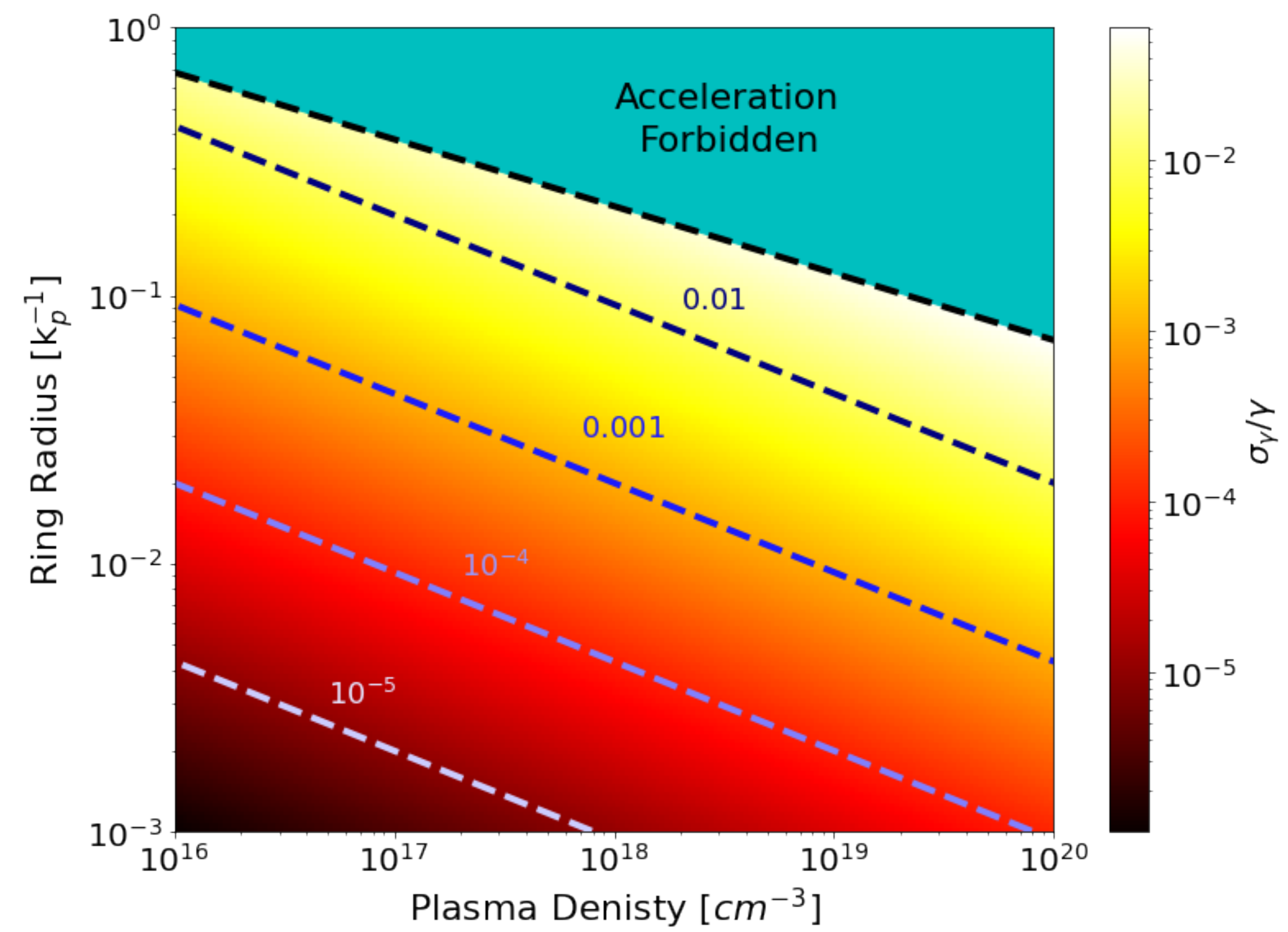
Ring distribution to reduce radiation induced energy spread



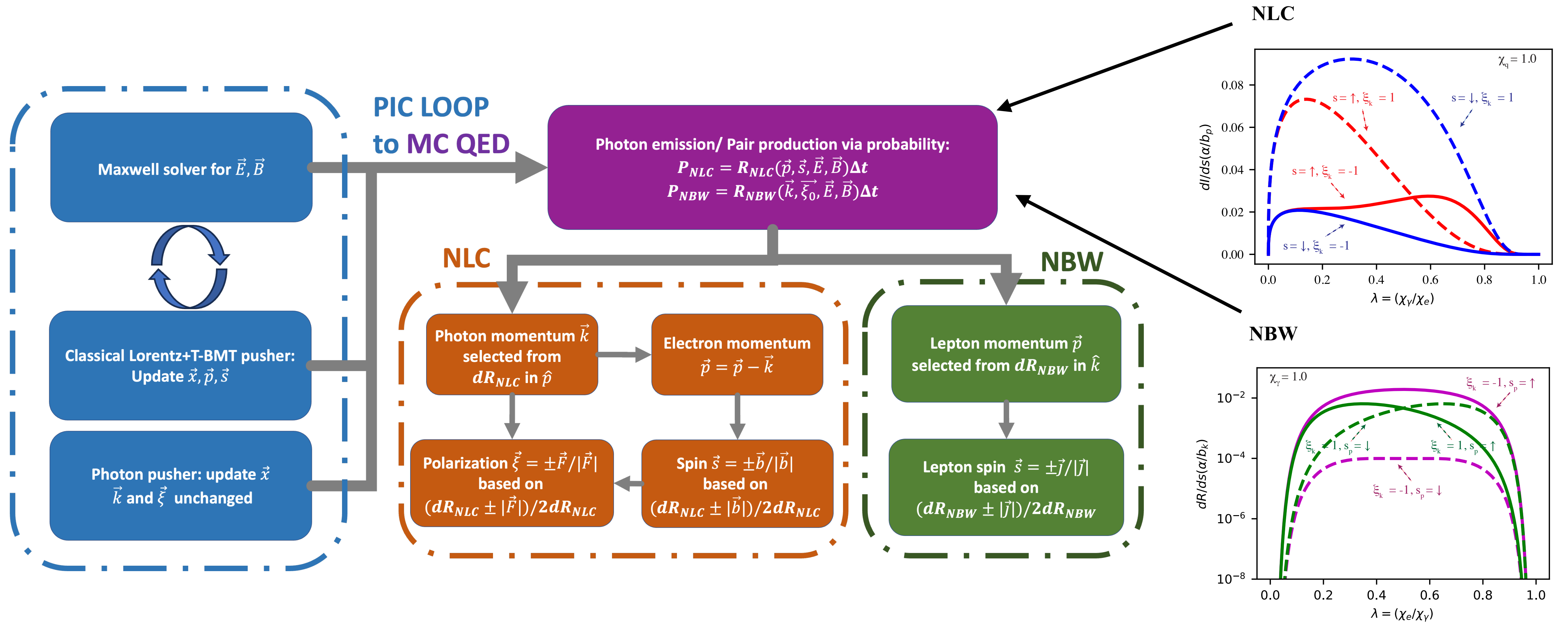
χ_e for ring shaped beam at 1 TeV



Energy spread of ring shaped beam at 1 TeV



Flow chart of polarization resolved QED code



$$\frac{dS}{dt} = \left(\frac{dS}{dt} \right)_{NLC} + \left(\frac{dS}{dt} \right)_{BMT}$$

Spin change from radiation: $\left(\frac{dS}{dt} \right)_{NLC}$ Spin change from precession: $\left(\frac{dS}{dt} \right)_{BMT}$

Spin and polarization depends rate and the spin basis



Non-Linear Compton

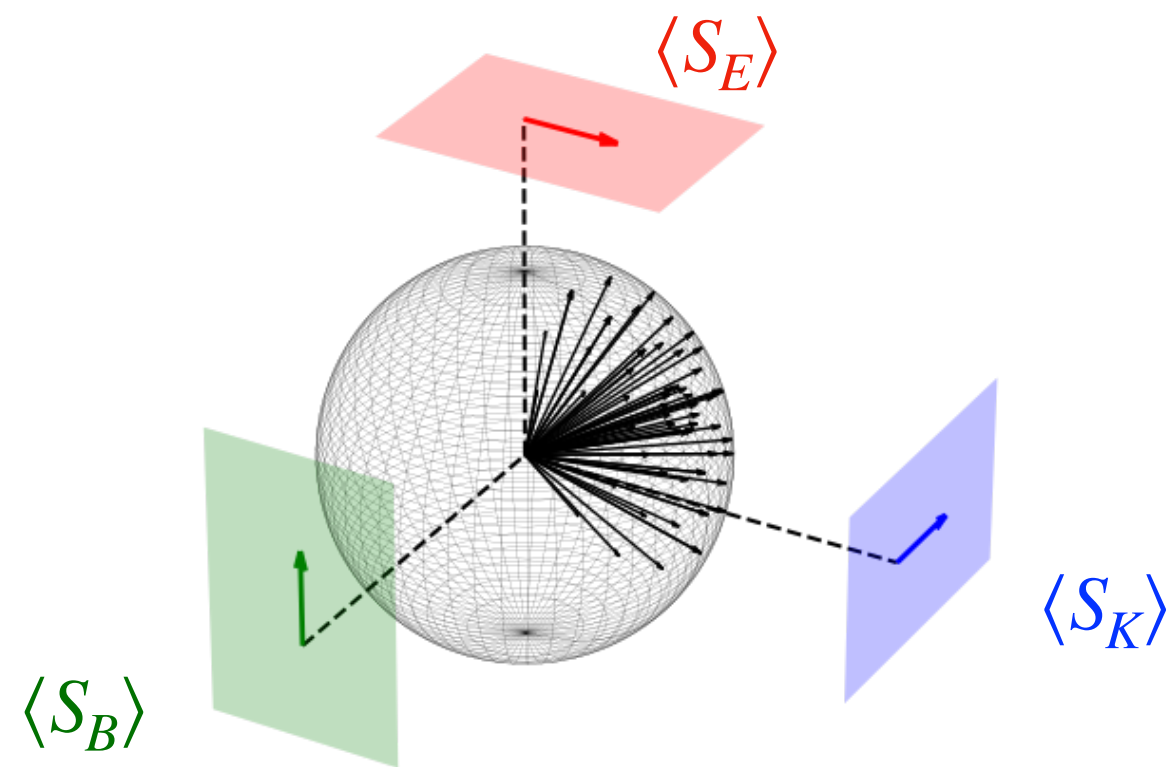
$$\begin{aligned}\frac{dR_{NLC}}{du} &= -(1+u)IntK_{1/3}(u') + (u^2 + 2u + 2)K_{2/3}(u') \\ &\quad - \vec{s} \cdot \hat{e}_2 u K_{1/3}(u') \\ \vec{b} &= -(1+u) [IntK_{1/3}(u') - 2K_{2/3}(u')] \vec{s} - u(1+u)K_{1/3}(u')\hat{e}_2 \\ &\quad - u^2 [IntK_{1/3}(u') - K_{2/3}(u')] (\vec{s} \cdot \hat{e}_v) \hat{e}_v\end{aligned}$$

Here, $\hat{e}_v = \vec{v}/|\vec{v}|$ is the direction of motion, $\hat{a} = \vec{v}/|\vec{v}|$ is the direction of acceleration. $\hat{e}_2 = (\hat{e}_v \times \hat{a})/|\hat{e}_v \times \hat{a}|$

Non-Linear Breit-Wheeler

$$\begin{aligned}\frac{dR_{NBW}}{d\varepsilon_+} &= \left(\frac{\omega_\gamma^2}{\varepsilon_+\varepsilon_-} - 2\right)K_{2/3}(\rho) + IntK_{1/3}(\rho) - \xi_3 K_{2/3}(\rho) \\ \vec{j}_+ &= -\xi_1 \frac{\omega_\gamma}{\varepsilon_-} K_{1/3}(\rho) \hat{e}_1 - K_{1/3}(\rho) \left(\frac{\omega_\gamma}{\varepsilon_+} - \xi_3 \frac{\omega_\gamma}{\varepsilon_-}\right) \hat{e}_2 \\ &\quad + \xi_2 \left[\frac{\omega_\gamma}{\varepsilon_+} IntK_{1/3}(\rho) + \frac{\varepsilon_+^2 - \varepsilon_-^2}{\varepsilon_+\varepsilon_-} K_{2/3}(\rho) \right] \hat{e}_v\end{aligned}$$

$\vec{\xi} = (\xi_1, \xi_2, \xi_3)$ is the photon polarization vector, ξ_i ($i = 1, 2, 3$) is the stokes parameters defined with respect to basis $\hat{e}_1$ and $\hat{e}_2$.



Schematic of spin measurement

Expected spin vector:

$$\vec{b}/R_0 = \langle S_E \rangle \hat{e}_1 + \langle S_B \rangle \hat{e}_2 + \langle S_K \rangle \hat{e}_2$$

Since $|\vec{b}/R_0| < 1$ To preserve spin length = 1, We project spin along or opposite to the spin quantization axis:

$$\vec{S}_{SQA} = \vec{b}/|\vec{b}|$$

Sokolov Ternov effect



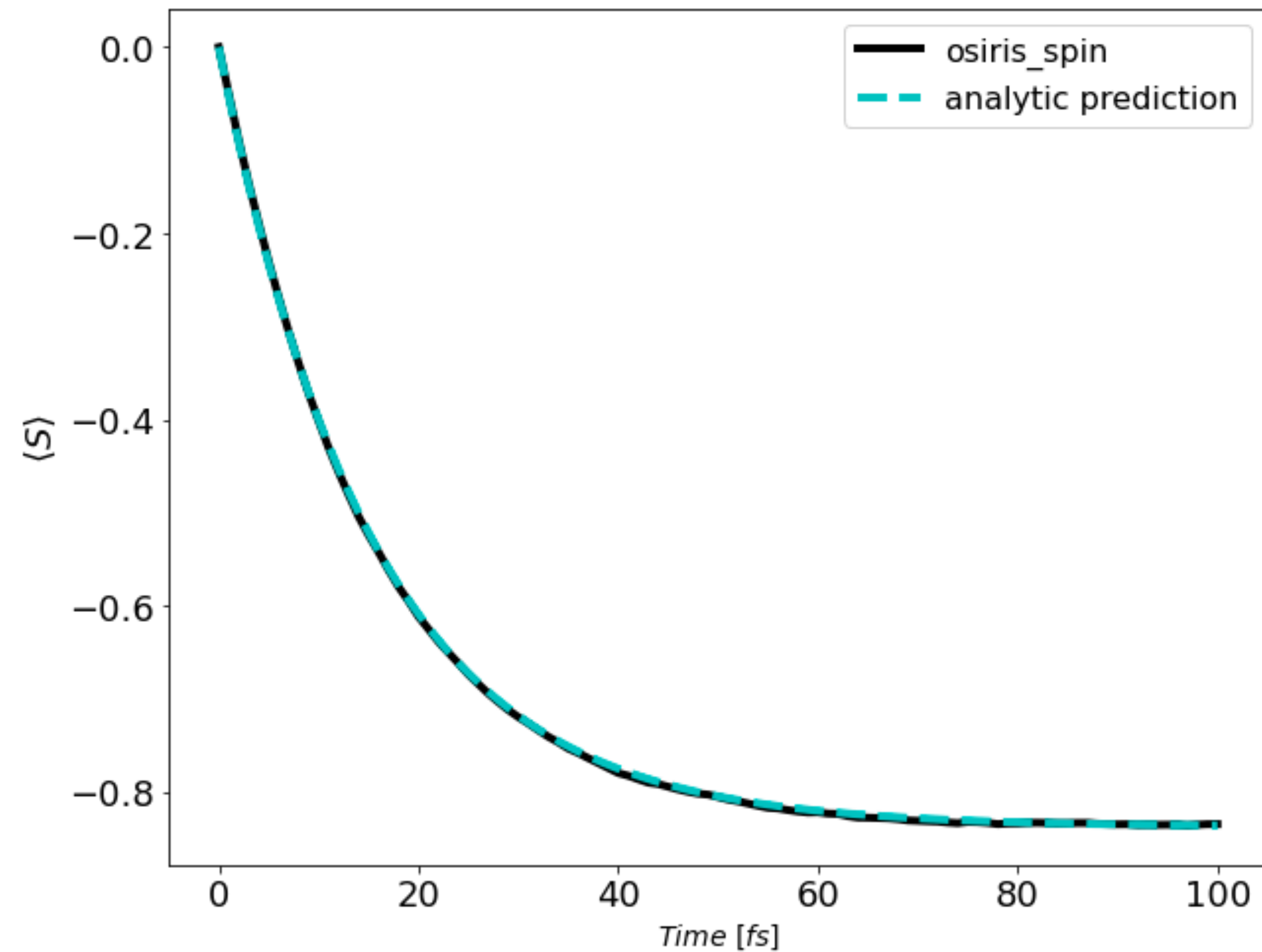
B field amplitude = 2.255 MT
Electron energy = 1 GeV (Mono)

Electron beam is unpolarized

B field stay in z direction,
Quantum parameter $\chi_e = 1.0$

No back reaction

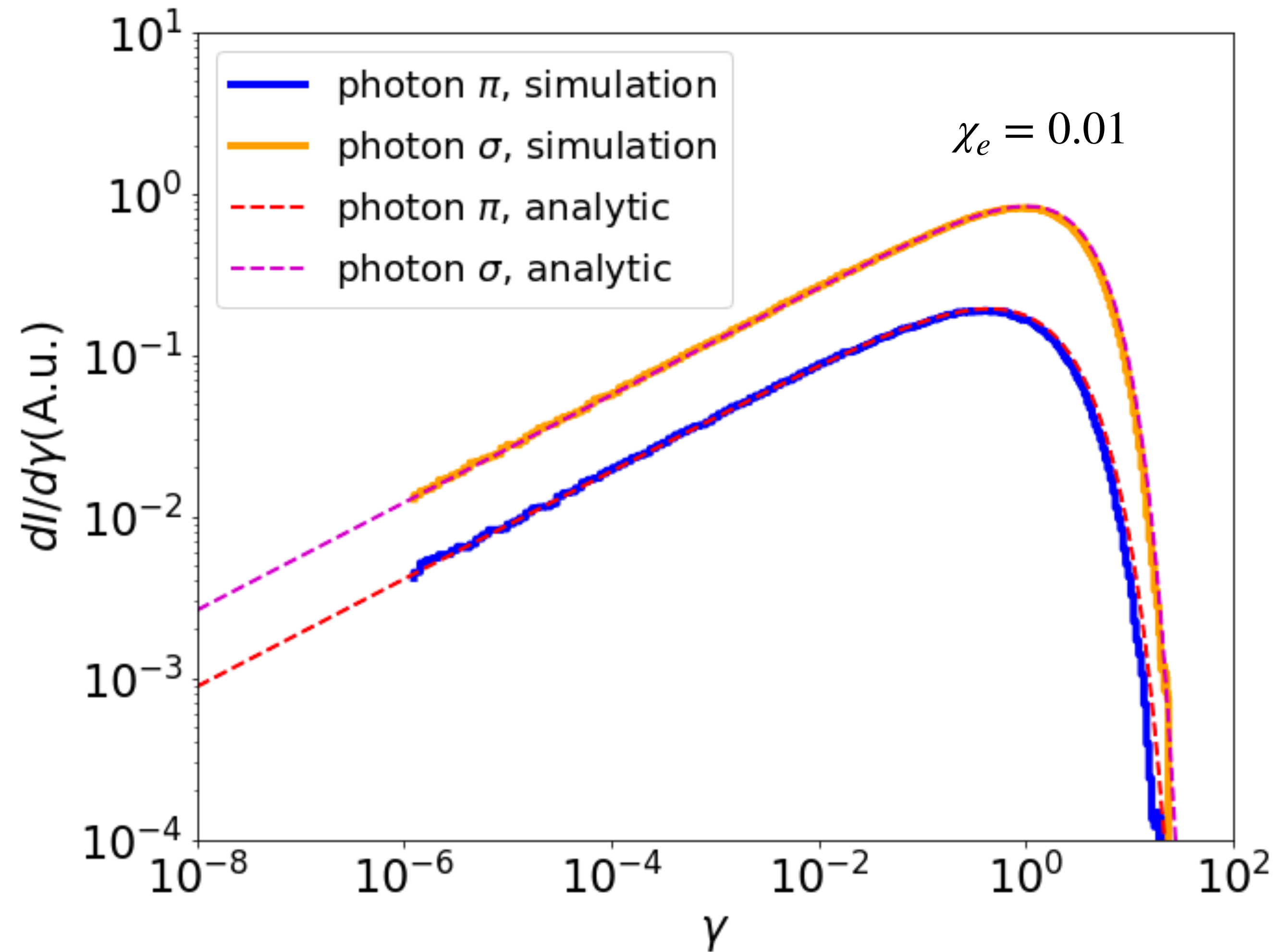
$$\frac{dS_\beta}{dt} = -P(\psi_1(\chi)S_\beta + \psi_3(\chi))$$



Photon Spectrum for $\chi_e = 0.01$, compare with classical spectrum



- $\chi_e \ll 1$
- Compare the spectrum from the simulation with the classical synchrotron radiation spectrum



From I M Ternov 1995 *Phys.-Usp.* **38** 409 :

$$W_{\sigma, \pi} = \frac{9\sqrt{3}}{16\pi} y \left[(l_{\sigma}^2 + l_{\pi}^2) \int_y^{\infty} K_{5/3}(x) dx + (l_{\sigma}^2 - l_{\pi}^2) K_{2/3}(y) \right] .$$

Benchmark against published result

