# Theory of Light Particles and Fields

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Yoni Kahn, University of Toronto

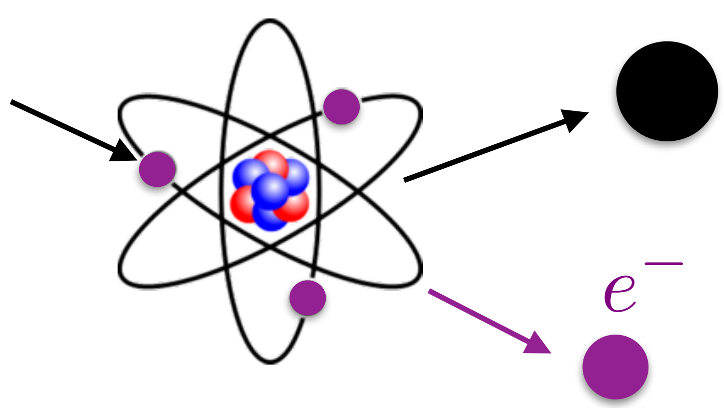
# Landscape of DM

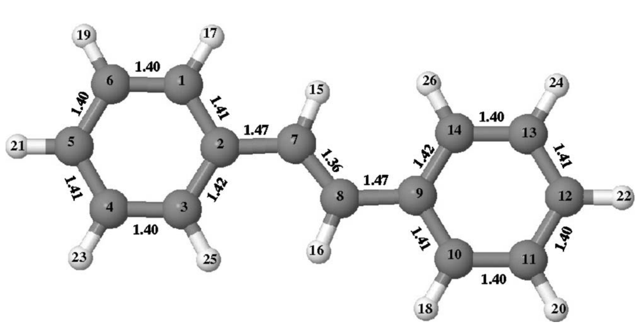
Wavelength fits in a galaxy

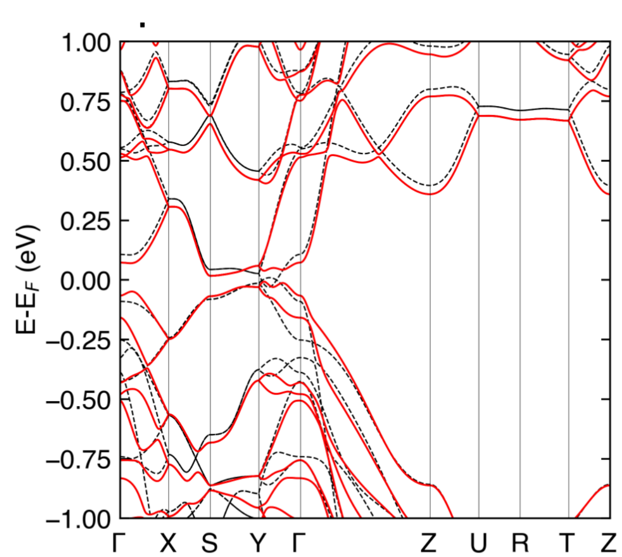
Elementary particle

$\sim 10^{-19}$ eV    1 eV    1 keV    1 GeV    100 TeV    $10^{19}$ GeV

"particle-like" DM
(Tongyan's lecture)






$$n_{\rm DM}\lambda_{\rm dB}^3 \ll 1$$

$$\mathrm{KE}_{\rm DM} = \frac{1}{2} m_{\rm DM} v_{\rm DM}^2$$

**Rare collisions, energy deposits scale with DM mass**

# Landscape of DM

Wavelength fits in a galaxy

Elementary particle

$\sim 10^{-19}$ eV   1 eV   1 keV   1 GeV   100 TeV   $10^{19}$ GeV

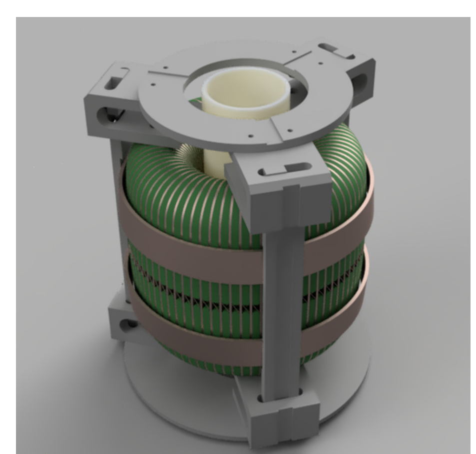
"wave-like" DM

$$n_{\rm DM}\lambda_{\rm dB}^3 \gg 1$$

$$\mathrm{KE}_{\rm DM} = \frac{1}{2} m_{\rm DM} v_{\rm DM}^2 \ll 1\ \mu\mathrm{eV}$$

**Collective effects**

Magnets, superconducting cavities, atomic clocks…DM acts like an oscillating external source of fixed but unknown frequency for any quantum-mechanical system



# Coherence length

We only know two things about DM: its mass density and its typical velocity

$$\rho \simeq 0.3 \ \mathrm{GeV/cm^3} \qquad v_0 \simeq 10^{-3}$$

$$\lambda_{\mathrm{dB}} = \frac{2\pi}{mv} \implies n\lambda_{\mathrm{dB}}^3 = \frac{\rho}{m}\left(\frac{2\pi}{mv_0}\right) \simeq 1 \times \left(\frac{26 \ \mathrm{eV}}{m}\right)^4$$

DM lighter than this can't be a fermion. Two possibilities:

- spin-0: axion
- spin-1: dark photon

Focus on axions this lecture: well-motivated, technically simpler (no polarization)

# Field description

Classically, spin-0 fields satisfy the Klein-Gordon equation:

$$(\Box + m_a^2)a(\mathbf{x}, t) = 0$$

If we look at the spatially-homogeneous part (set gradients to zero):

$$(\partial_t^2 + m_a^2)a(t) = 0 \implies a(t) = a_0 \cos(m_a t + \phi)$$

Fix $a_0$ by equating energy density in field to $\rho$:

$$a_0 = \frac{\sqrt{2\rho}}{m_a}$$ (KG Lagrangian -> energy-momentum tensor -> $T^{00} = \rho$)

"standing DM wave" oscillating at fixed but unknown frequency

# Coherence properties

Any plane wave is a solution to the KG equation, and therefore, so are their superpositions:

$$a = a_0 \left[ \cos \left\{ \left( m_a + \frac{1}{2} m_a v^2 \right) t + m_a \mathbf{v} \cdot \mathbf{x} \right\} + \cos \left\{ \left( m_a + \frac{1}{2} m_a (v + \Delta v)^2 \right) t + m_a (\mathbf{v} + \Delta \mathbf{v}) \cdot \mathbf{x} \right\} \right]$$

At a fixed time (say t=0), at what distance do they dephase?

$$\lambda_c \sim \frac{2\pi}{m \Delta v}$$

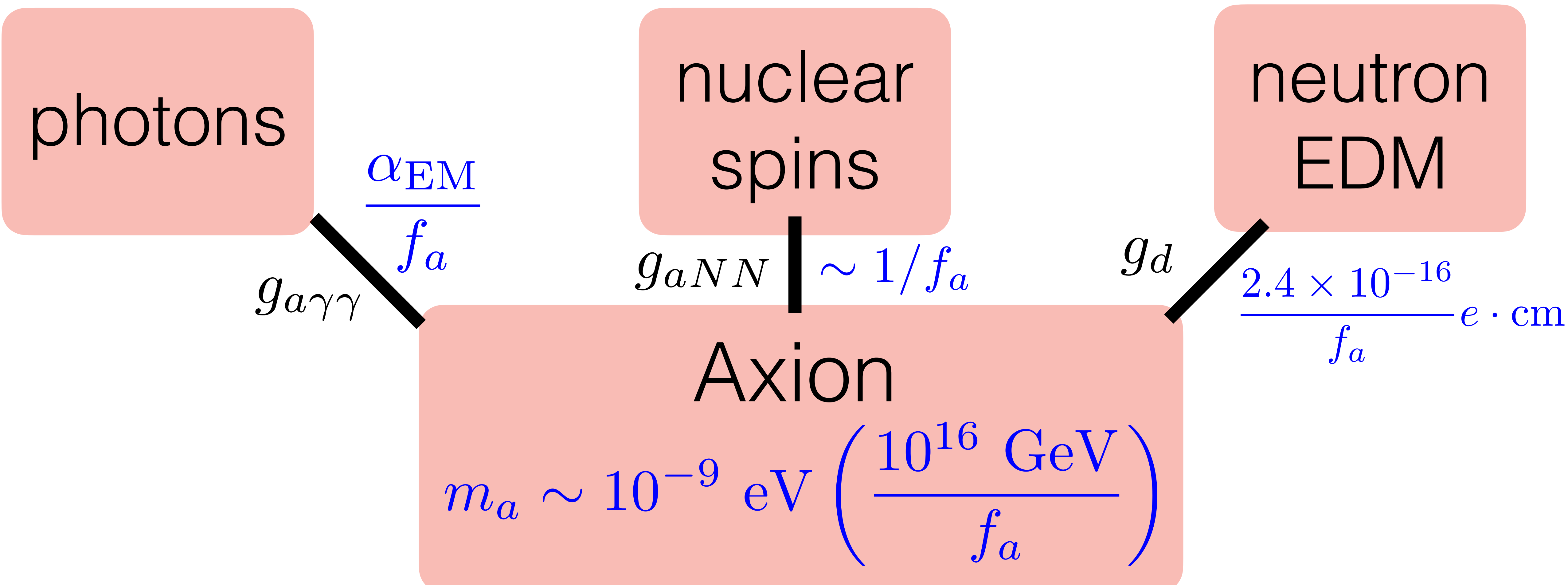
note: **not** the dB wavelength!! velocity **dispersion**

At a fixed place (say x=0), after what time do they dephase?

$$\tau_c \sim \frac{2\pi}{m v \Delta v}$$

In typical virialized galaxies, $v \sim \Delta v \sim v_0$.
But we will see when this distinction matters!

# How do axions interact?




For QCD axion, only one free parameter!

While strong-CP is a great motivation, let's think broadly.
For this lecture, "axion" can vary mass/coupling relation

# What should we measure?

$$a(\mathbf{x}, t) = \frac{\sqrt{2\rho}}{m} \cos(mt + \mathcal{O}(v)\mathbf{x})$$

In axion DM background, get oscillating observables:

$$\nabla \times \mathbf{B}_a = \frac{\partial \mathbf{E}_a}{\partial t} - g_{a\gamma\gamma}\left(\mathbf{E}_0 \times \nabla a - \mathbf{B}_0 \frac{\partial a}{\partial t}\right)$$

$$\nabla \cdot \mathbf{E}_a = -g_{a\gamma\gamma}\mathbf{B}_0 \cdot \nabla a$$

Oscillating response from static fields

$$H_N \supset g_{aNN} \nabla a \cdot \vec{\sigma}_N$$

Spin-dependent force

$$d_n = g_d a$$

Time-varying EDM

Note: $\nabla a \sim v \sim 10^{-3}$ so some are easier than others

# Electromagnetic searches

$$\underbrace{\nabla \times \mathbf{B}_a = \frac{\partial \mathbf{E}_a}{\partial t}} - g_{a\gamma\gamma}\mathbf{B}_0\frac{\partial a}{\partial t}$$

Cavity regime: $\lambda_{\mathrm{Comp}} \sim R_{\mathrm{exp}}$

e.g. ADMX

$$\nabla \times \mathbf{B}_a = \cancel{\frac{\partial \mathbf{E}_a}{\partial t}} \underbrace{- g_{a\gamma\gamma}\mathbf{B}_0\frac{\partial a}{\partial t}}_{\mathbf{J}_{\mathrm{eff}}}$$

Quasistatic regime: $\lambda_{\mathrm{Comp}} \gg R_{\mathrm{exp}}$

e.g. DM-Radio

$$\cancel{\nabla \times \mathbf{B}_a} = \frac{\partial \mathbf{E}_a}{\partial t} - g_{a\gamma\gamma}\mathbf{B}_0\frac{\partial a}{\partial t}$$

Radiation regime: $\lambda_{\mathrm{Comp}} \ll R_{\mathrm{exp}}$

e.g. MADMAX

DM deposits power in an experiment, so the object we need is the power spectral density (PSD).
Following arXiv:2408.04696, let's calculate that carefully.

# Quantum vs. classical

The universe is described by quantum field theory.
The axion field is a quantum field, like everything else:

$$a(t,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left( \hat{a}_{\mathbf{k}} e^{-ik\cdot x} + \mathrm{h.c.} \right)$$

This is a bunch of harmonic oscillators, so a convenient (overcomplete) basis is that of coherent states (eigenstates of annihilation operator):

$$\hat{a}_{\mathbf{k}} |\alpha_{\mathbf{k}}\rangle = \alpha_{\mathbf{k}} |\alpha_{\mathbf{k}}\rangle$$

For each mode $\mathbf{k}$, write the density matrix in this basis:

$$\hat{\rho}_{\mathbf{k}} = \int d\alpha_{\mathbf{k}}\, P(\alpha_{\mathbf{k}}) |\alpha_{\mathbf{k}}\rangle\langle\alpha_{\mathbf{k}}|$$

not guaranteed positive-definite!
not a probability!

# Quantum vs. classical

However, arXiv:2408.04696 and follow-ups showed that every detector effectively coarse-grains the field such that the observed P-function **is** positive-definite:

$$\widehat{\rho}_{\boldsymbol{k}} = \int d^2\alpha_{\boldsymbol{k}} \, \underbrace{\frac{e^{-|\alpha_{\boldsymbol{k}}|^2/N_{\boldsymbol{k}}}}{\pi N_{\boldsymbol{k}}}}_{P(\alpha_{\boldsymbol{k}})} \, |\alpha_{\boldsymbol{k}}\rangle\langle\alpha_{\boldsymbol{k}}|, \qquad N_{\boldsymbol{k}} \gg 1$$

$$a(t,\boldsymbol{x}) = \sum_{\boldsymbol{k}} \frac{1}{\sqrt{V\omega_{\boldsymbol{k}}}} \left( \alpha_{\boldsymbol{k}} e^{-i(\omega_{\boldsymbol{k}} t - \boldsymbol{k}\cdot\boldsymbol{x})} + \text{c.c.} \right),$$

$\langle|\alpha_{\boldsymbol{k}}|^2\rangle = N_{\boldsymbol{k}}$ free field has **Gaussian statistics**

Wave-like DM is a "classical field": operators $\hat{a}_{\mathbf{k}}$ replaced by c-numbers $\alpha_{\mathbf{k}}$

$$N_{\mathbf{k}} = (2\pi)^3 \frac{\rho}{m_a} p(\mathbf{k})$$

momentum distribution -> velocity distribution in non-relativistic limit

# Autocorrelation function

As a step toward computing the PSD, let's compute
how the axion field is correlated with itself in time and space

$$a(x^\mu) = \sum_{\mathbf{k}} \frac{1}{\sqrt{V\omega_{\mathbf{k}}}} \left(\alpha_{\mathbf{k}} e^{-ik\cdot x} + \alpha^*_{\mathbf{k}} e^{ik\cdot x}\right)$$

$$x^\mu = (t, \mathbf{x})$$

$$\langle \Gamma(\tau, \mathbf{d}) \rangle = \langle a(x^\mu) a(x^\mu + d^\mu) \rangle$$

$$d^\mu = (\tau, \mathbf{d})$$

$$= \sum_{\mathbf{k},\mathbf{q}} \frac{1}{2V\sqrt{\omega_{\mathbf{k}}\omega_{\mathbf{q}}}} \langle (\alpha_{\mathbf{k}} + \alpha^*_{\mathbf{k}})(\alpha_{\mathbf{q}} e^{-iq\cdot d} + \alpha^*_{\mathbf{q}} e^{iq\cdot d}) \rangle$$

$$= \sum_{\mathbf{k}} \frac{\langle |\alpha_{\mathbf{k}}|^2 \rangle}{V\omega_{\mathbf{k}}} \cos(k\cdot d)$$

each mode is statistically independent

$$= \sum_{\mathbf{k}} \frac{N_{\mathbf{k}}}{V\omega_{\mathbf{k}}} \cos(k\cdot d)$$

# Continuum limit and PSD

$$\langle \Gamma(\tau, \mathbf{d}) \rangle = \sum_{\mathbf{k}} \frac{N_{\mathbf{k}}}{V\omega_{\mathbf{k}}} \cos(k \cdot d) \longrightarrow \frac{\rho}{m_a} \int d^3\mathbf{k}\, \frac{p(\mathbf{k})}{\omega_{\mathbf{k}}} \cos(\omega_{\mathbf{k}}\tau - \mathbf{k} \cdot \mathbf{d})$$

At zero spatial separation ($\mathbf{d} = \mathbf{0}$) and non-relativistic limit ($\omega_{\mathbf{k}} \approx m_a$, $\mathbf{k} \approx m_a \mathbf{v}$) we have

$$\langle \Gamma(\tau) \rangle = \frac{\rho}{m_a} \int dv\, f(\mathbf{v}) \cos\left( m_a \left(1 + \frac{v^2}{2}\right) \tau \right)$$

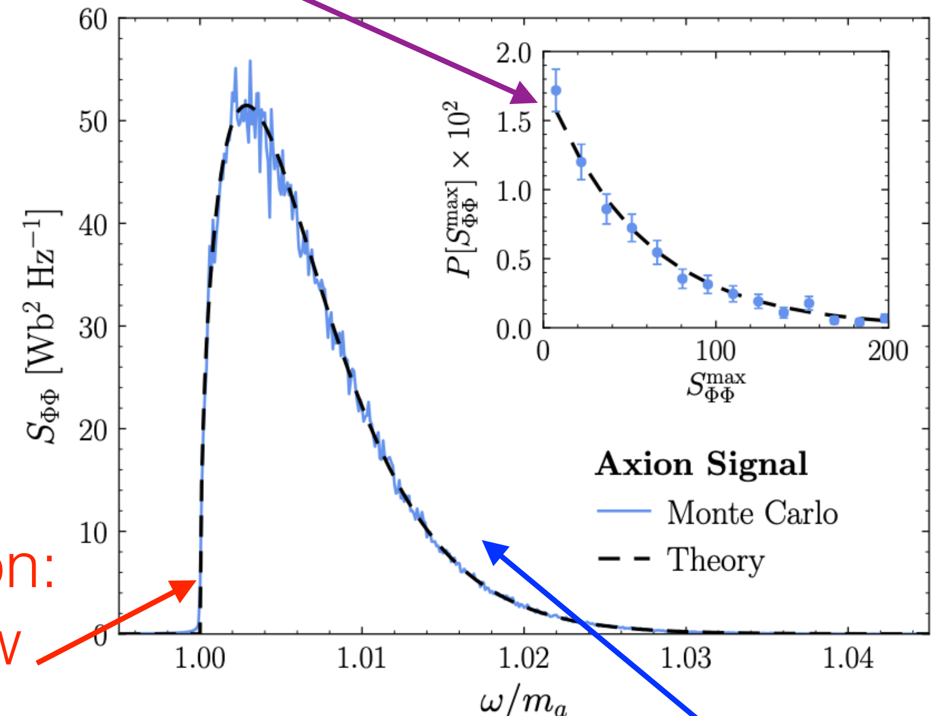
Wiener-Khinchin theorem from spectral analysis gives average PSD:

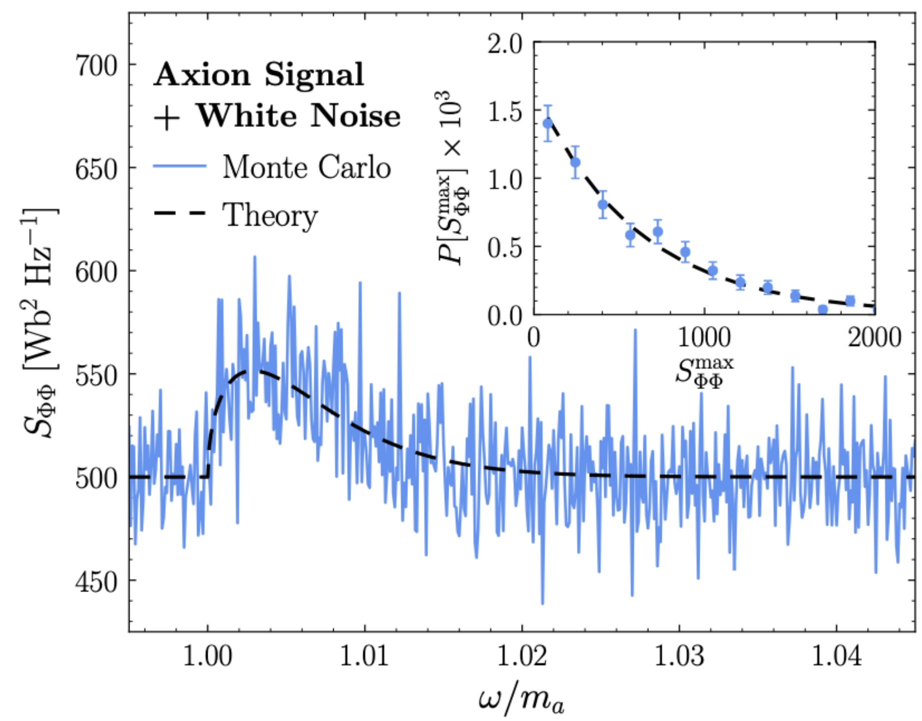
$$\langle S(\omega) \rangle = 2 \int_{-\infty}^{\infty} d\tau\, \langle \Gamma(\tau) \rangle e^{i\omega\tau} = \frac{2\pi \rho f(v_\omega)}{m_a^3 v_\omega} \qquad v_\omega = \sqrt{2\,(\omega/m_a - 1)}$$

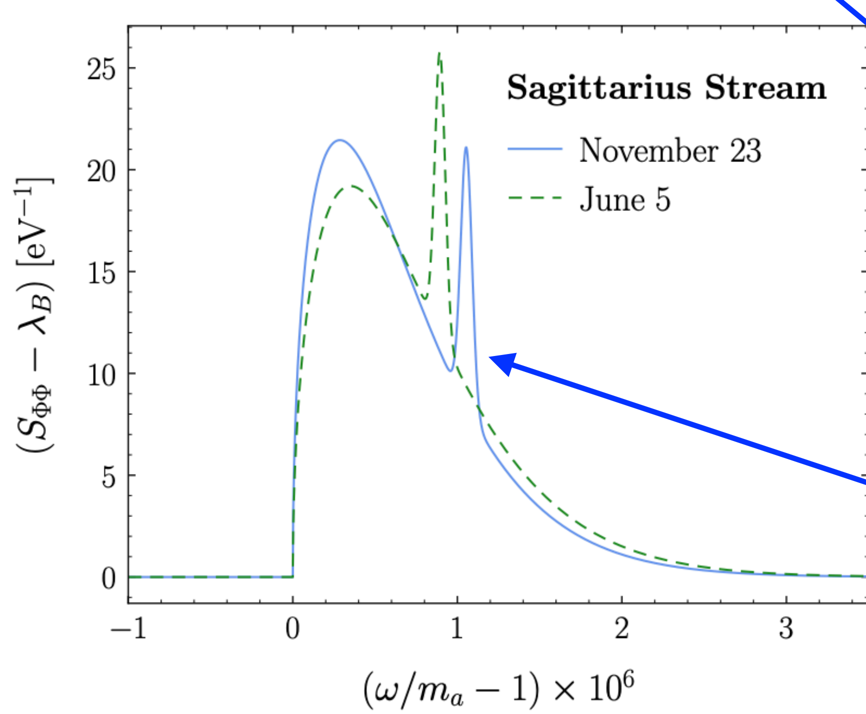
# Galactic archaeology with the PSD

exponentially-distributed amplitude

energy conservation: zero below $\omega = m_a$




traces out speed distribution, including annual modulation!

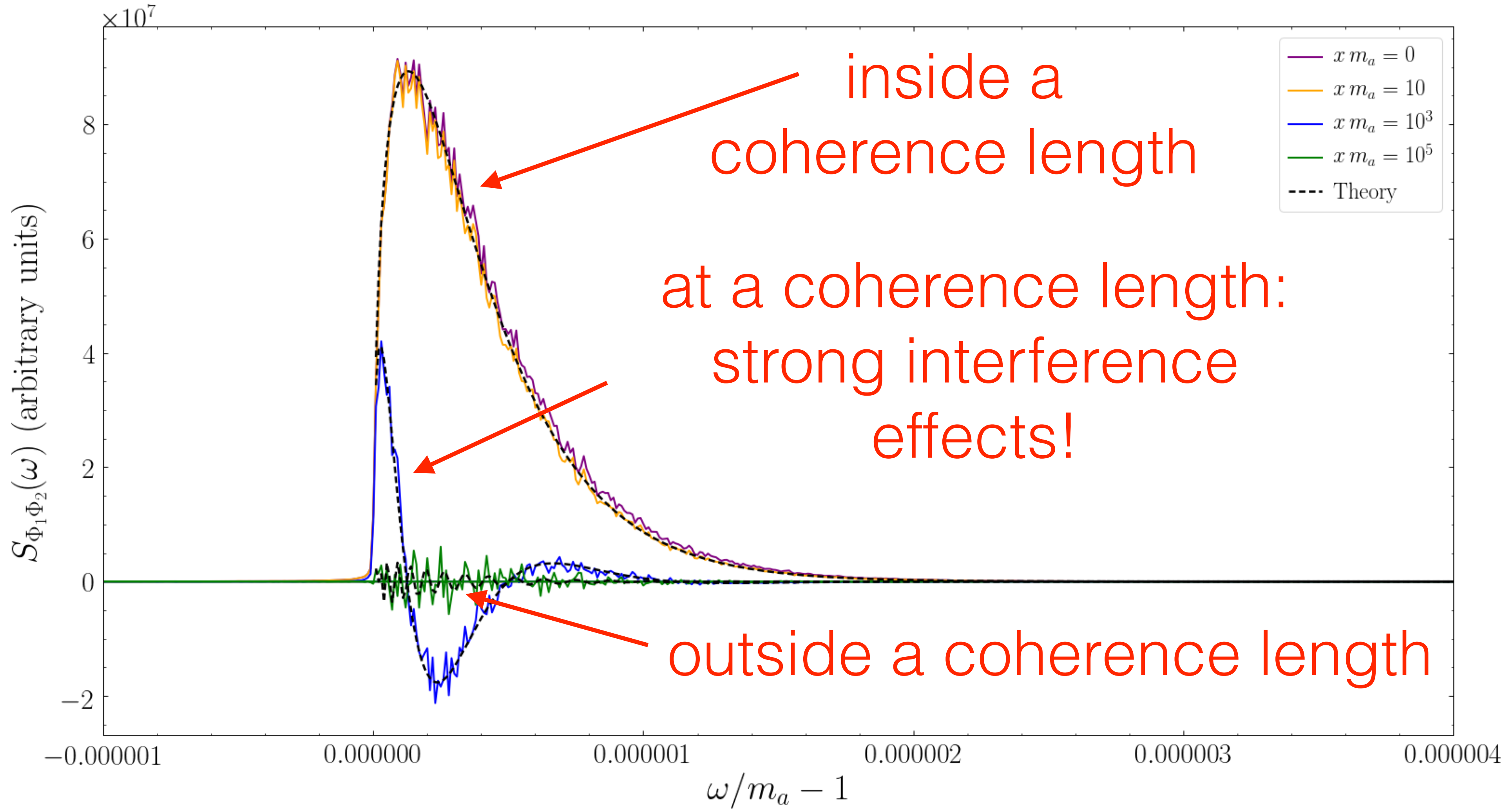
Of course, the PSD in an experiment will be multiplied by the axion-photon coupling $g_{a\gamma\gamma}$, which is VERY small! Hence the need for quantum probes to detect a tiny power

# What about two detectors?

$$\langle S_{\Phi_1\Phi_2}(\omega)\rangle \propto \int d\Omega\, v\, f(\mathbf{v}) \cos(m_a \mathbf{v}\cdot\mathbf{x})\Big|_{v=\sqrt{2\omega/m_a - 2}}$$

Cross-spectrum depends on separation compared to coherence length:

$\tau \sim 2\pi/(m_a v) \sim$ 1700 km for $m_a = 10^{-9}$ eV



# Axion interferometry

Compute the multi-detector PSD by discrete Fourier transform this time:

$$a_j(\omega) = \frac{\Delta t}{\sqrt{T}} \sum_{n=0}^{N-1} a(n\Delta t, \mathbf{x}_j) e^{-i\omega n \Delta t}$$

Expand correlation in real and imaginary parts:

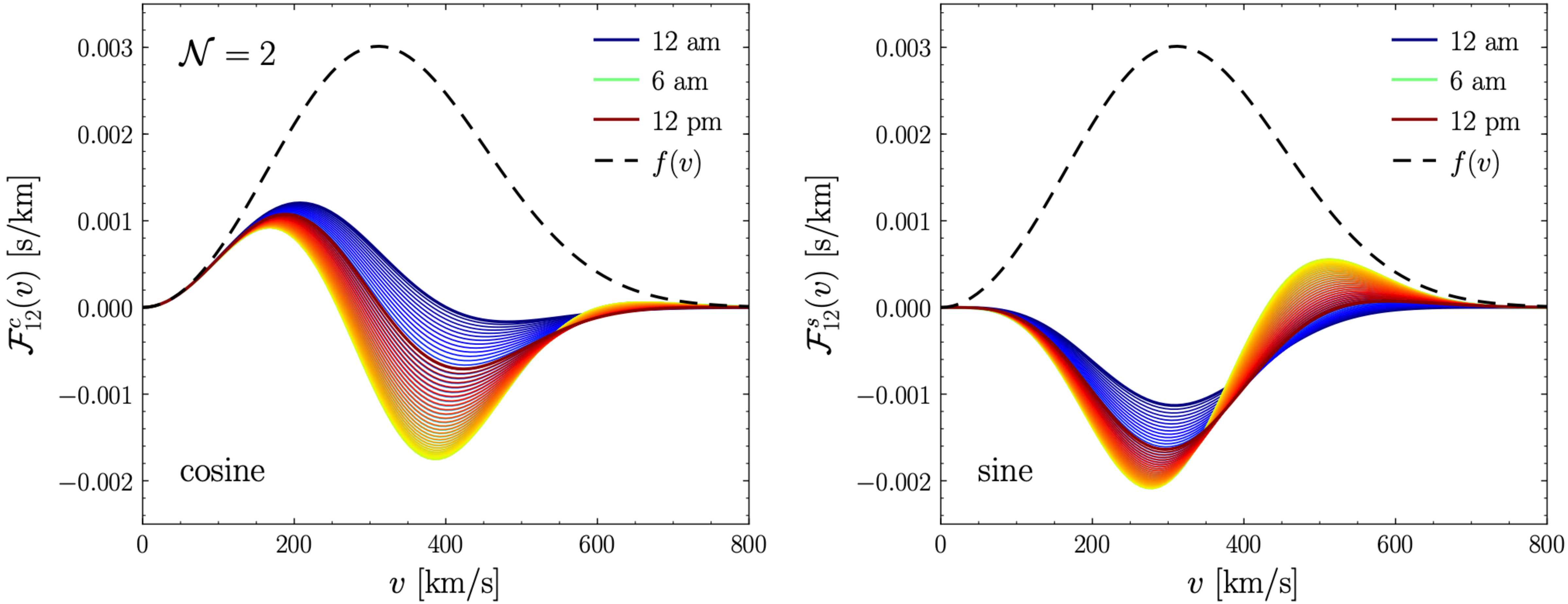
$$\mathbf{\Sigma}_k = \begin{bmatrix} \langle R_k(\mathbf{x}_1) R_k(\mathbf{x}_1)\rangle & \langle R_k(\mathbf{x}_1) I_k(\mathbf{x}_1)\rangle & \langle R_k(\mathbf{x}_1) R_k(\mathbf{x}_2)\rangle & \dots & \langle R_k(\mathbf{x}_1) I_k(\mathbf{x}_\mathcal{N})\rangle \\ \langle I_k(\mathbf{x}_1) R_k(\mathbf{x}_1)\rangle & \langle I_k(\mathbf{x}_1) I_k(\mathbf{x}_1)\rangle & \langle I_k(\mathbf{x}_1) R_k(\mathbf{x}_2)\rangle & \dots & \langle I_k(\mathbf{x}_1) I_k(\mathbf{x}_\mathcal{N})\rangle \\ \vdots & & \ddots & & \\ \langle I_k(\mathbf{x}_\mathcal{N}) R_k(\mathbf{x}_1)\rangle & \langle I_k(\mathbf{x}_\mathcal{N}) I_k(\mathbf{x}_1)\rangle & \langle I_k(\mathbf{x}_\mathcal{N}) R_k(\mathbf{x}_2)\rangle & \dots & \langle I_k(\mathbf{x}_\mathcal{N}) I_k(\mathbf{x}_\mathcal{N})\rangle \end{bmatrix}$$

The diagonal pieces just give the speed distribution. The off-diagonals are new:

$$\mathcal{F}^c_{ij}(v) = \int d^3\mathbf{v} f(\mathbf{v}) \cos(m_a \mathbf{v}\cdot\mathbf{x}_{ij}) \delta[|\mathbf{v}| - v], \quad \mathcal{F}^s_{ij}(v) = \int d^3\mathbf{v} f(\mathbf{v}) \sin(m_a \mathbf{v}\cdot\mathbf{x}_{ij}) \delta[|\mathbf{v}| - v].$$

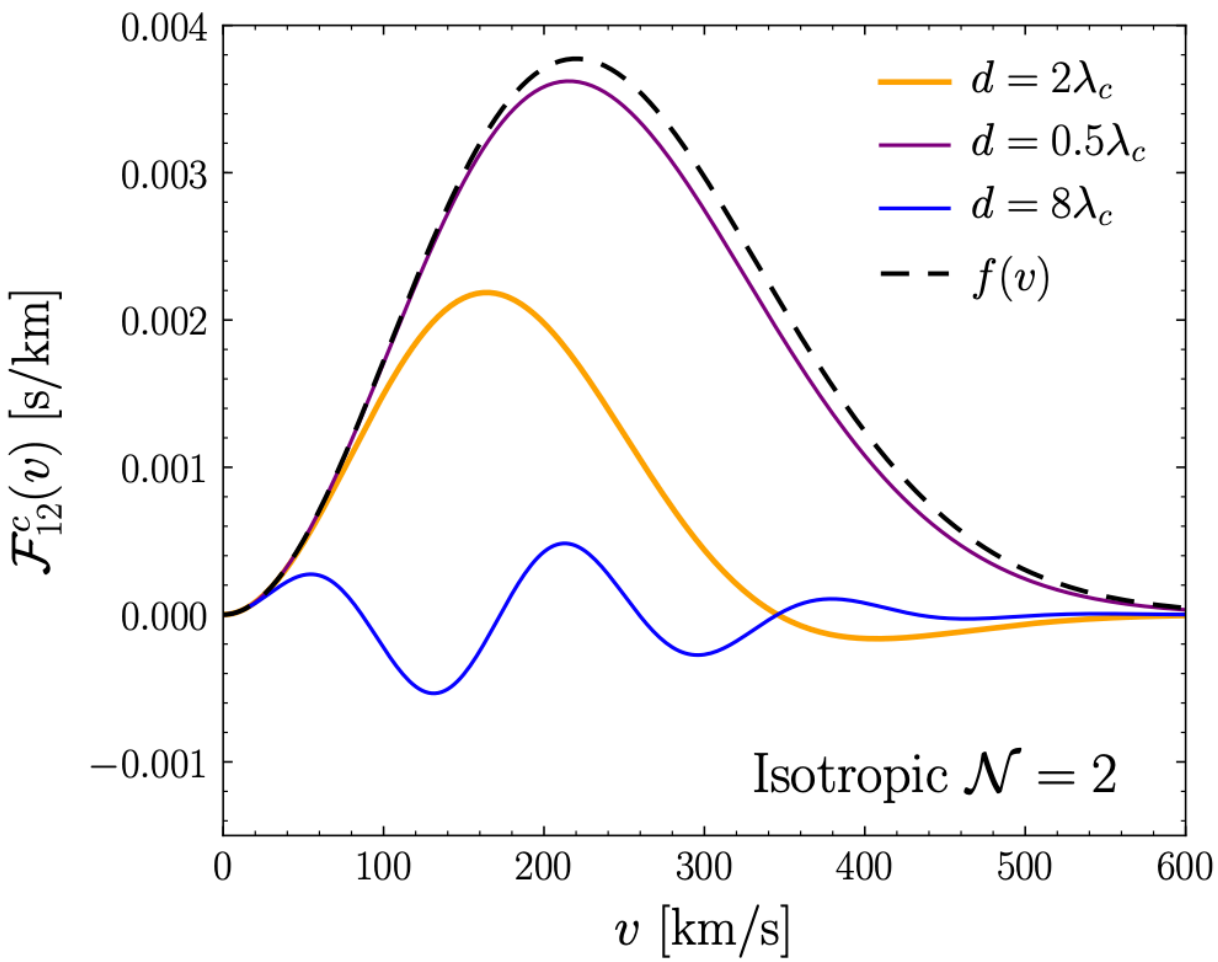
# Daily modulation

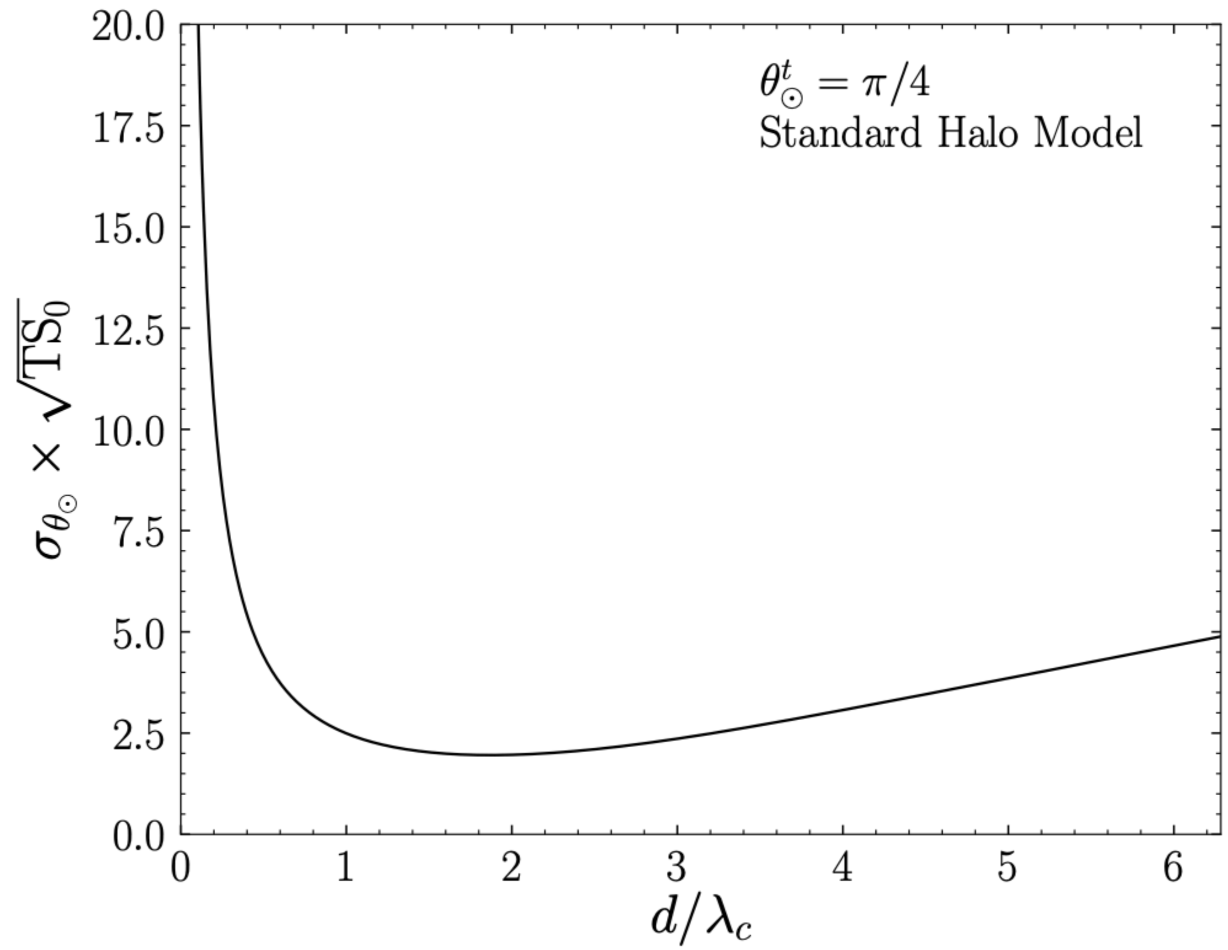
Since the off-diagonal covariance depends on the direction of **v**, they change as the DM wind rotates throughout the day!



# Parameter estimation

Say we see a signal at two detectors. How well can we measure the polar angle of the DM's boost by the solar system velocity?



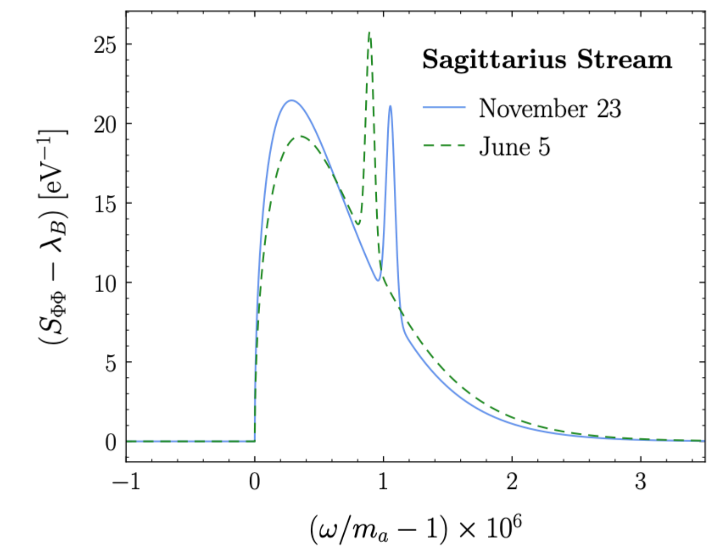

Best sensitivity when detectors are separated by $\mathcal{O}(1)$ coherence length

# Summary

Ultralight bosonic DM with macroscopic phase space occupancy is a classical Gaussian random field

$$a(x^\mu) = \sum_{\mathbf{k}} \frac{1}{\sqrt{V\omega_{\mathbf{k}}}} \left(\alpha_{\mathbf{k}} e^{-ik\cdot x} + \alpha^*_{\mathbf{k}} e^{ik\cdot x}\right)$$

Single-detector PSD traces out speed distribution, sensitive to annual modulation




Multi-detector PSD sensitive to velocity distribution and daily modulation, if separated by a (mass-dependent) coherence length