PROPERTIES OF QCD AXION DARK MATTER FROM COSMOLOGICAL LATTICE SIMULATIONS

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based on JCAP10(2024)043 with K. Saikawa, J. Redondo & A. Vaquero

and work in preparation with K. Chathirathas, C. Eröncel, J. Foster, J. Redondo, I. Rybak & K. Saikawa

Theory Seminar
SLAC, October 15th 2025

Background: G. Pierobon

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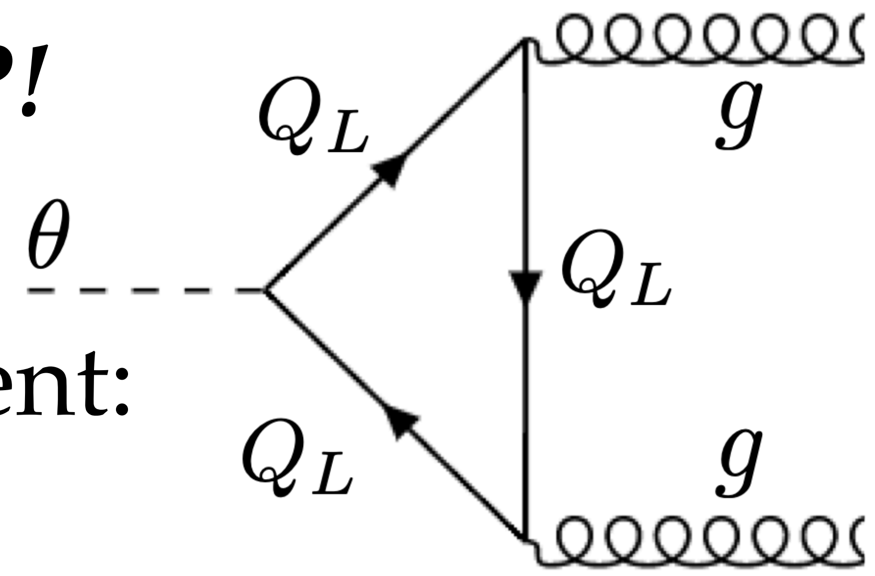
Strong-CP Problem and Peccei-Quinn Solution

- Theory of strong interactions (QCD) allows for an additional, **topological** term:

$$\mathcal{L} \supset \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$$

For a review, cf. Peccei [0607.268] or
TASI lectures by A. Hook

- Violates discrete **P** and **T** symmetries, but conserves **C** invariance, so it **violates CP***!*



e.g Marsh [2308.16003]

- Induces a neutron electric dipole moment, which is strongly bounded by experiment:

$$|d_n| < 3 \times 10^{-26}\ \mathrm{e \cdot cm} \quad \rightarrow \quad \theta < 10^{-9}$$

Baker+ [0602.020]

- **Peccei + Quinn:** Extend SM by additional, global $U(1)$ symmetry which is *spontaneously broken*, effectively replacing the static ***θ*-angle** with a dynamical, CP-conserving field, the **axion**!

Peccei&Quinn [PRL 38 (1977), PRD 16 (1977)]
at Stanford! :-)

$$\theta \rightarrow \frac{a}{f_a}$$

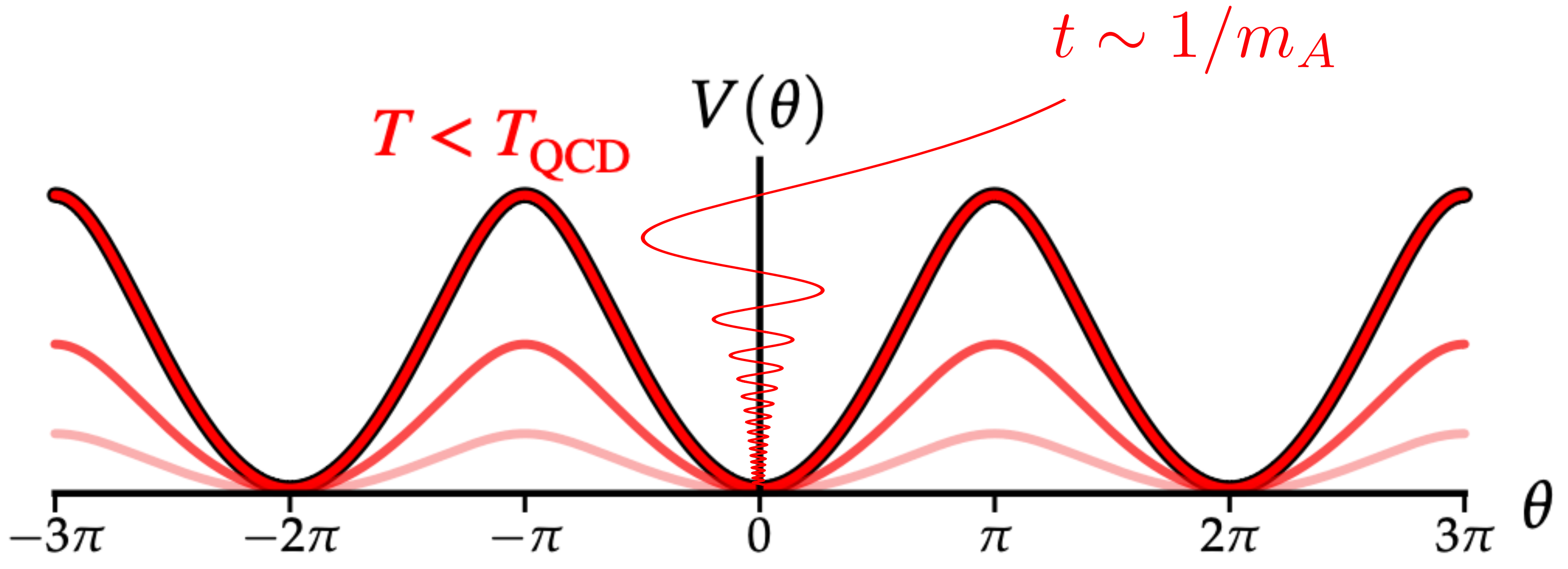
* It's actually even worse if we consider chiral transformations on the θ-vacuum, the real coefficient then reads $\bar{\theta} = \theta + \mathrm{Arg\,Det}\, Y_u Y_d$

QCD Axion

- The QCD Axion
  - Pseudo Nambu-Goldstone boson associated with the spontaneous breaking of the global Peccei-Quinn (PQ) $U(1)$ symmetry at the high-energy scale f_a.
  - Dynamical solution to the strong-CP problem.
  - Suitable candidate for Cold Dark Matter.
- Acquires a mass below the QCD scale.

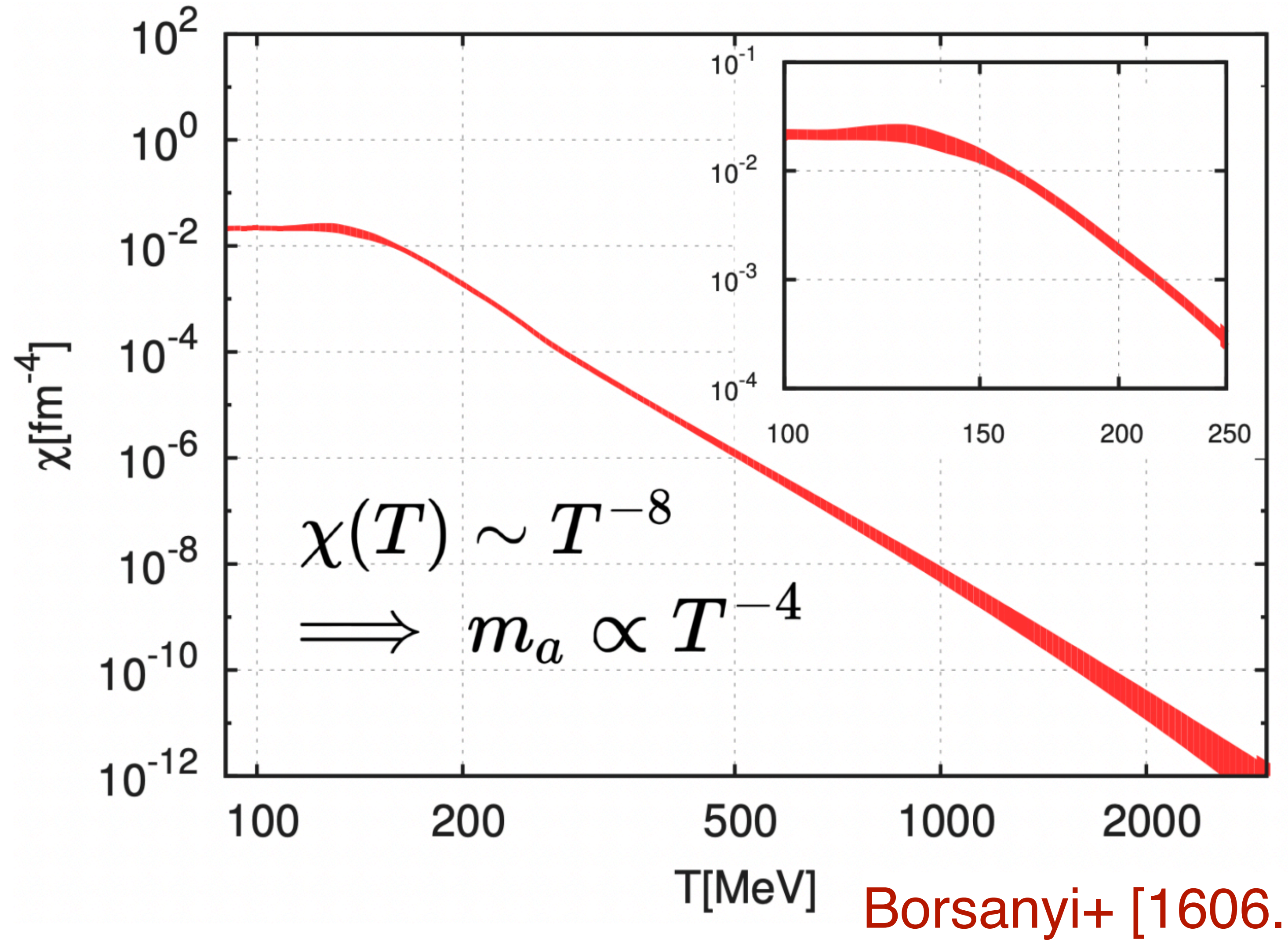
* Throughout this talk, when we refer to the axion, we implicitly mean the **QCD axion** (i.e. solves the strong CP problem)

$$V(\theta) \approx \chi(T)(1-\cos\theta) = m_a^2(T) f_a^2 (1-\cos\theta)$$



O'Hare [2403.17697]

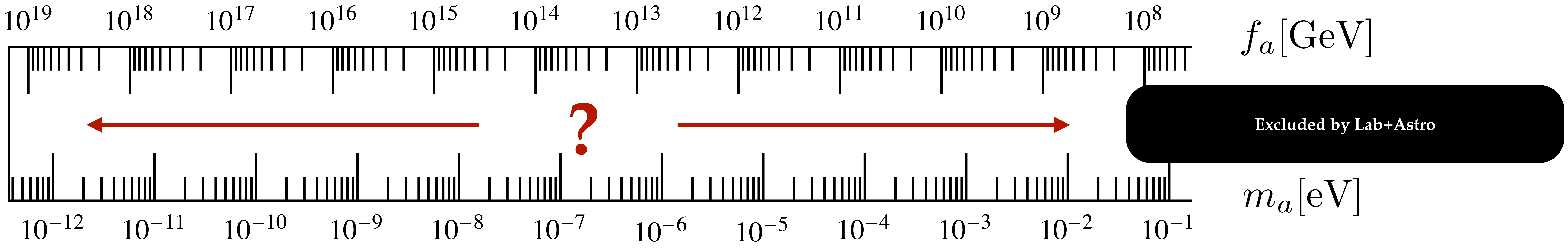
$$m_A |\theta|^2 R^3 = \frac{\rho_A}{m_A} R^3 \equiv N_A$$



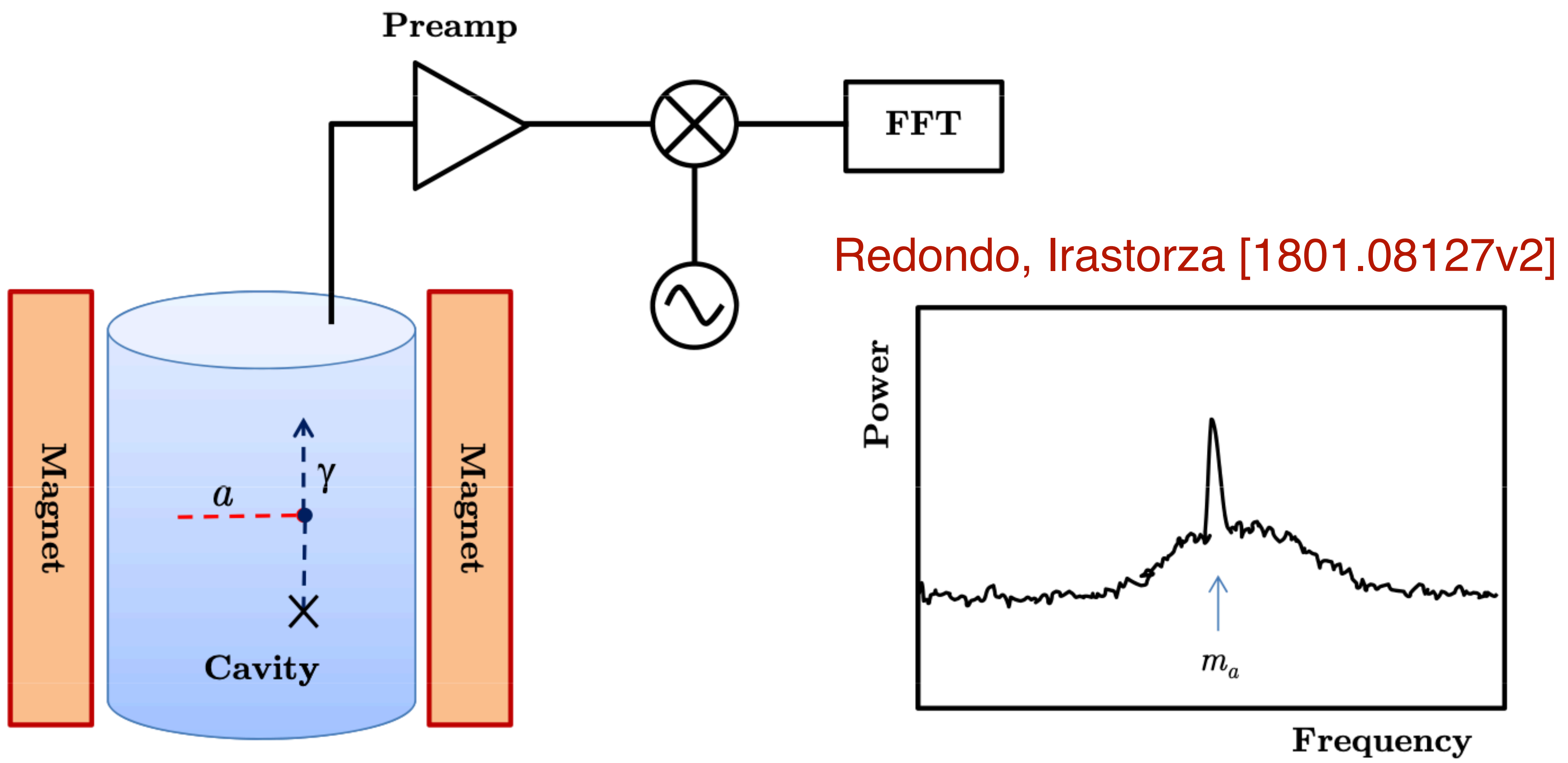
Borsanyi+ [1606.07494]

Axion Dark Matter Mass

- What is the “typical mass” of QCD axion dark matter?



- How can theory inform experimental searches?
- Information on theoretical predictions could also be used to interpret future experimental results.



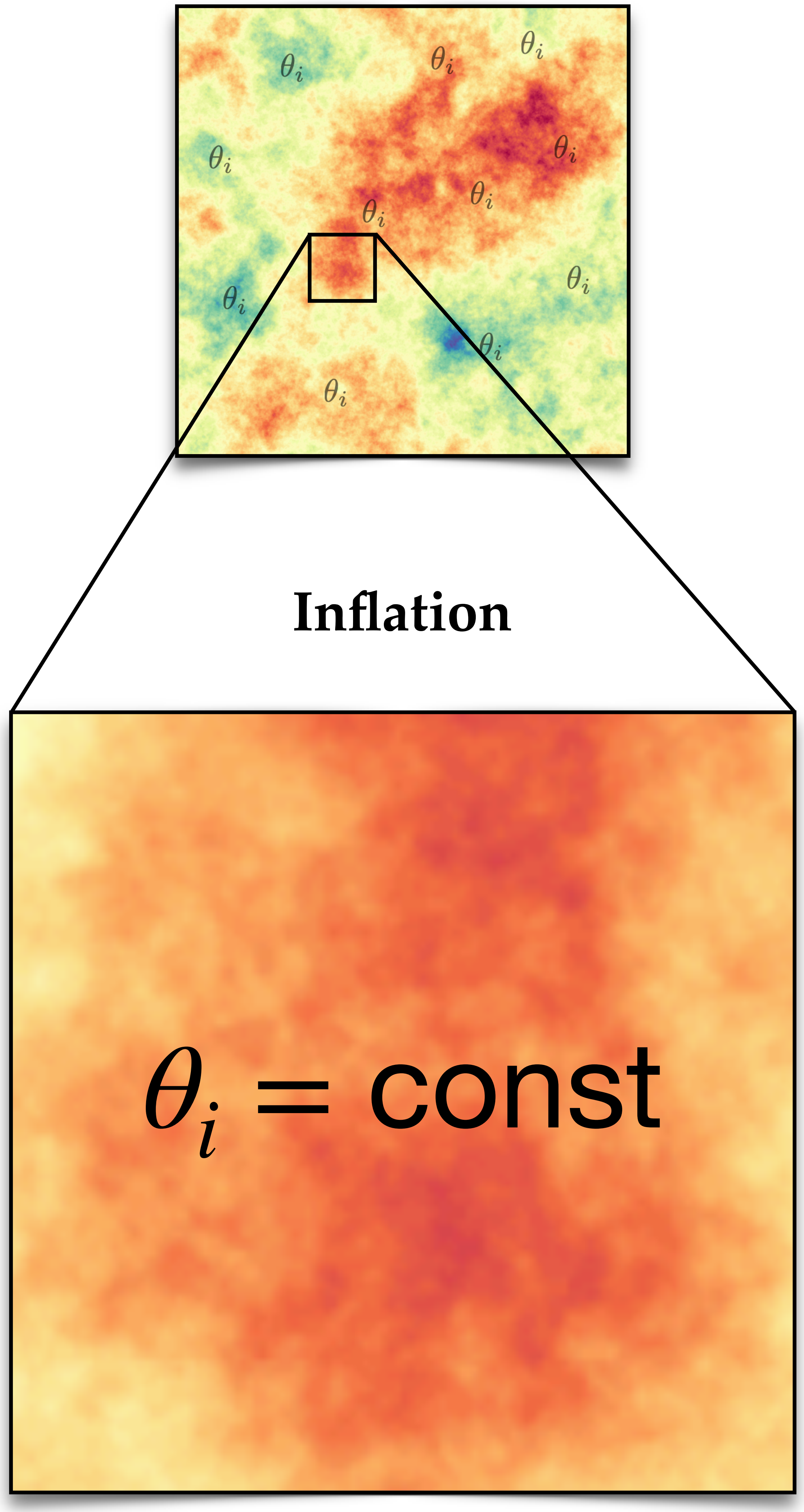
Redondo, Irastorza [1801.08127v2]

$$P_s = \kappa \left(\frac{Q}{m_a}\right) g_{a\gamma}^2 B_e^2 |\mathcal{G}_m|^2 V \varrho_a$$

When did Inflation happen?

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation



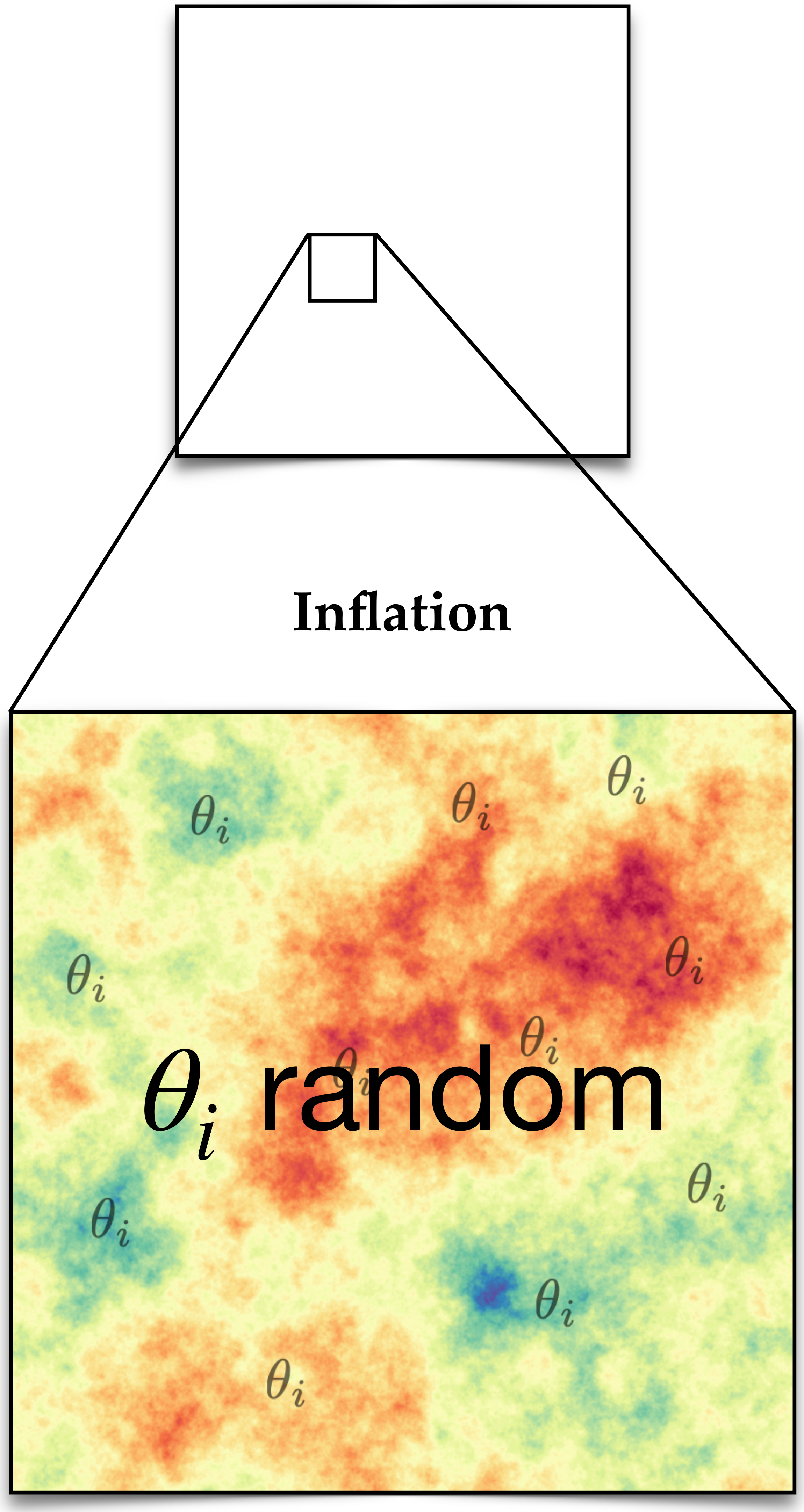
Inflation

$\theta_i = \text{const}$

$$\sqrt{\langle\theta_i\rangle} = ???$$

Post-Inflationary Scenario

PQ broken **after** inflation



Inflation

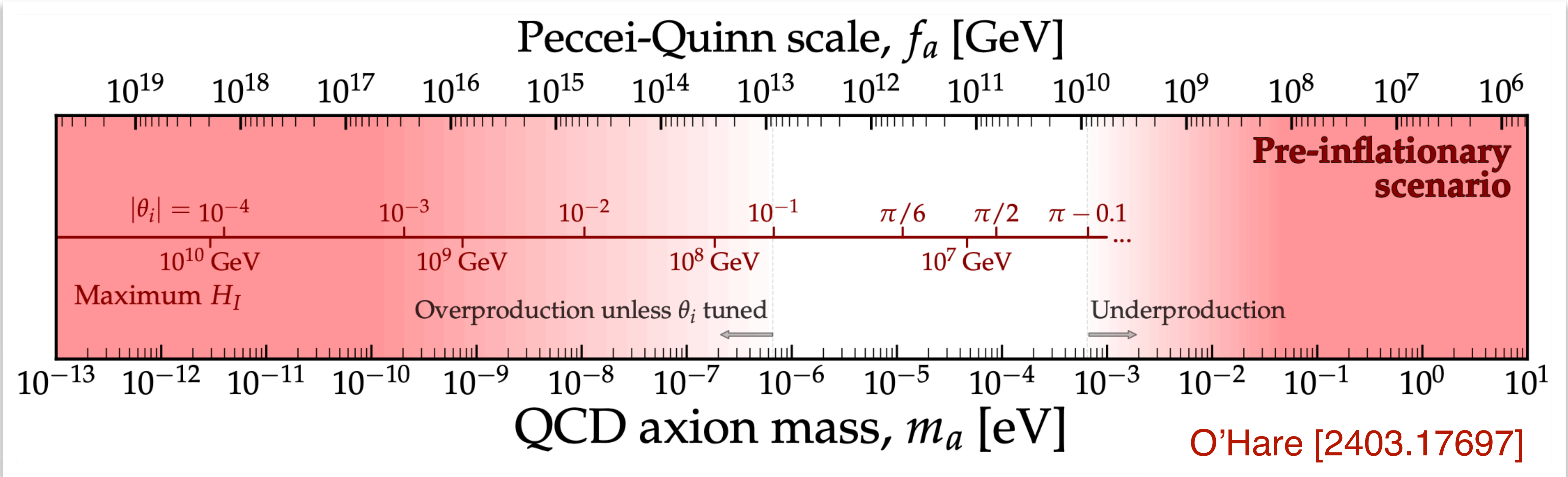
θ_i random

$$\sqrt{\langle\theta_i\rangle} \sim 2$$

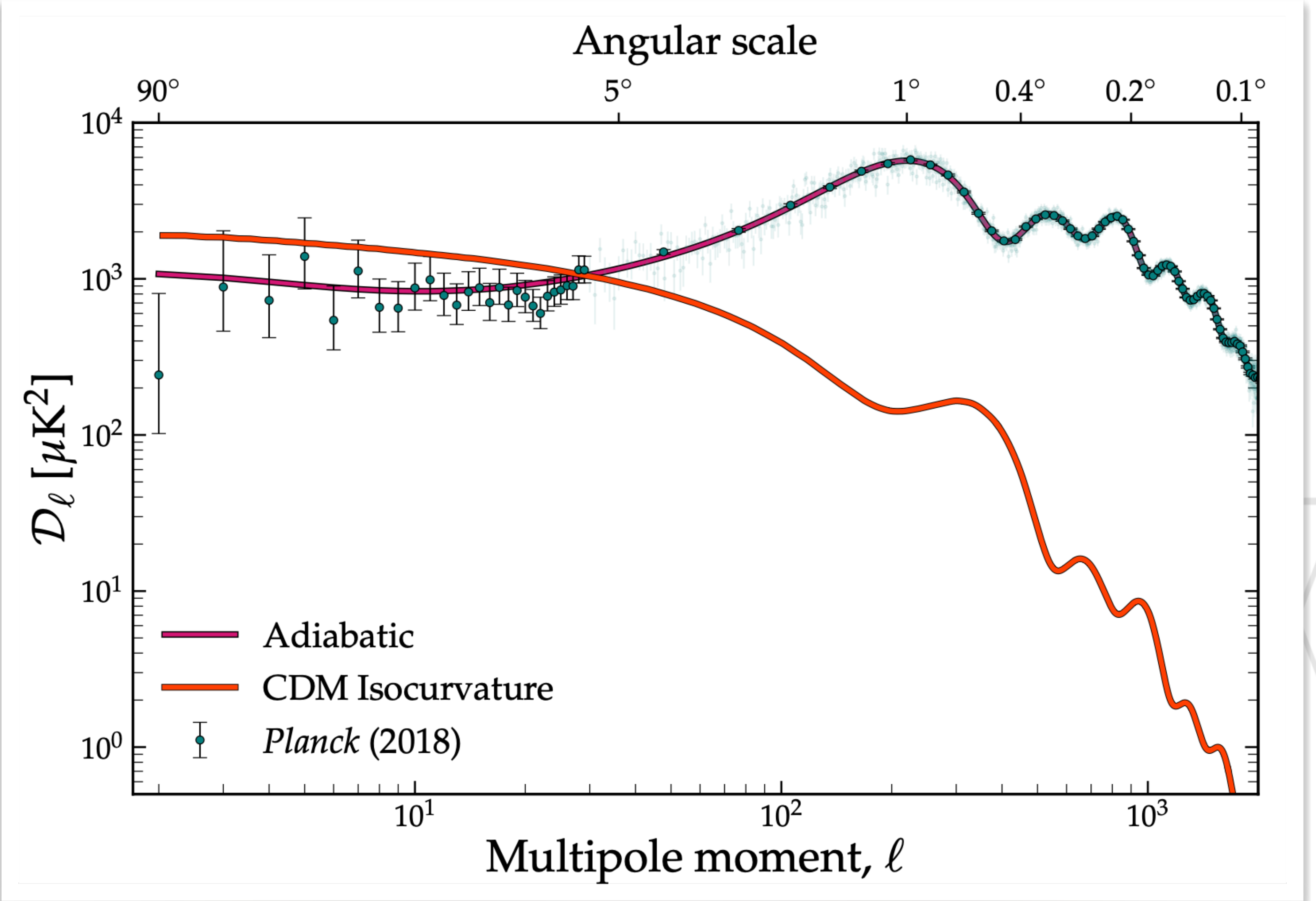
When did Inflation happen?

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation



O'Hare [2403.17697]

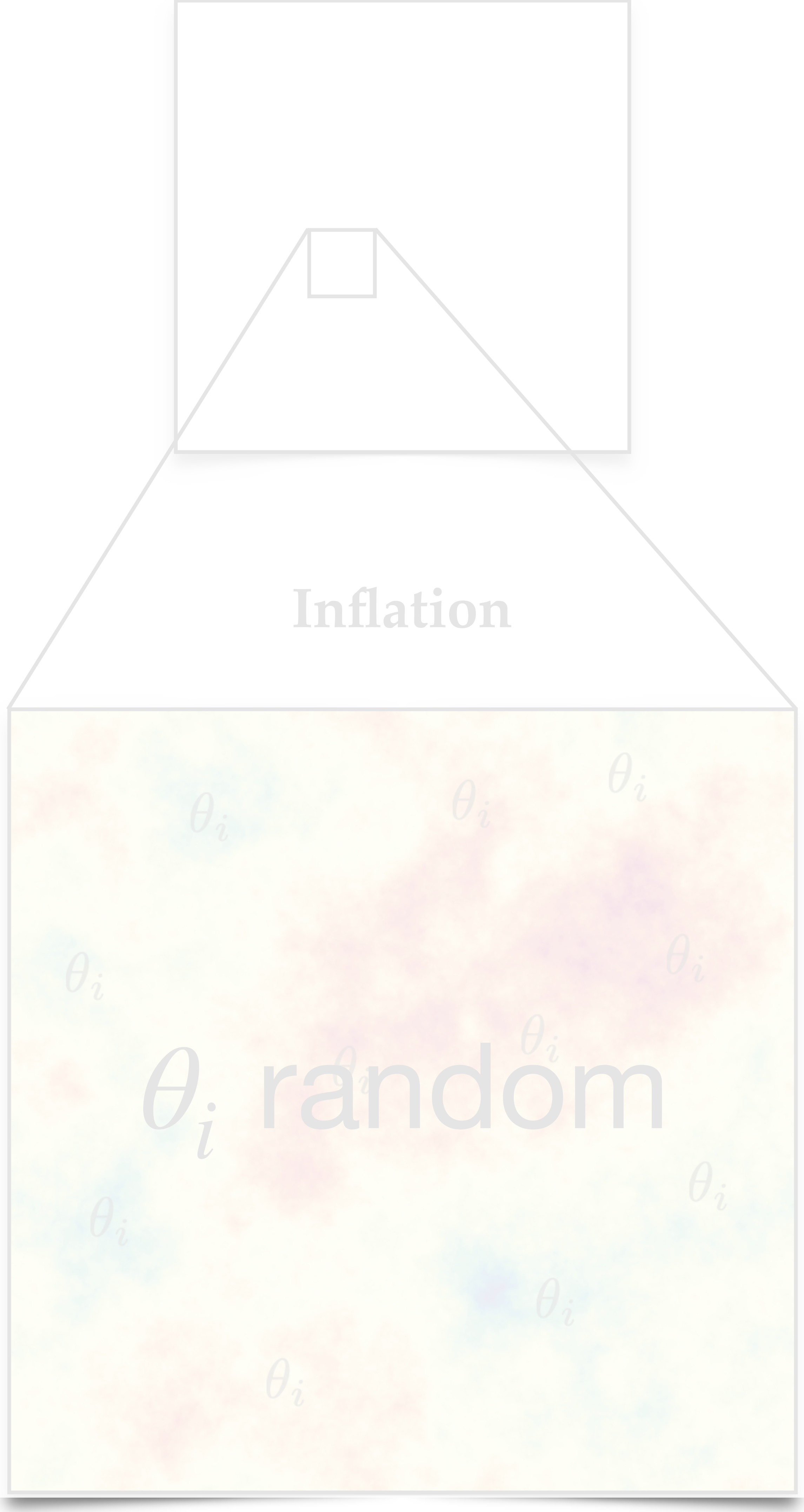


PLANCK [1907.12875]

$\sqrt{\langle\theta_i\rangle} = ???$

Post-Inflationary Scenario

PQ broken **after** inflation



$\sqrt{\langle\theta_i\rangle} \sim 2$

When did Inflation happen?

Pre-Inflationary Scenario

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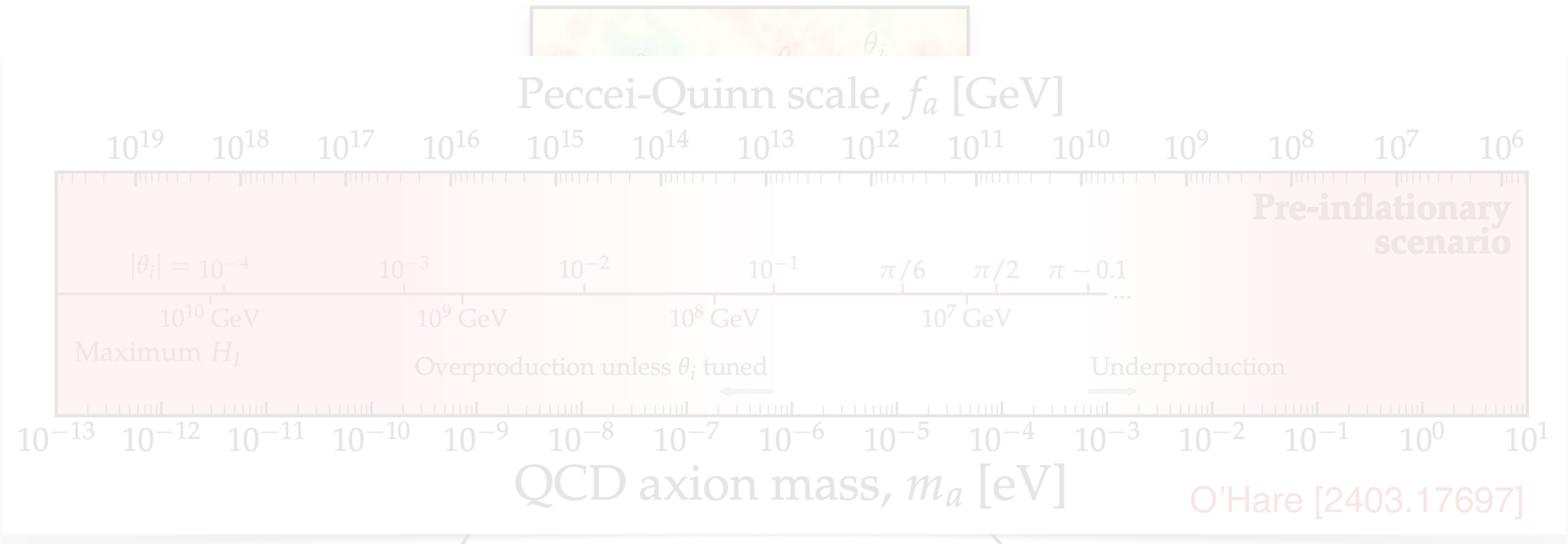
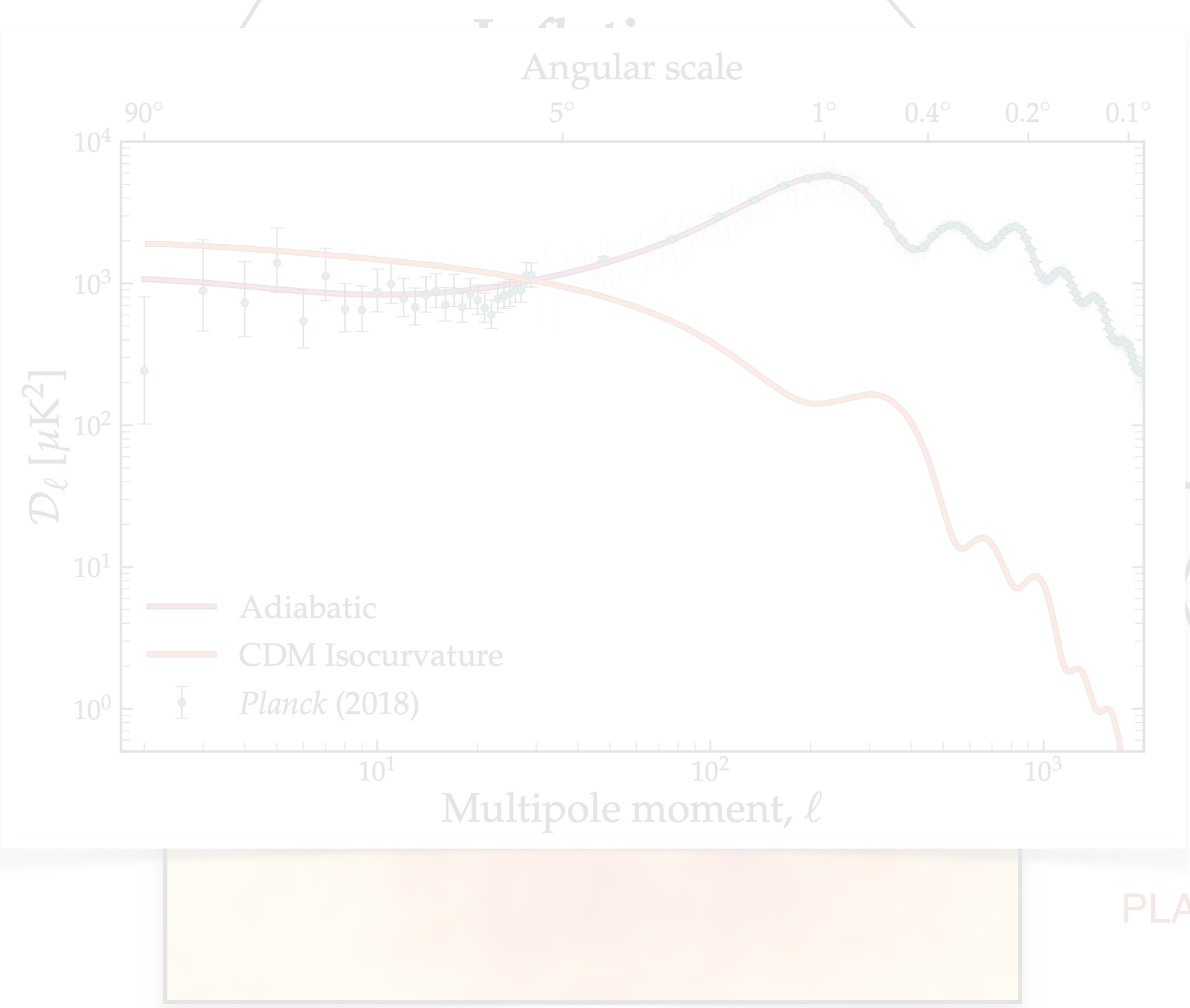


O'Hare [2403.17697]



$$\langle \theta_i \rangle = ???$$

PLANCK [1907.12875]

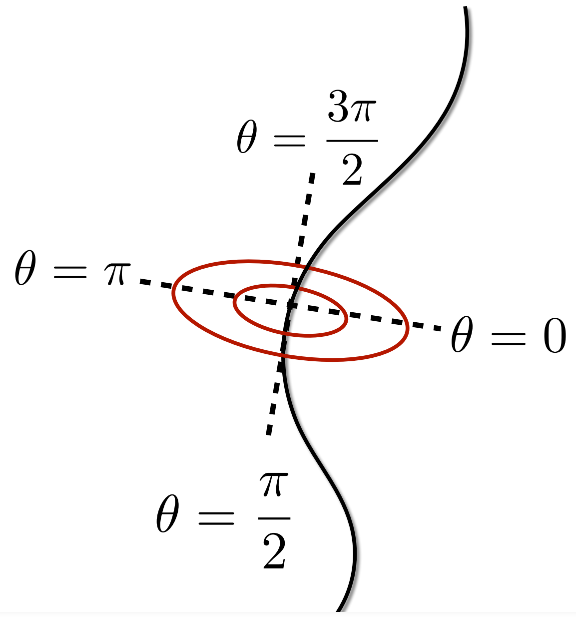
Post-Inflationary Scenario

PQ broken **after** inflation

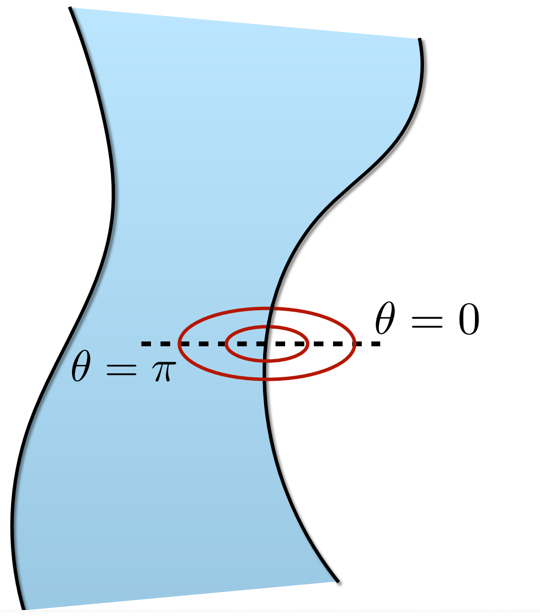
- Allows in principle for precise axion mass prediction

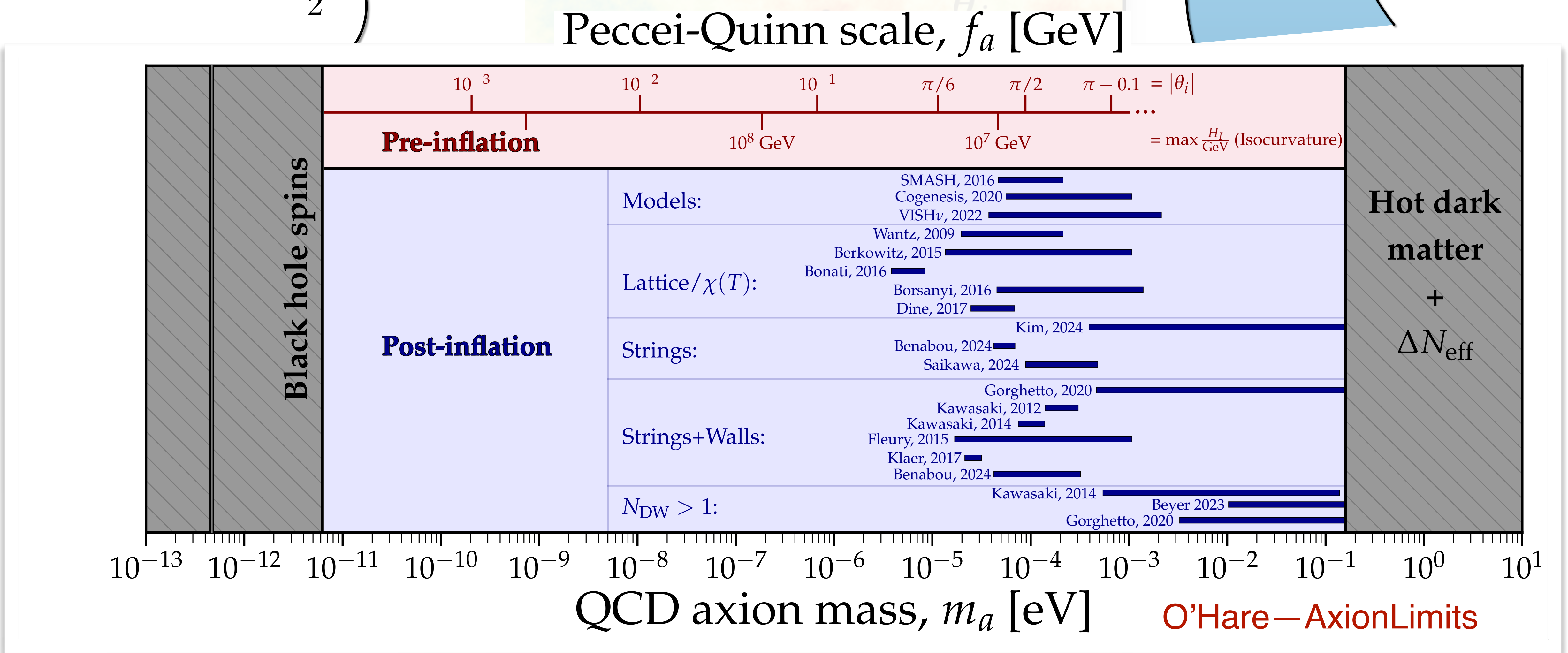
$$\Omega_a h^2 = 0.12 \quad \Rightarrow \quad m_a = ???\mu\text{eV}$$

- **Subtlety:** Formation of Strings (+ Domain Walls)



Buschmann, MK+ [to appear]





O'Hare—AxionLimits

Pre-Inflation!

Part I:

Phenomenology of Axion Misalignment

Based on 251X.XXXX (soon!) with K. Chathirathas, C. Eröncel, J. Redondo & K. Saikawa

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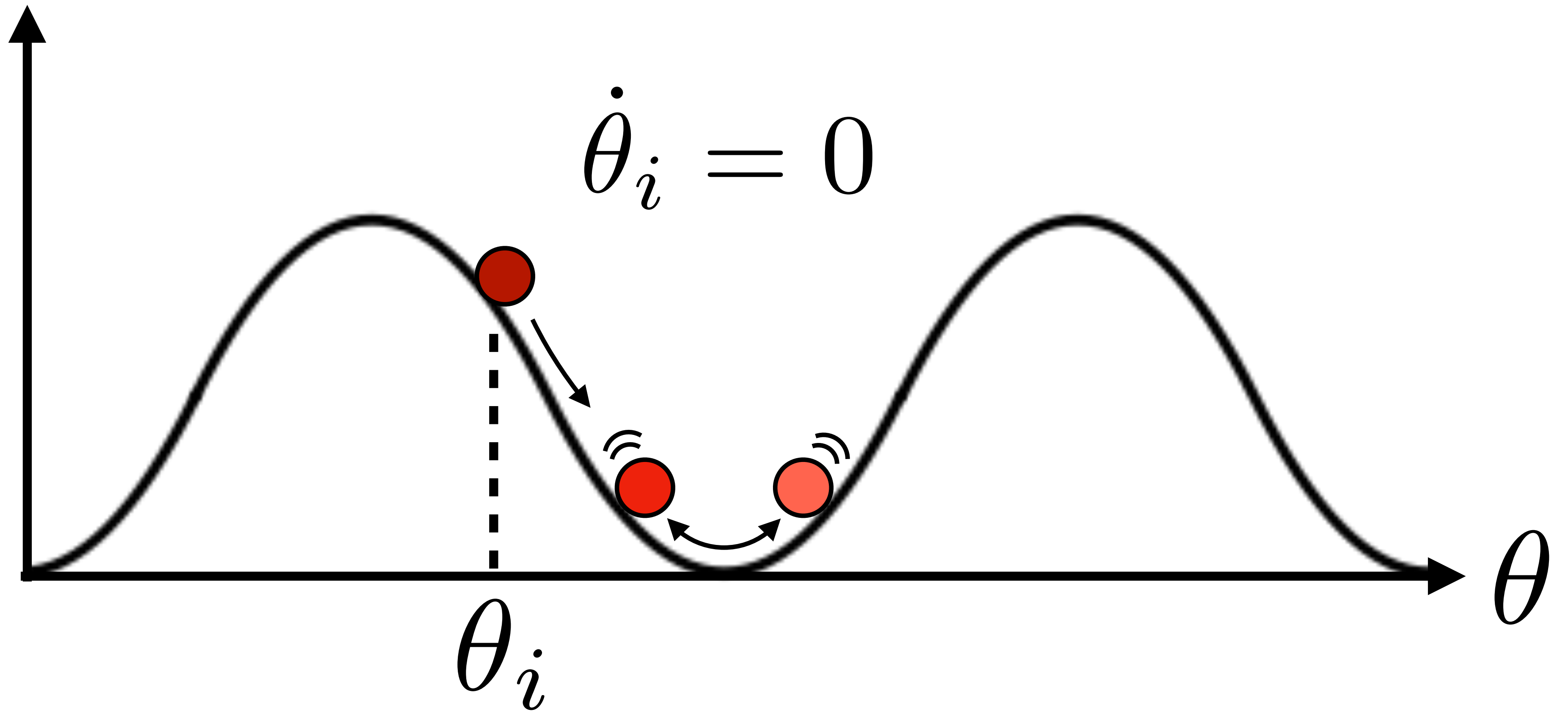
Axion Misalignment Mechanism

$$V(\theta) \sim \chi(T)\,(1-\cos\theta) = m_a(T)^2\,(1-\cos\theta)$$

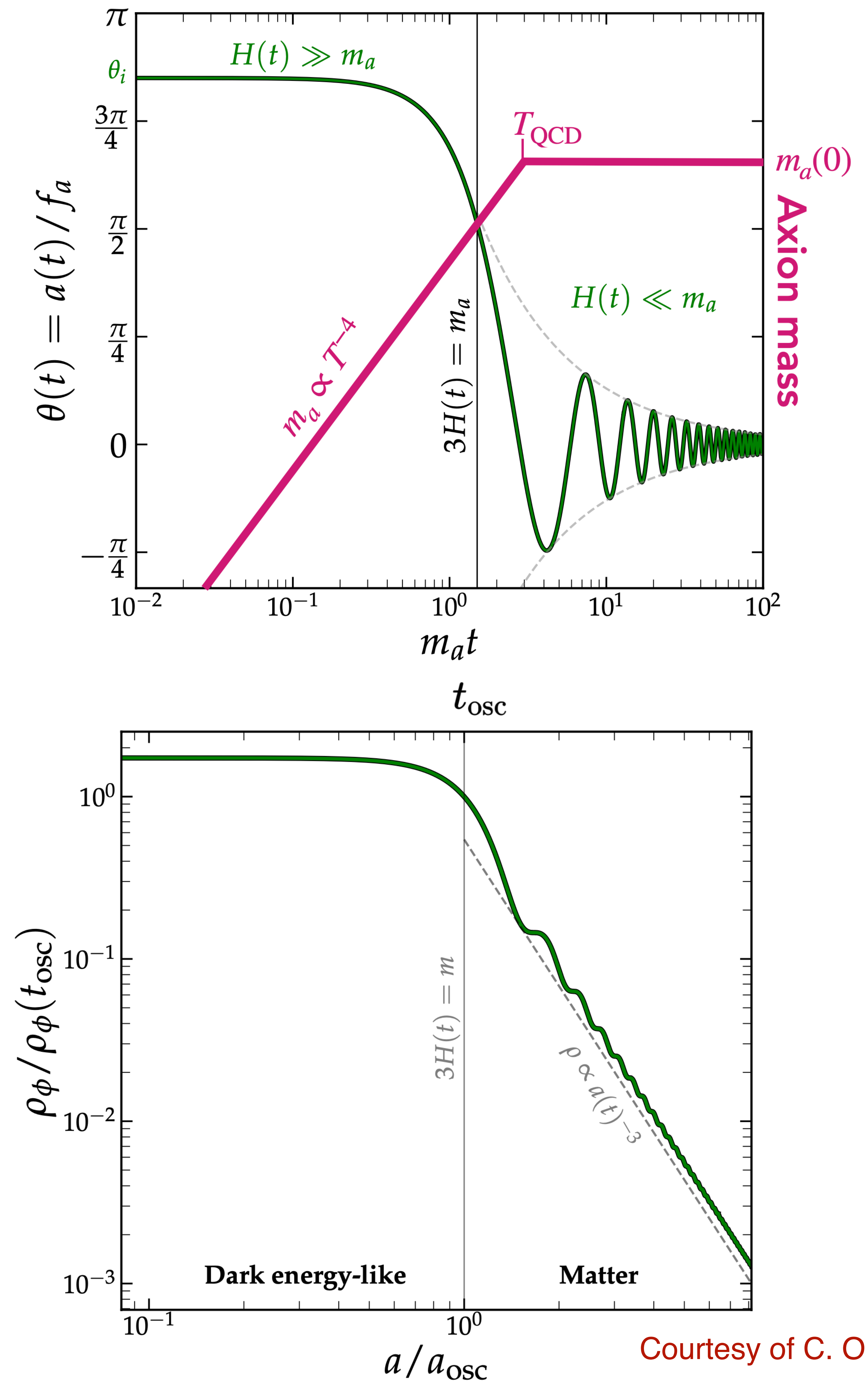


- Field initially displaced from minimum, starts (damped) oscillations:

$$\ddot{\theta} + 3H\dot{\theta} + m_a(T)^2 \sin\theta = 0$$

- Axion Dark Matter abundance from energy dissipation:

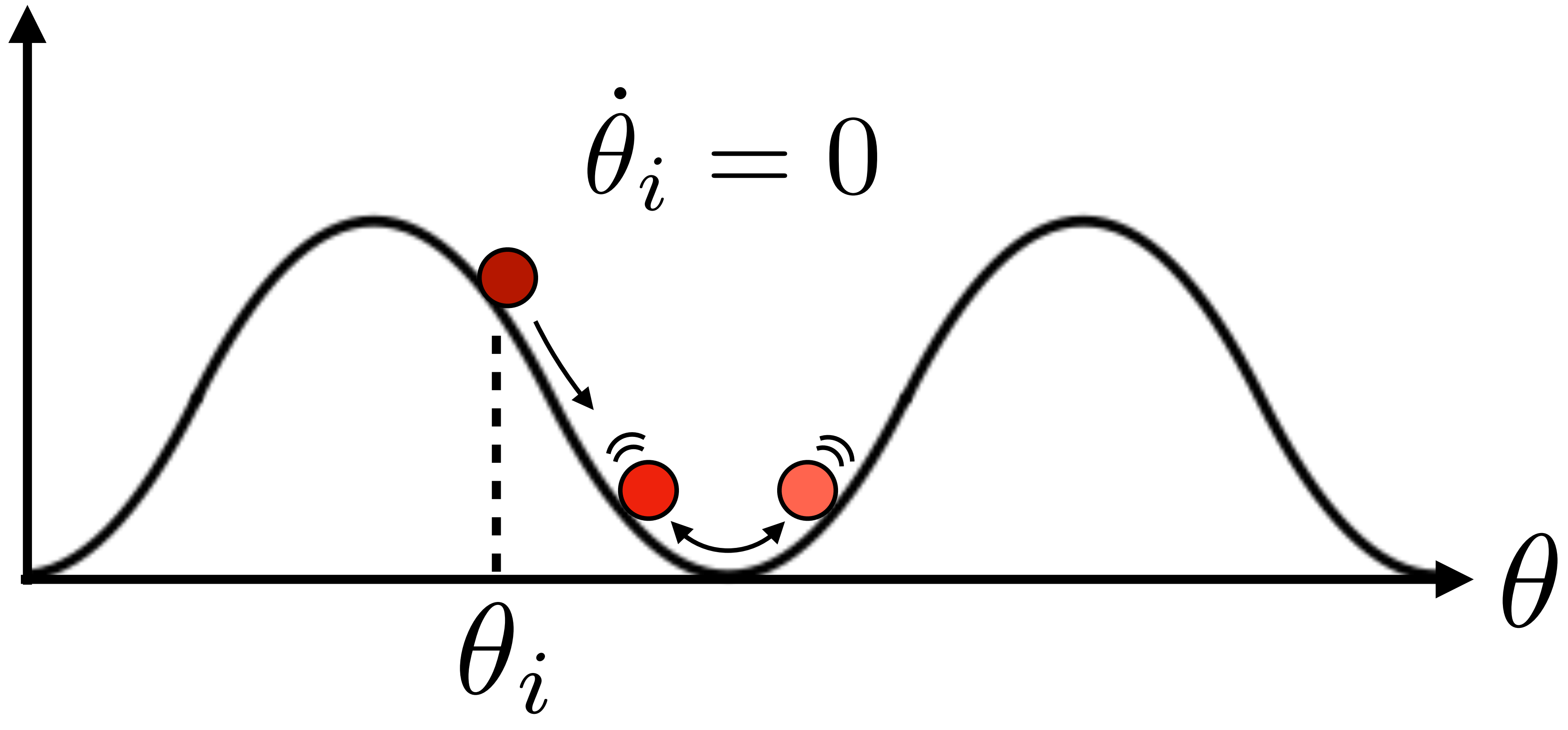
$$\Omega_a h^2 = \frac{\rho_a}{\rho_c} h^2 = \frac{\rho_a}{3H_0^2 M_{\rm Pl}^2} h^2 \to 0.12$$



Courtesy of C. O'Hare

Axion Misalignment Mechanism

$$V(\theta) \sim \chi(T)\,(1-\cos\theta) = m_a(T)^2\,(1-\cos\theta)$$



- Field initially displaced from minimum, starts (damped) oscillations:

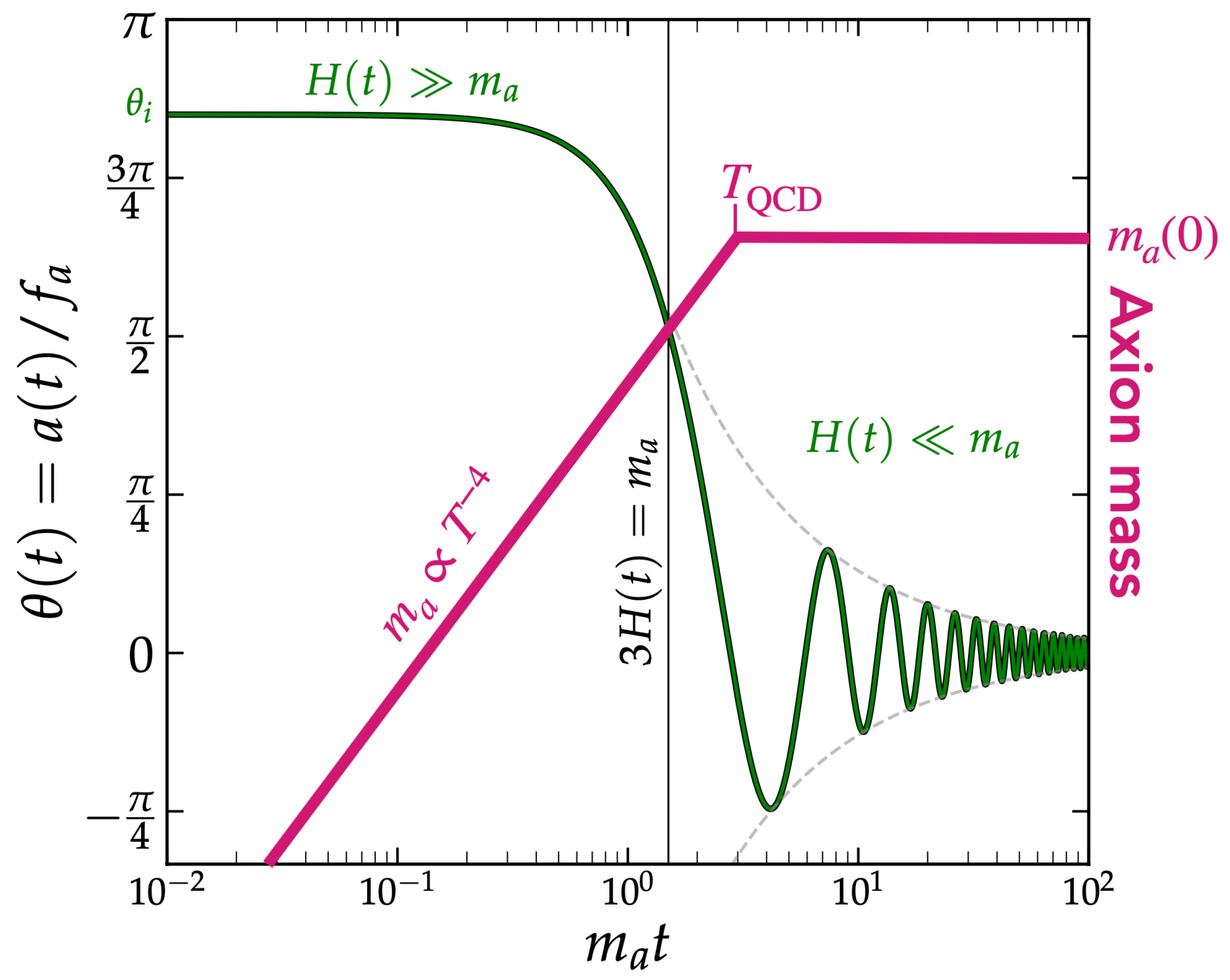
$$\ddot{\theta} + 3H\dot{\theta} + m_a(T)^2 \sin\theta = 0$$

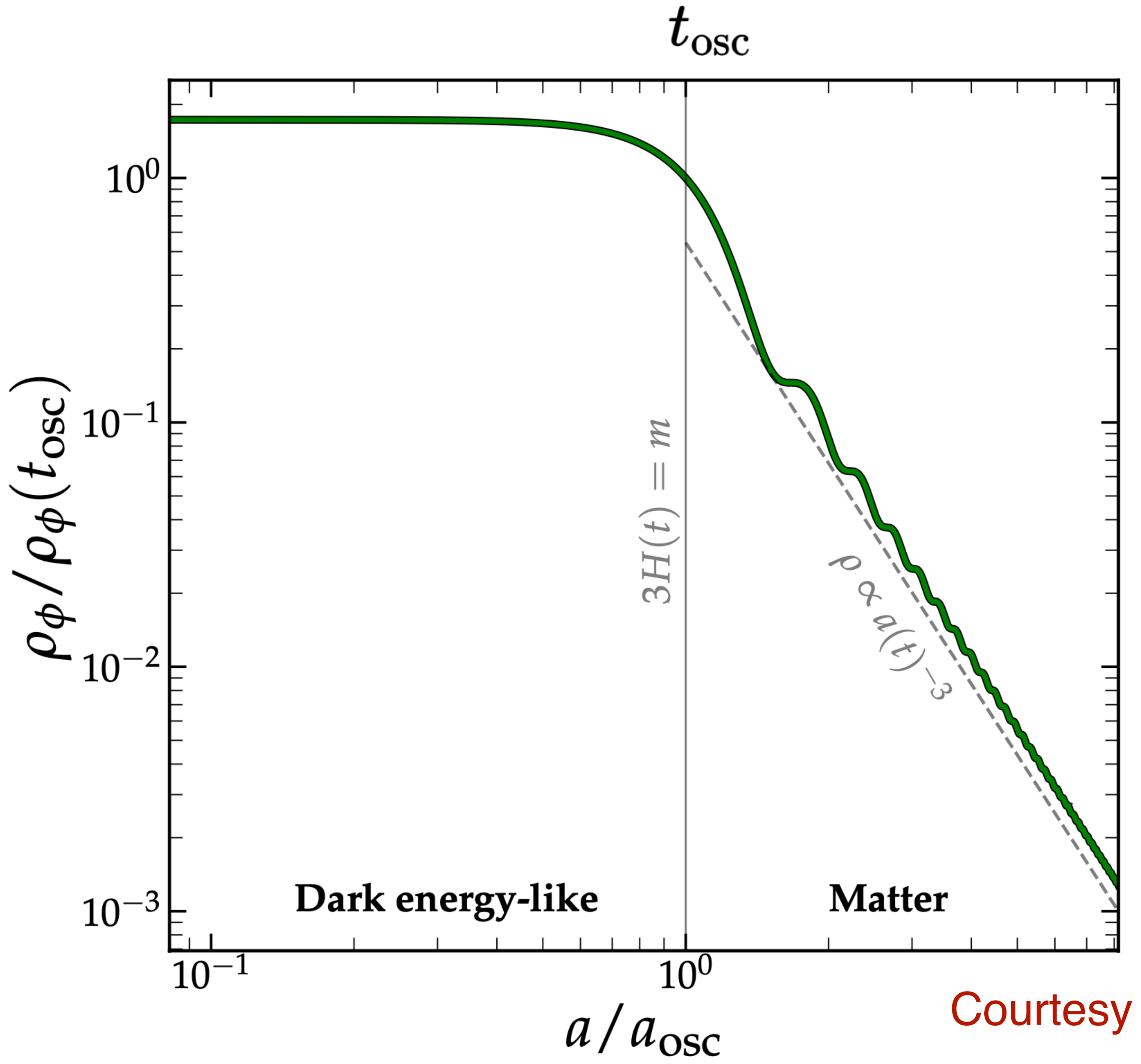
- Axion Dark Matter abundance from energy dissipation:

$$\boxed{\Omega_a h^2 \approx 0.12\theta_i^2 \left(\frac{7.26\mu\text{eV}}{m_a}\right)^{\frac{n+6}{n+4}}}$$

$n \sim 8$

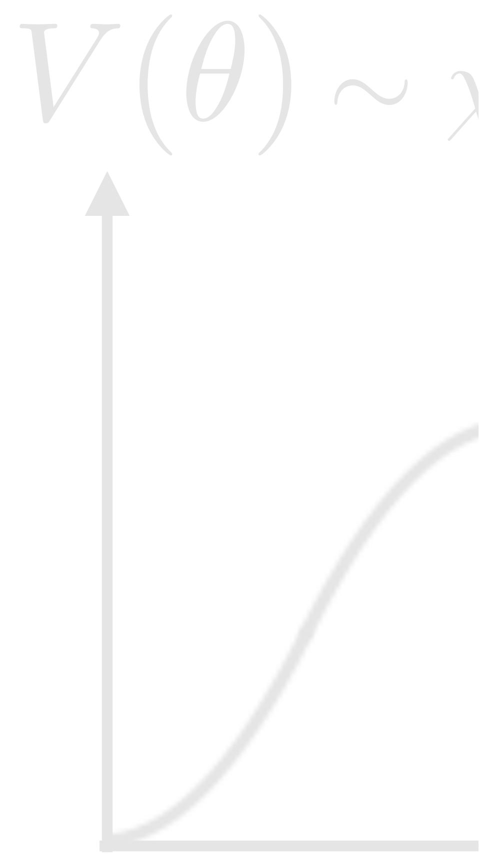
Borsanyi+ [1606.07494]





Courtesy of C. O'Hare

Canonical Mass Prediction from Misalignment

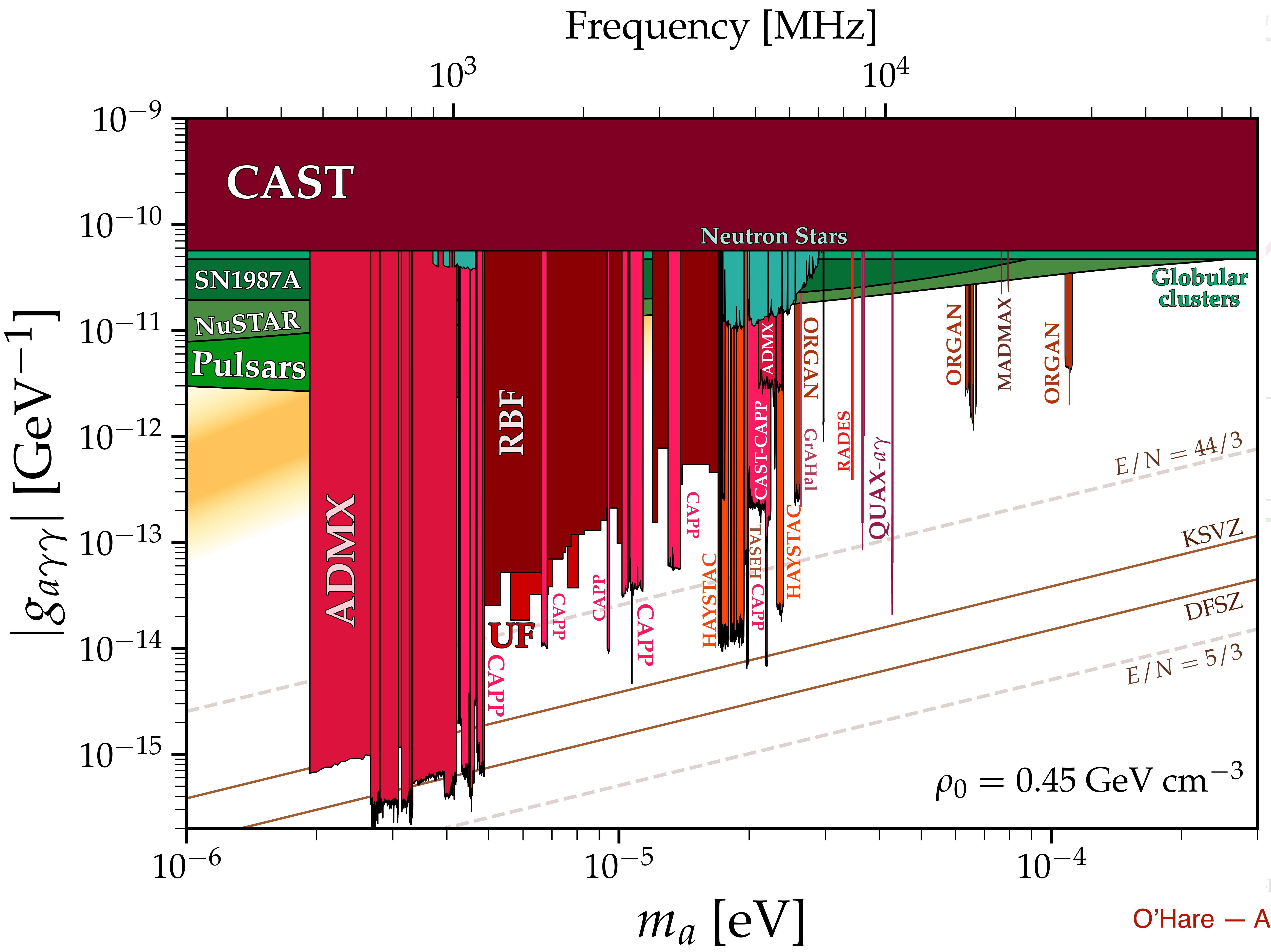


- Field initi
  (damped)

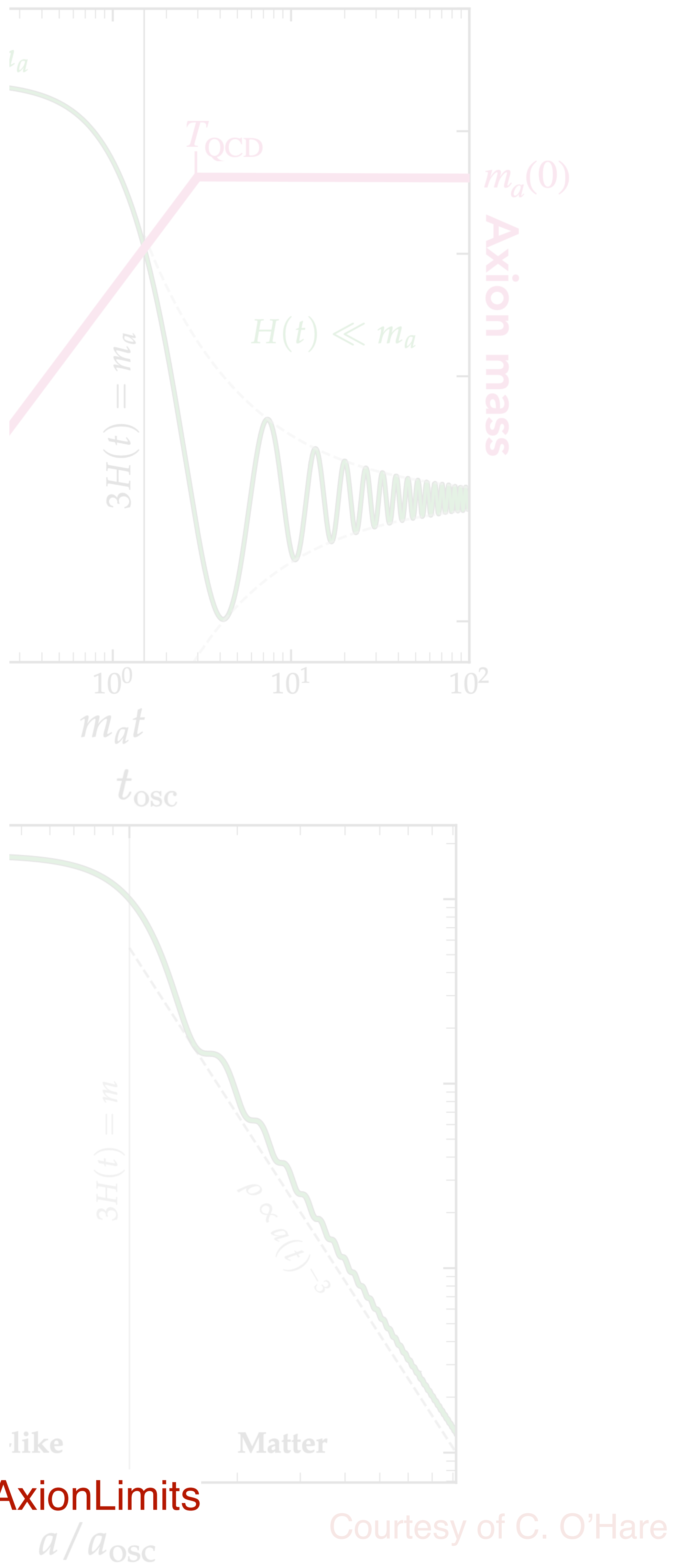
$\ddot{\theta}$ –

- Axion Da

$\Omega_a h^2$



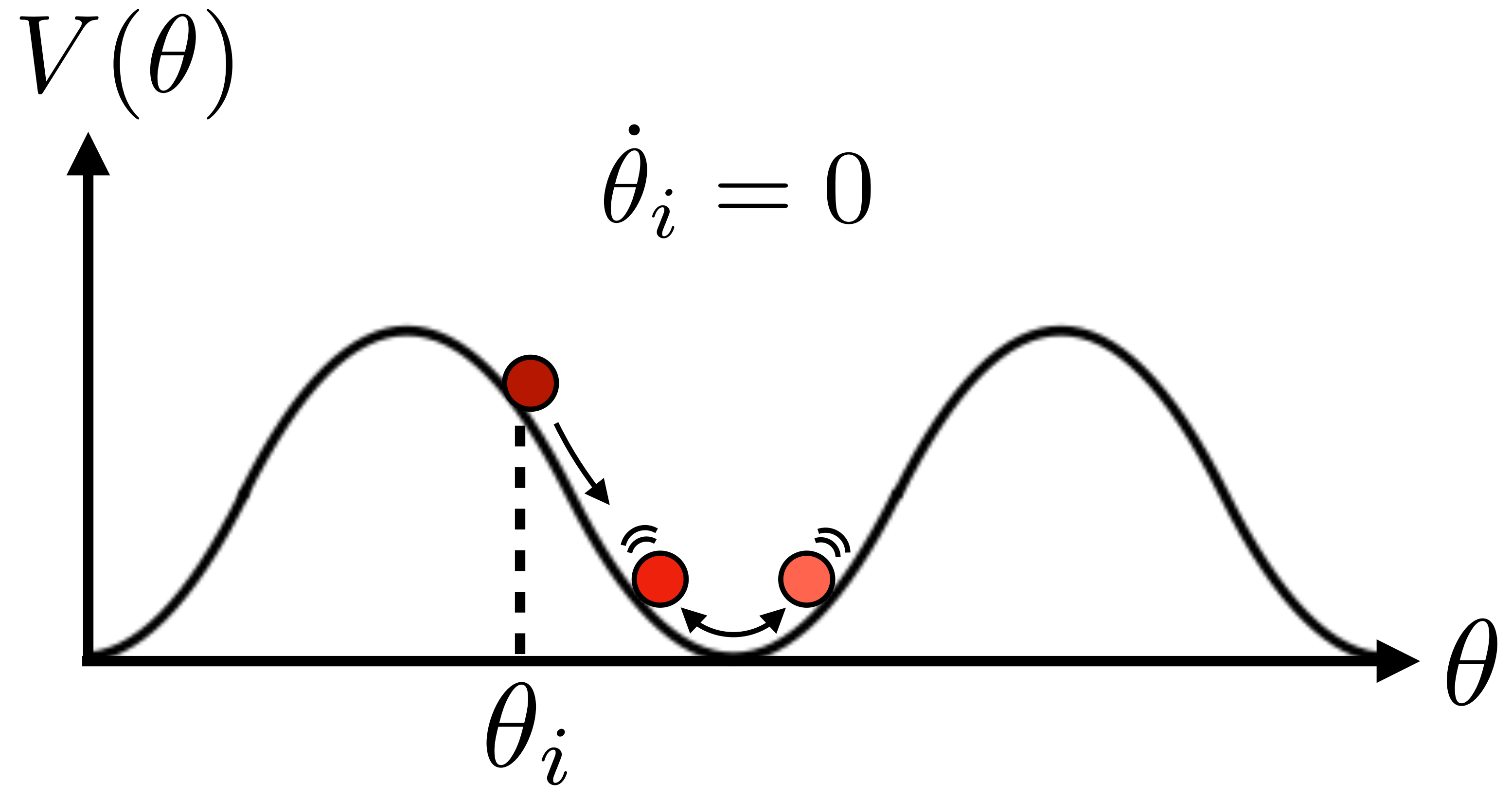
O'Hare — AxionLimits



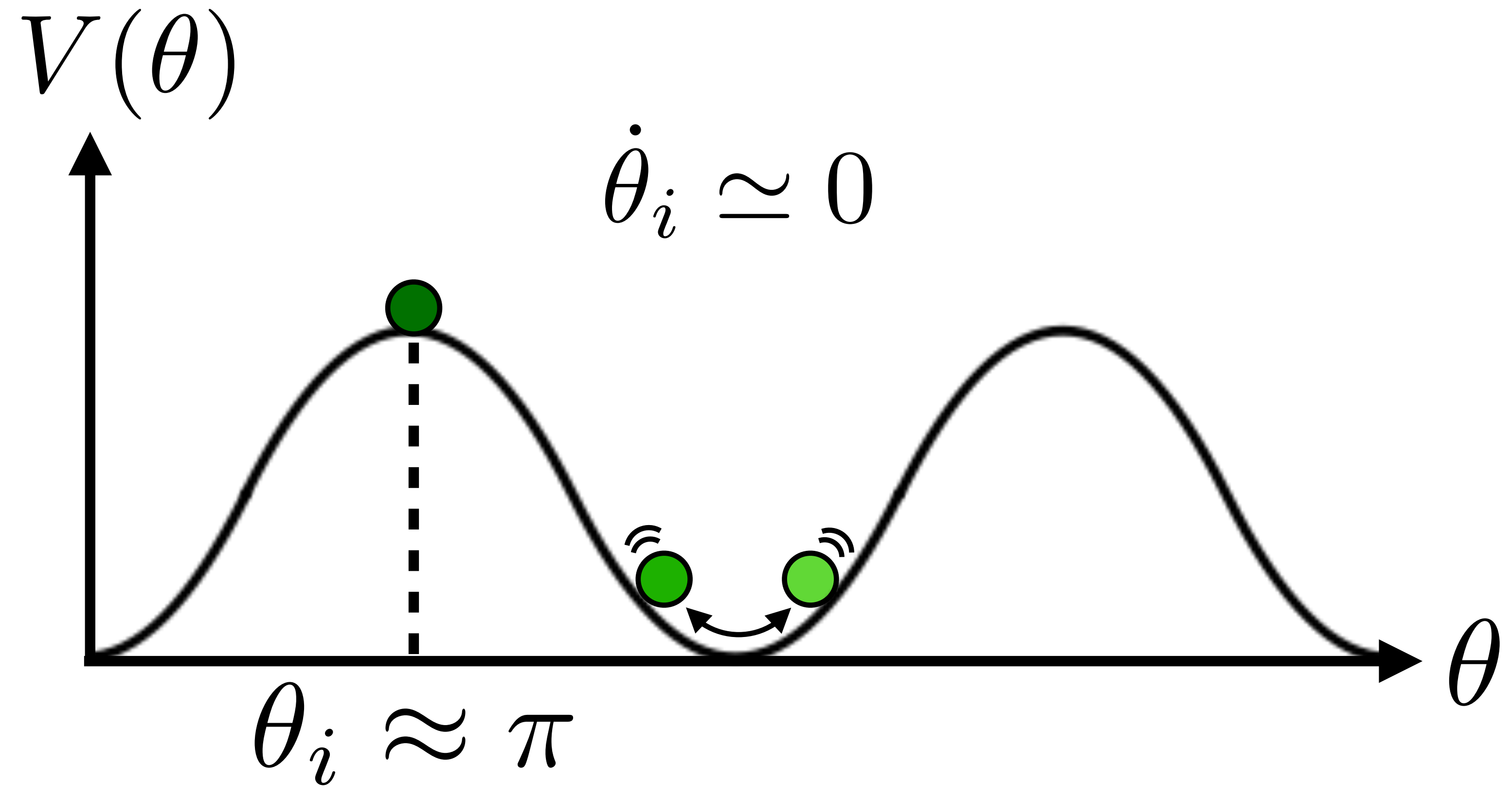
Courtesy of C. O'Hare

The Importance of Initial Conditions

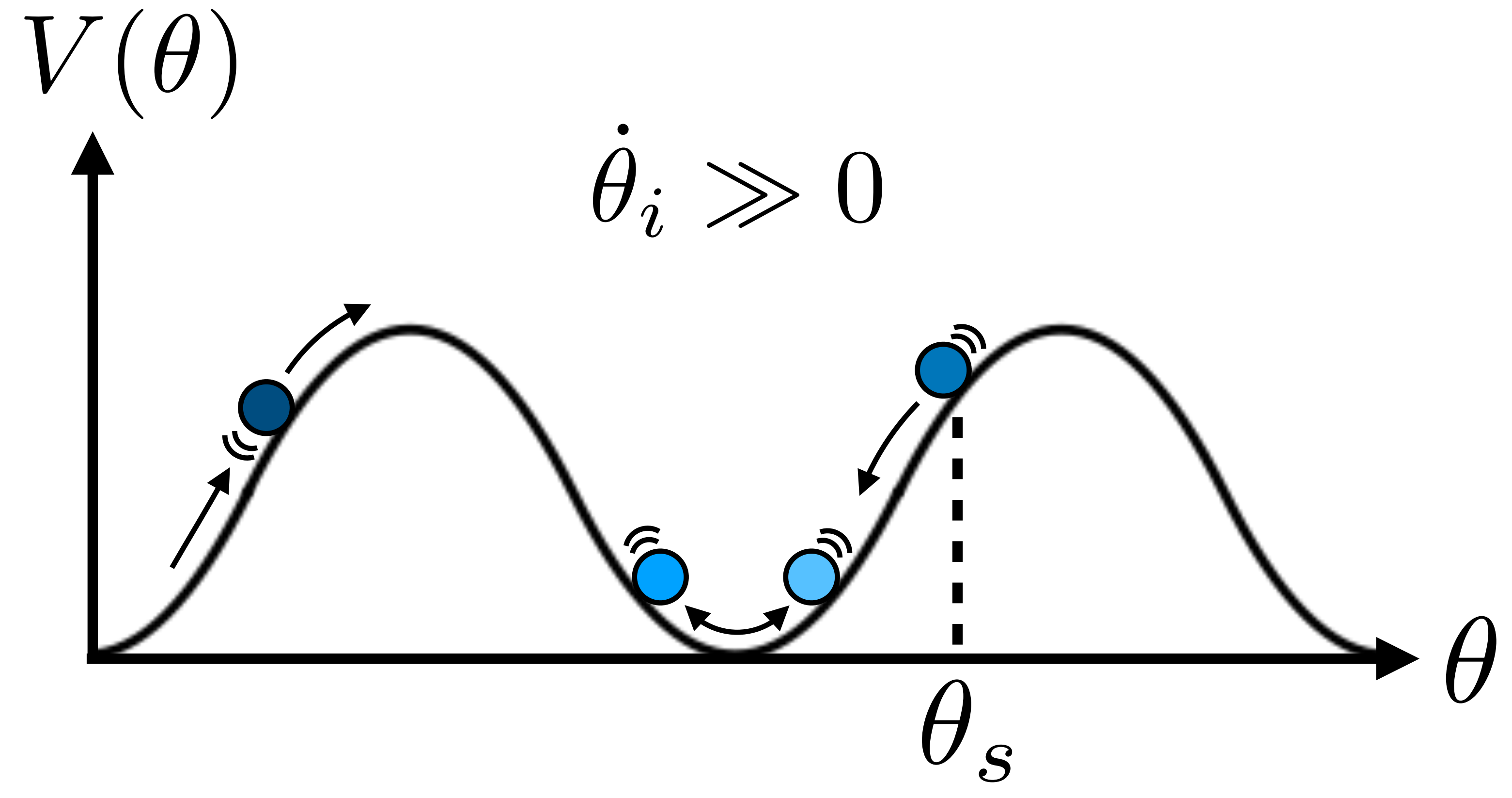
Standard



Large Misalignment Angle



Large Kinetic Misalignment



- Depending on the exact initial conditions, the onset of the adiabatic oscillations (*trapping*) can be quite different!
- The trapping temperature that yields all of CDM can be precisely computed:

$n \sim 8$ Borsanyi+ [1606.07494]

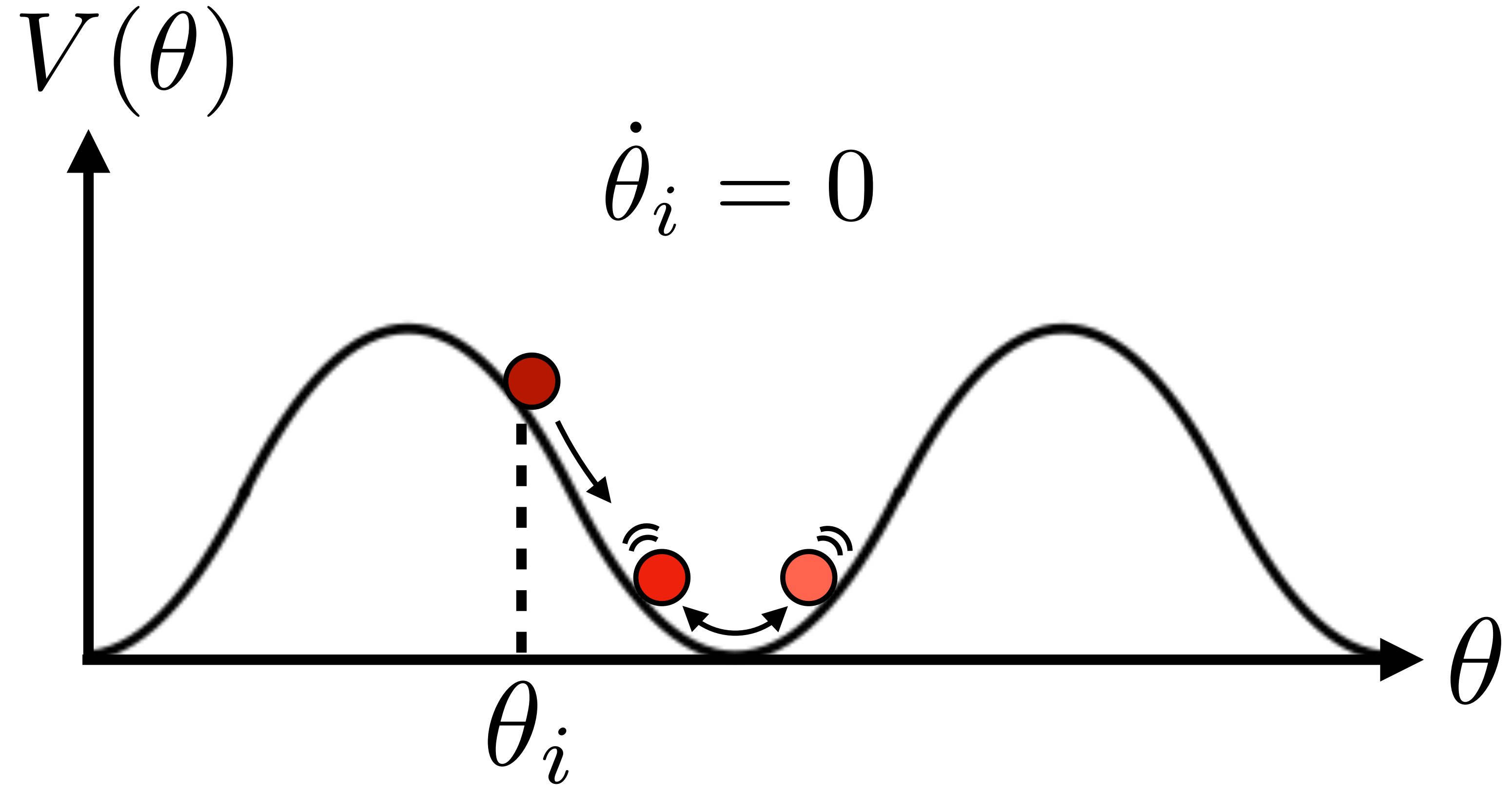
$$\Omega_A h^2 = (1-\cos\theta_M)\frac{\sqrt{\chi(T_s)\chi(T_0)}\,\frac{g_S(T_0)T_0^3}{g_S(T_s)T_s^3}}{\rho_c/h^2} = 0.12$$

$$T_s \sim 1.03\ \mathrm{GeV}(1-\cos\theta_s)^{2/(n+6)}$$

Arvanitaki+ [1909.11665], Co+ [1910.14152, …], Eröncel+ [2206.14259, 2207.10111], Fasiello+ [2507.01822], Chathirathas, MK+ [*to appear*]

Standard Axion Misalignment

Standard



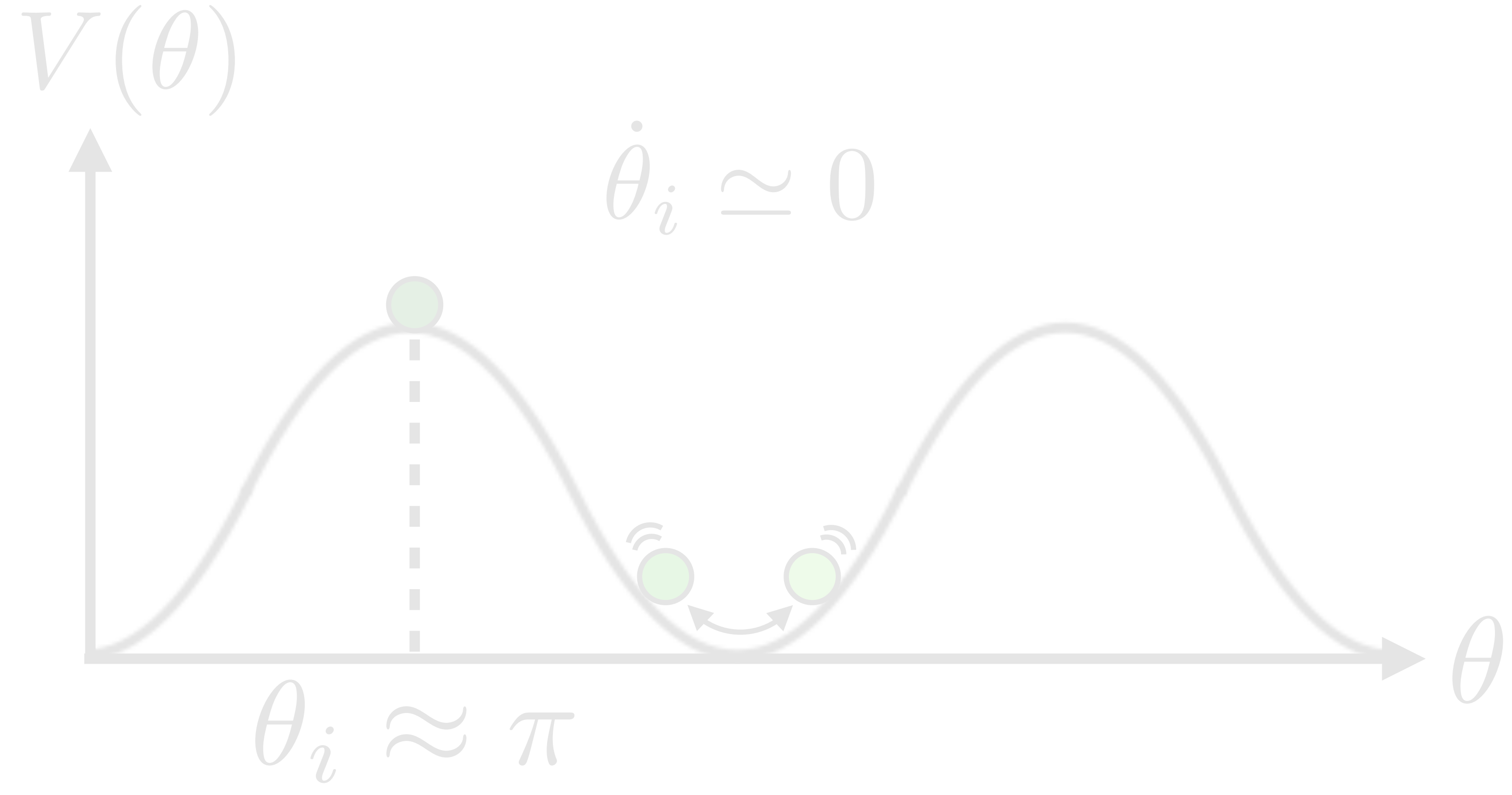
$$\ddot{\theta} + 3H\dot{\theta} + m_a(T)^2 \sin\theta = 0$$

- Characteristic timescale:

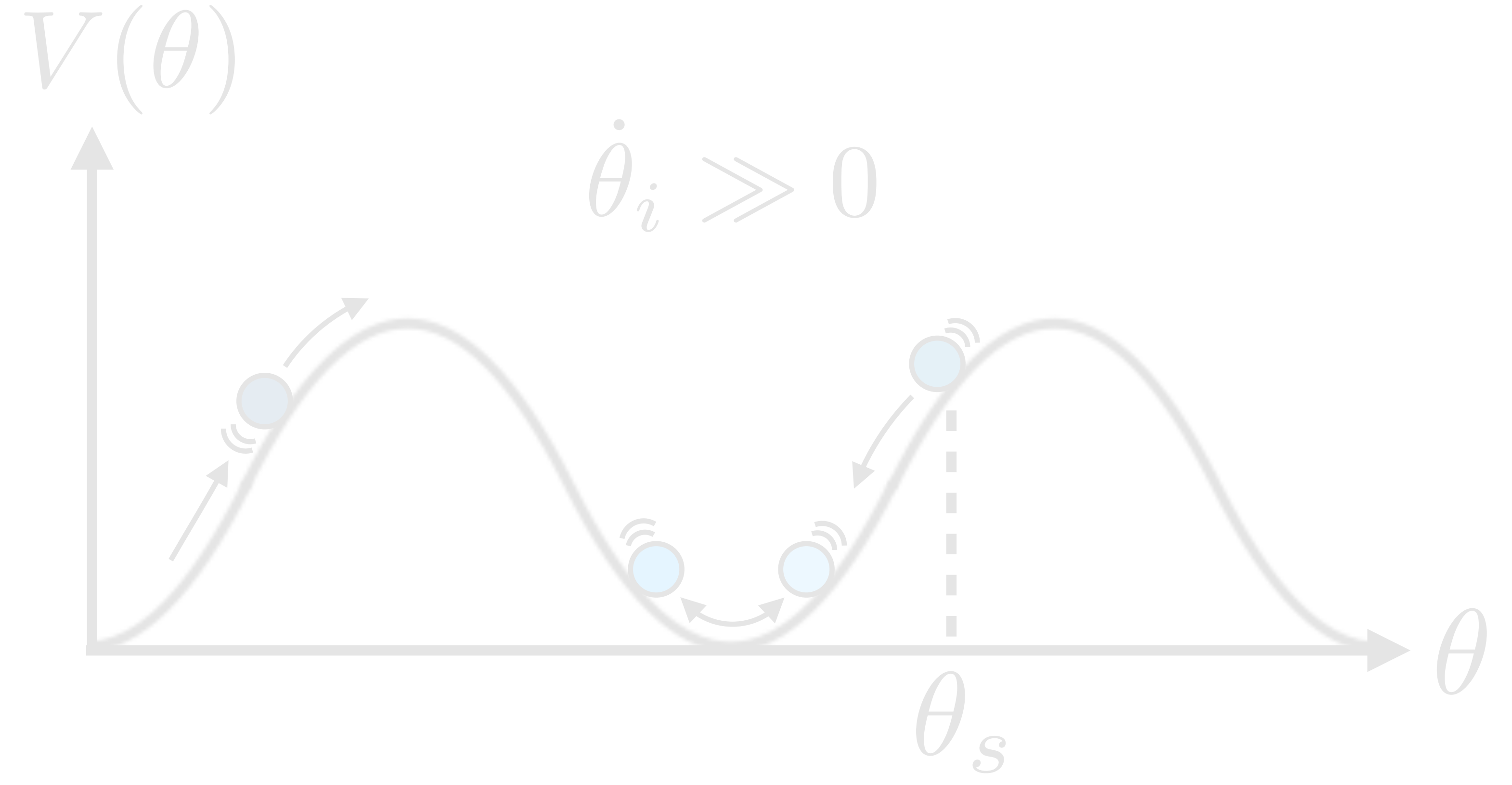
$$m_A\,(T_1) \cdot t_1 = 1$$

- Trapping at $T_s \sim T_1$
- Field stops at $\theta_s \lesssim \theta_i$

Large Misalignment Angle

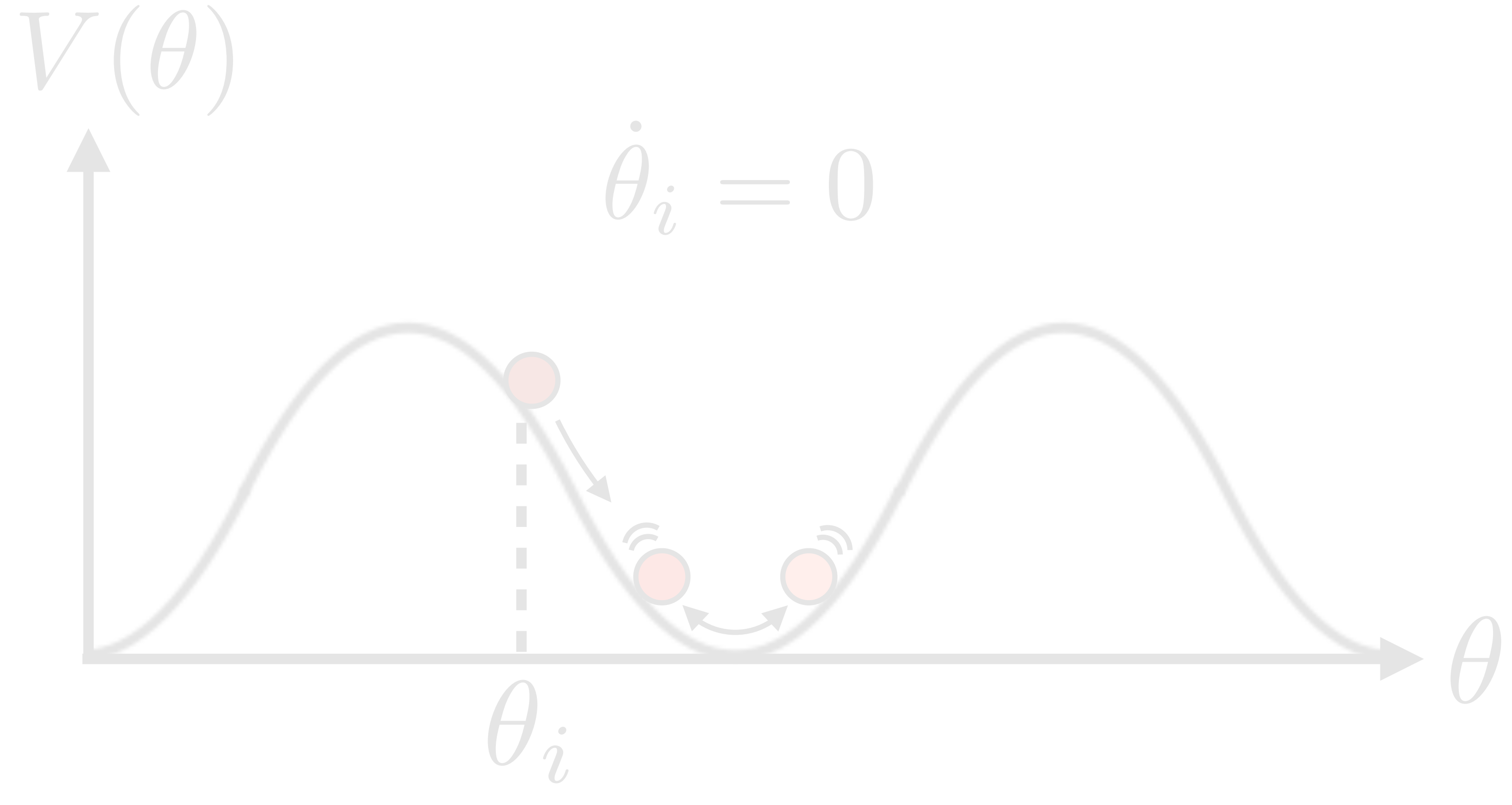


Large Kinetic Misalignment



Large Misalignment Angle

Standard



$$\ddot\theta + 3H\dot\theta + m_a(T)^2 \sin\theta = 0$$

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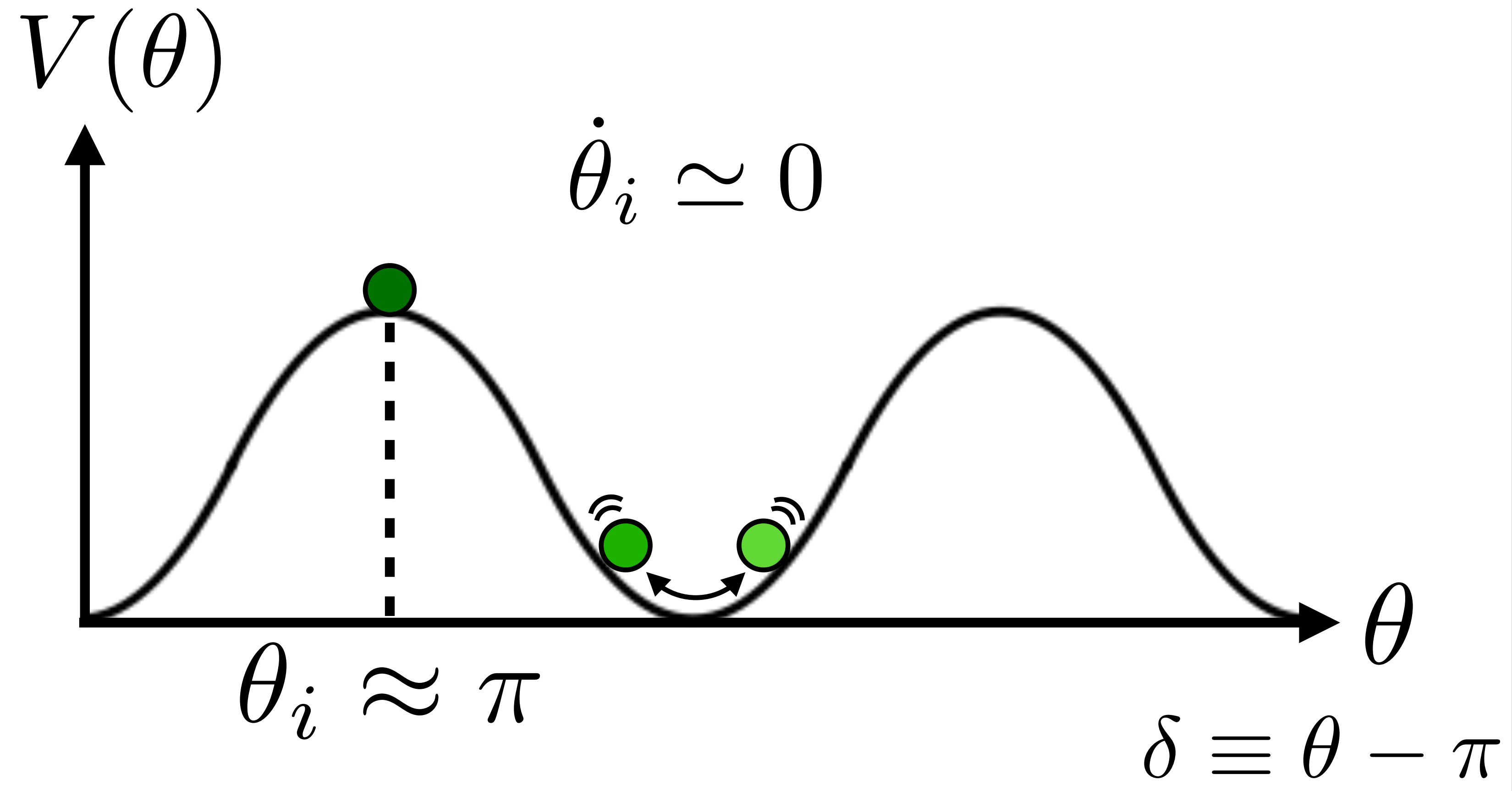
Large Misalignment Angle



$$\ddot\delta + 3H\dot\delta - m_A^2\delta = 0$$

- Exponential growth:

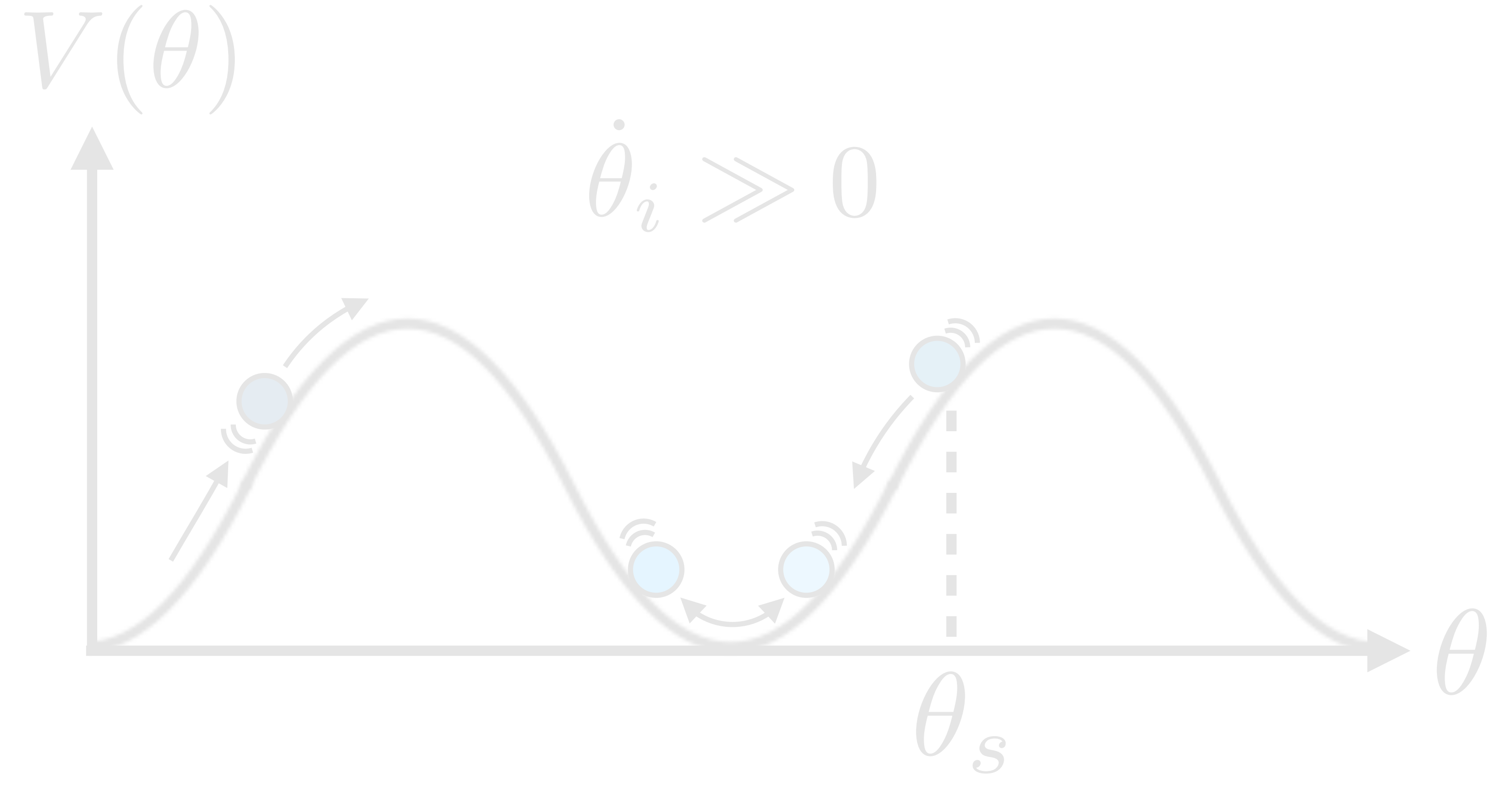
$$\delta(t) \simeq \delta_i \exp\left(\frac{t^6}{39+3t^3}\right)$$

- Characteristic timescale:

$$T_1/T_s \sim \left(6\log\delta^{-1}\right)^{1/6}$$

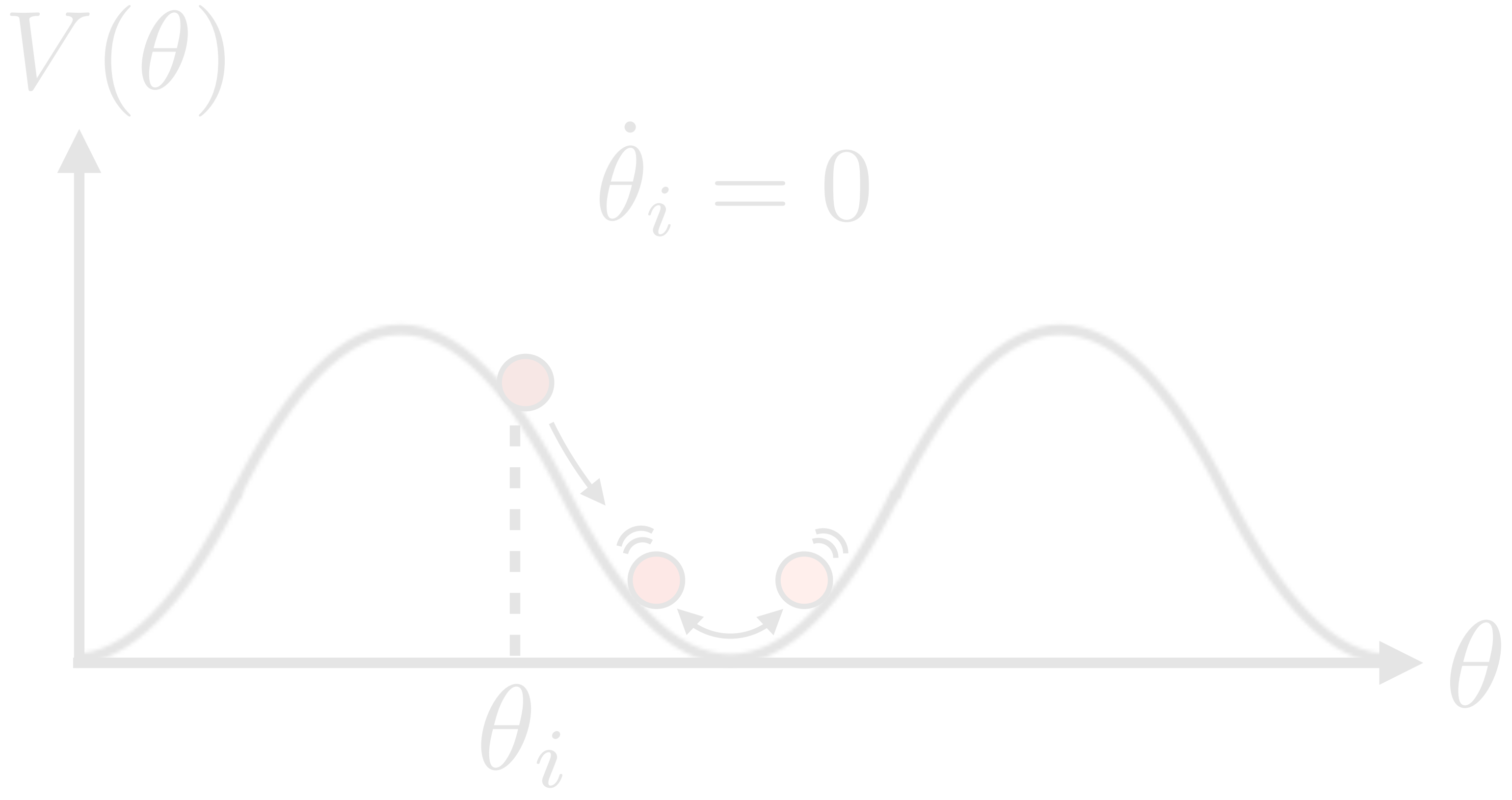
- Field stops at $\theta_s \sim 2$

Large Kinetic Misalignment



Large Kinetic Misalignment

Standard



$$\ddot{\theta} + 3H\dot{\theta} + m_a(T)^2 \sin\theta = 0$$

- Characteristic timescale:

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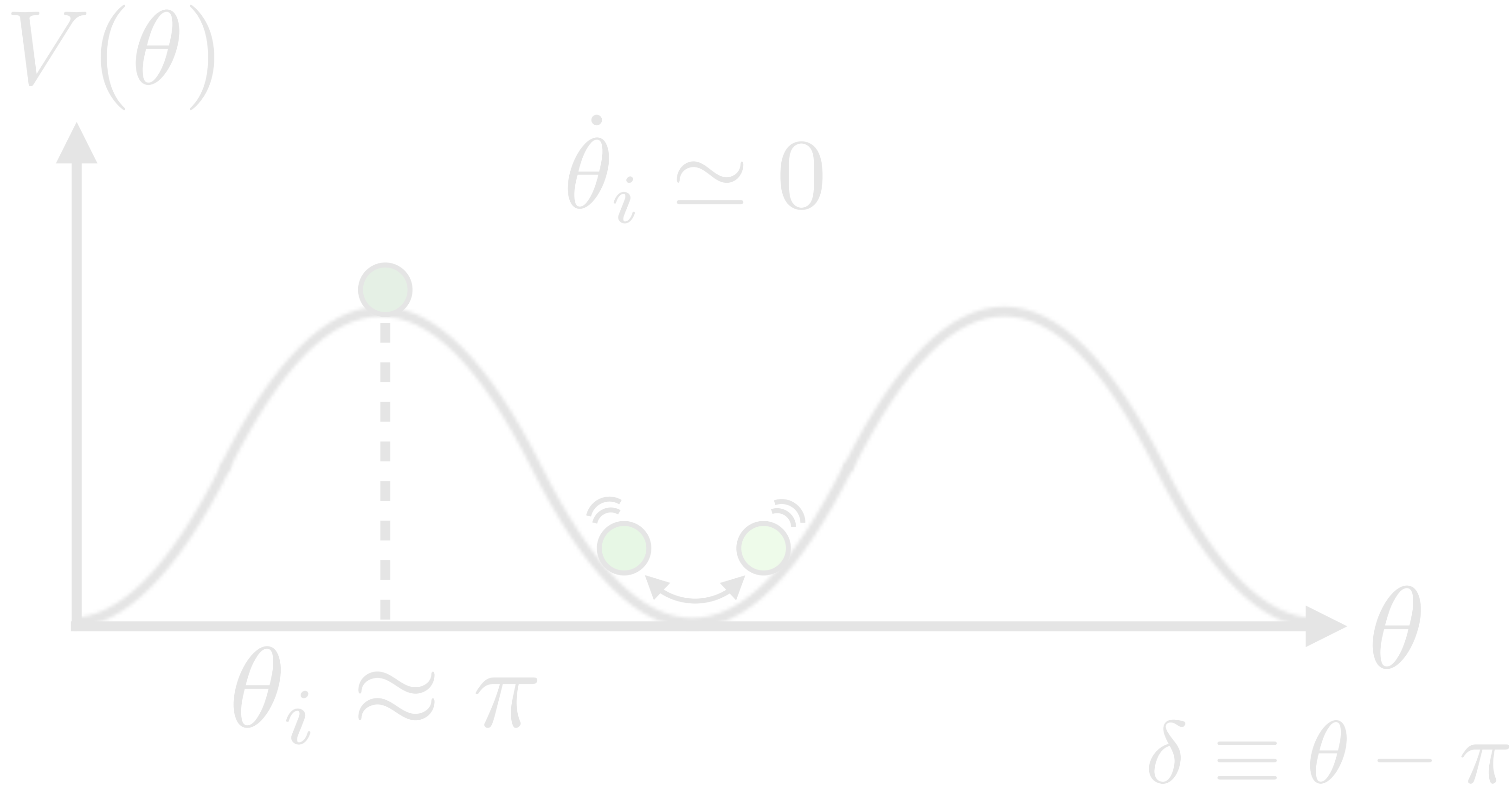
Large Misalignment Angle



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- Exponential growth:

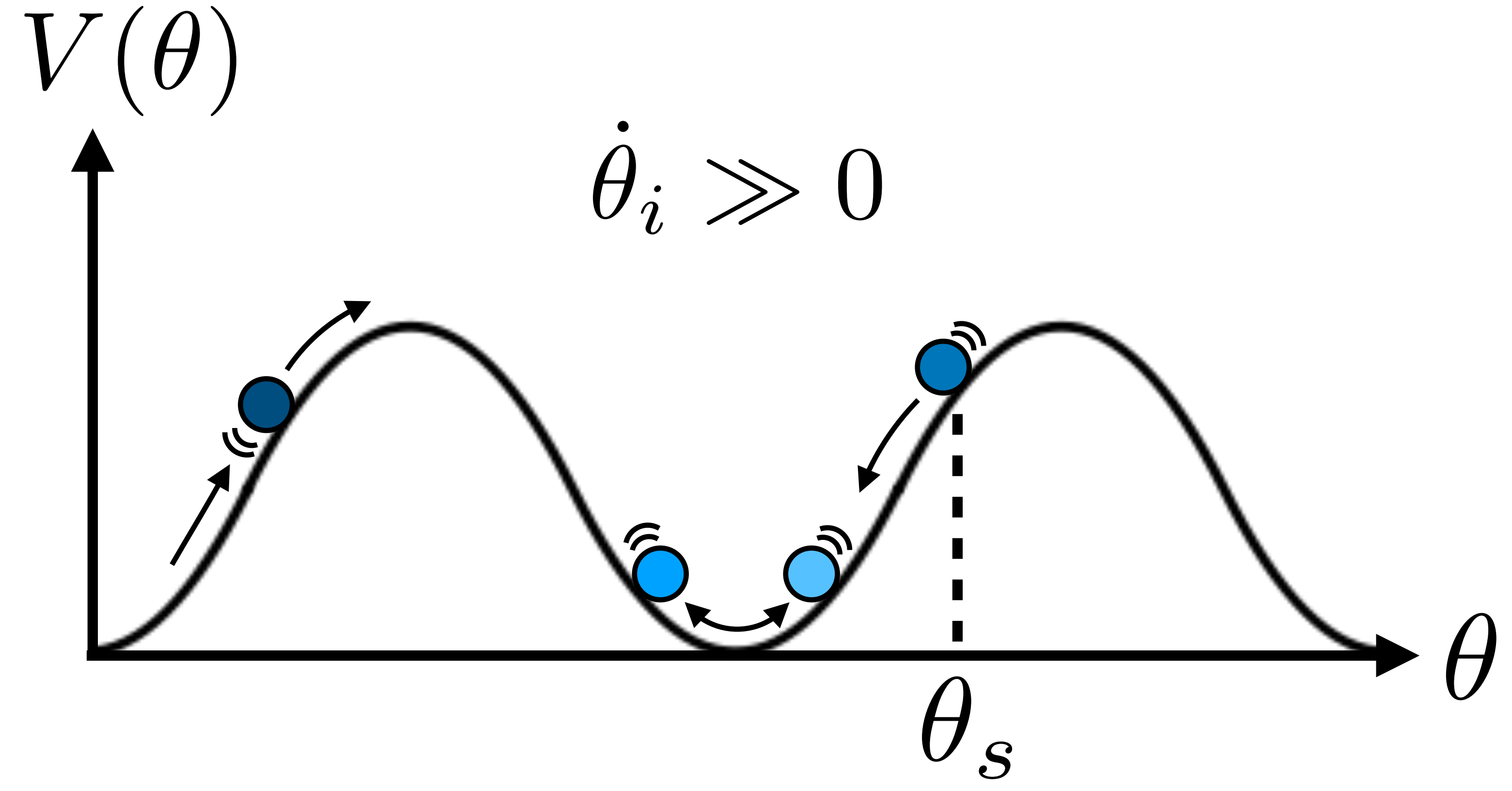
$$\delta(t) \simeq \delta_i \exp\left(\frac{t^6}{39 + 3t^3}\right)$$

- Characteristic timescale:

$$T_1/T_s \sim \left(6\log\left(\theta_i - \pi\right)^{-1}\right)^{1/6}$$

- Field stops at $\theta_s \sim 2$

Large Kinetic Misalignment



$$\ddot{\theta} + 3H\dot{\theta} + m_a(T)^2 \sin\theta = 0$$

- Characteristic timescale:

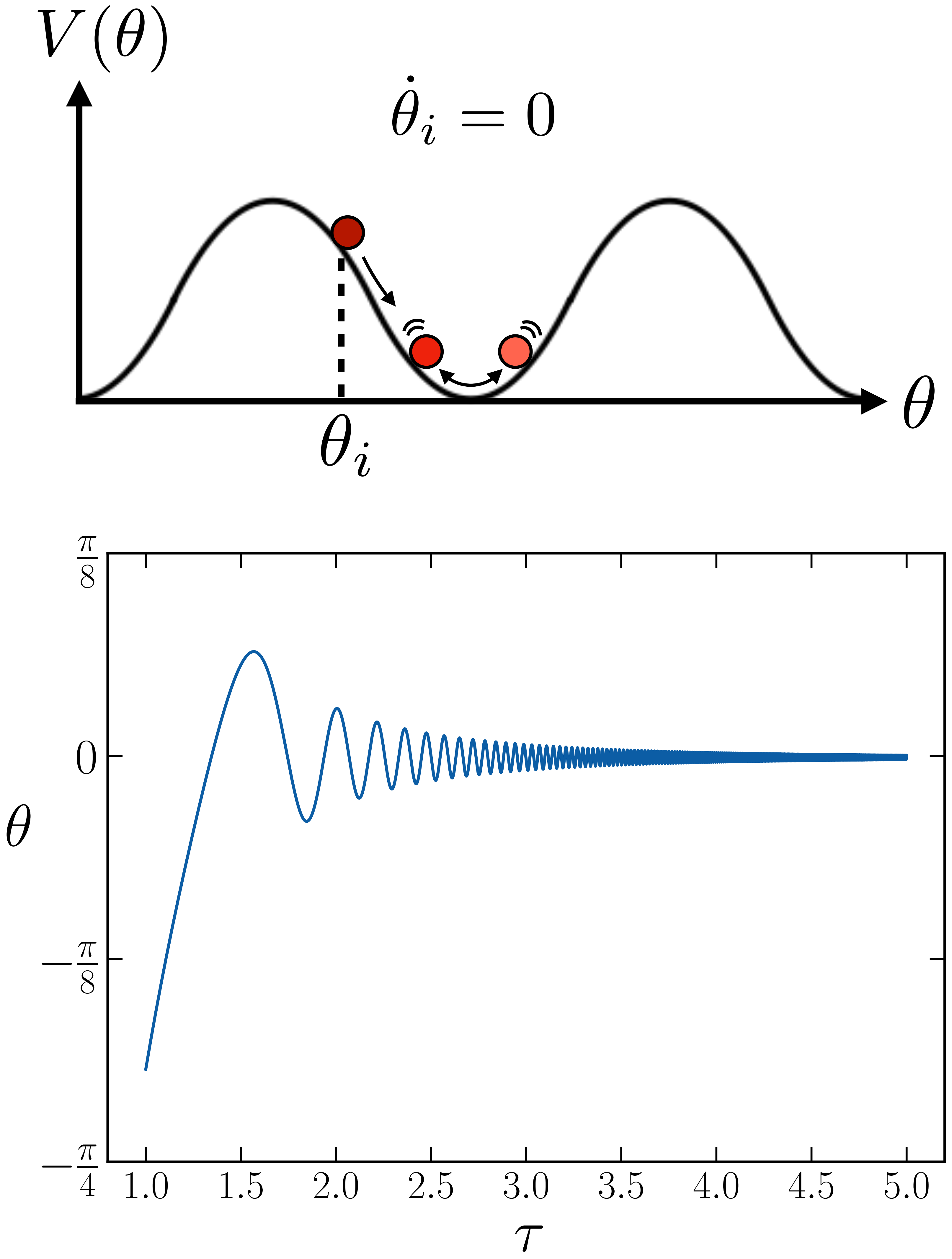
$$T_1/T_* \sim \left(\dot{\theta}_1/H_1\right)^{\frac{2}{n+6}}$$

- Field stops at $\theta_s \geq 1.35$
- Yield: $Y_\theta = \dfrac{\dot{\theta} f_a^2}{s}$

$$\Omega_a h^2 \simeq \Omega_{\rm DM} h^2 \left(\frac{10^9\ \mathrm{GeV}}{f_a}\right)\left(\frac{Y_\theta}{40}\right)$$

The Importance of Initial Conditions

Standard



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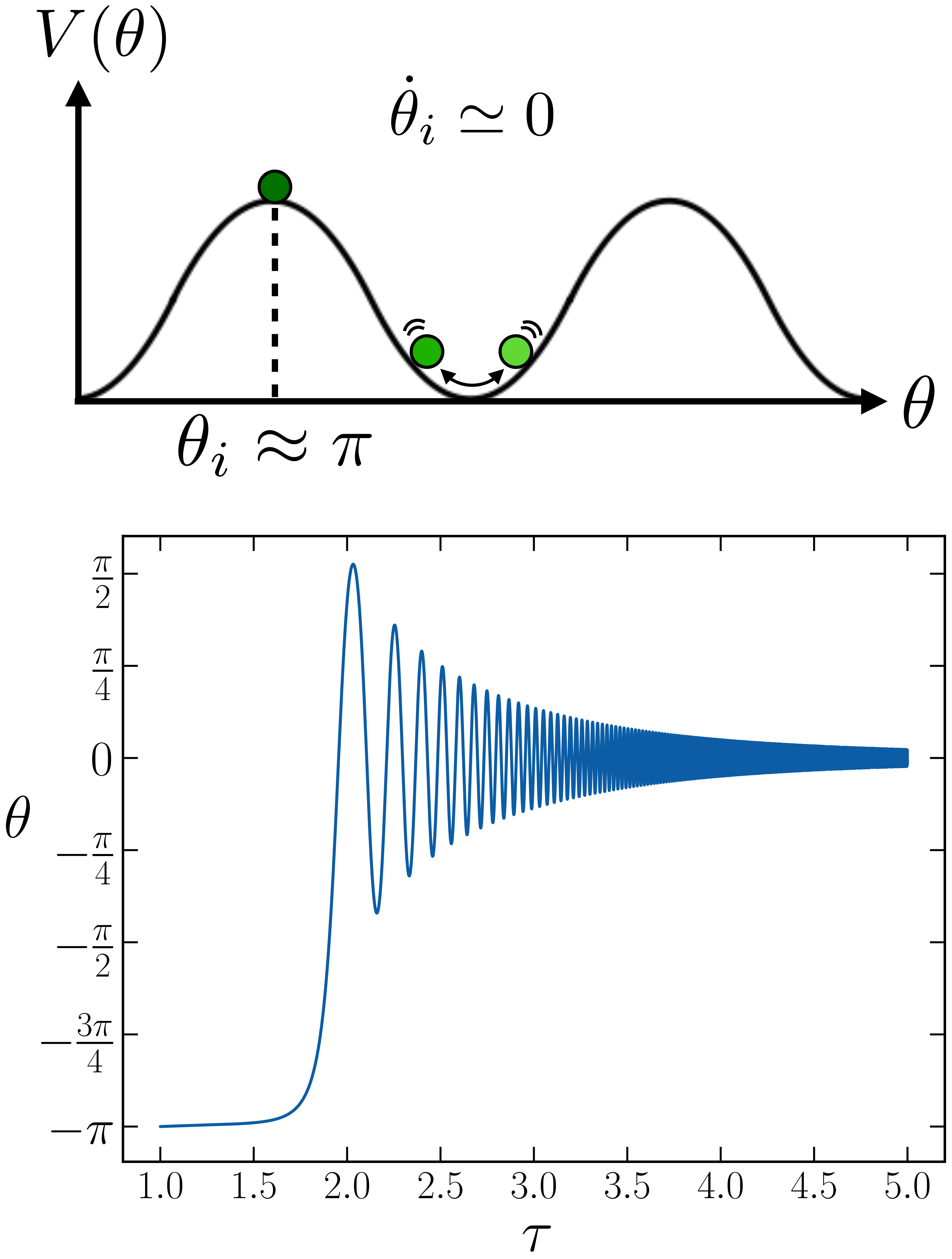

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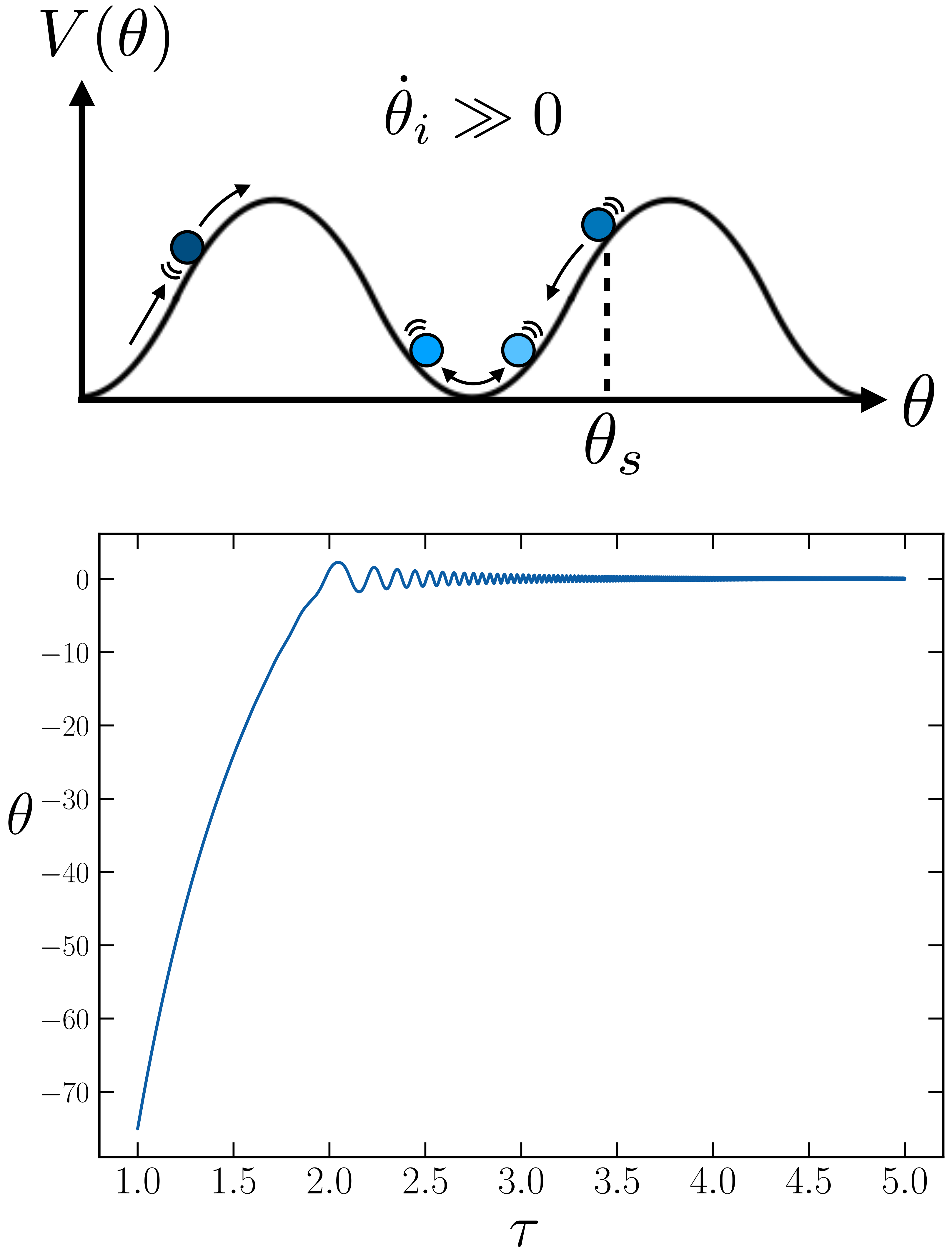


Arvanitaki+ [1909.11665], Co+ [1910.14152, …], Eröncel+ [2206.14259, 2207.10111], Fasiello+ [2507.01822], Chathirathas, MK+ [*to appear*]

Initial Conditions: Zero-Mode & Fluctuations

- Consider fluctuations around the homogeneous solution: $\theta = \theta_0 + \int dk^3 \theta_k$
- At first order the EoM for the modes reads $\ddot{\theta}_k + \left[\frac{k^2}{R^2} + m_a^2 \cos\theta_0\right]\theta_k = 0$
- General solution: $\theta_k(t) = \theta_{k+} e^{\mu_k t} + \theta_{k-} e^{-\mu_k t}$

Eröncel+ [2206.14259, 2207.10111]

- Amplification factor becomes time-integral

$$N_k(t) = \exp(\int^t dt' \mu(\varrho(t'), \kappa(t'))) \sim \exp(m_*/(2H_*)\mathcal{B}_k(t))$$

- **Fragmentation:** Initially small fluctuations can become $\mathcal{O}(1)$!

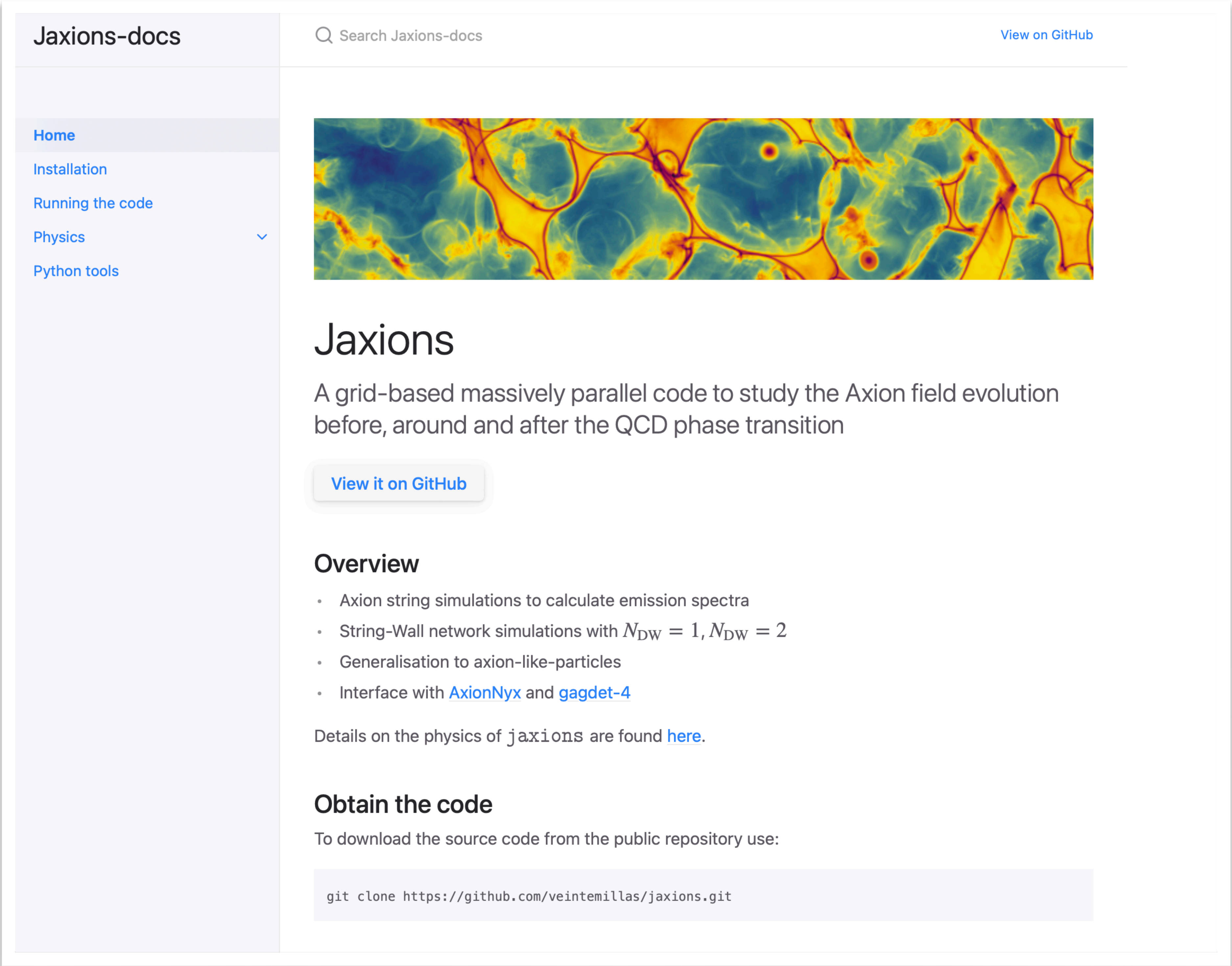
$A_s = 2 \times 10^{-9}, n_s = 0.965, k_\star = 0.05\ \mathrm{Mpc}^{-1}$

- In practice: Feed modes with PS $P_\theta(k) = |\theta_k|^2 = \frac{1}{V}\frac{1}{9}\left(\frac{2\pi^2}{k^3}\right) A_s \left(\frac{\dot{\theta}_0}{H}\right)^2 = A_s \left(\frac{k}{k_\star}\right)^{n_s - 1}$

Jaxions Code

- Highly parallelised `C++` code to simulate the evolution of the axion dark matter field in the early Universe

- Available on Github*



Jaxions

A grid-based massively parallel code to study the Axion field evolution before, around and after the QCD phase transition

Overview

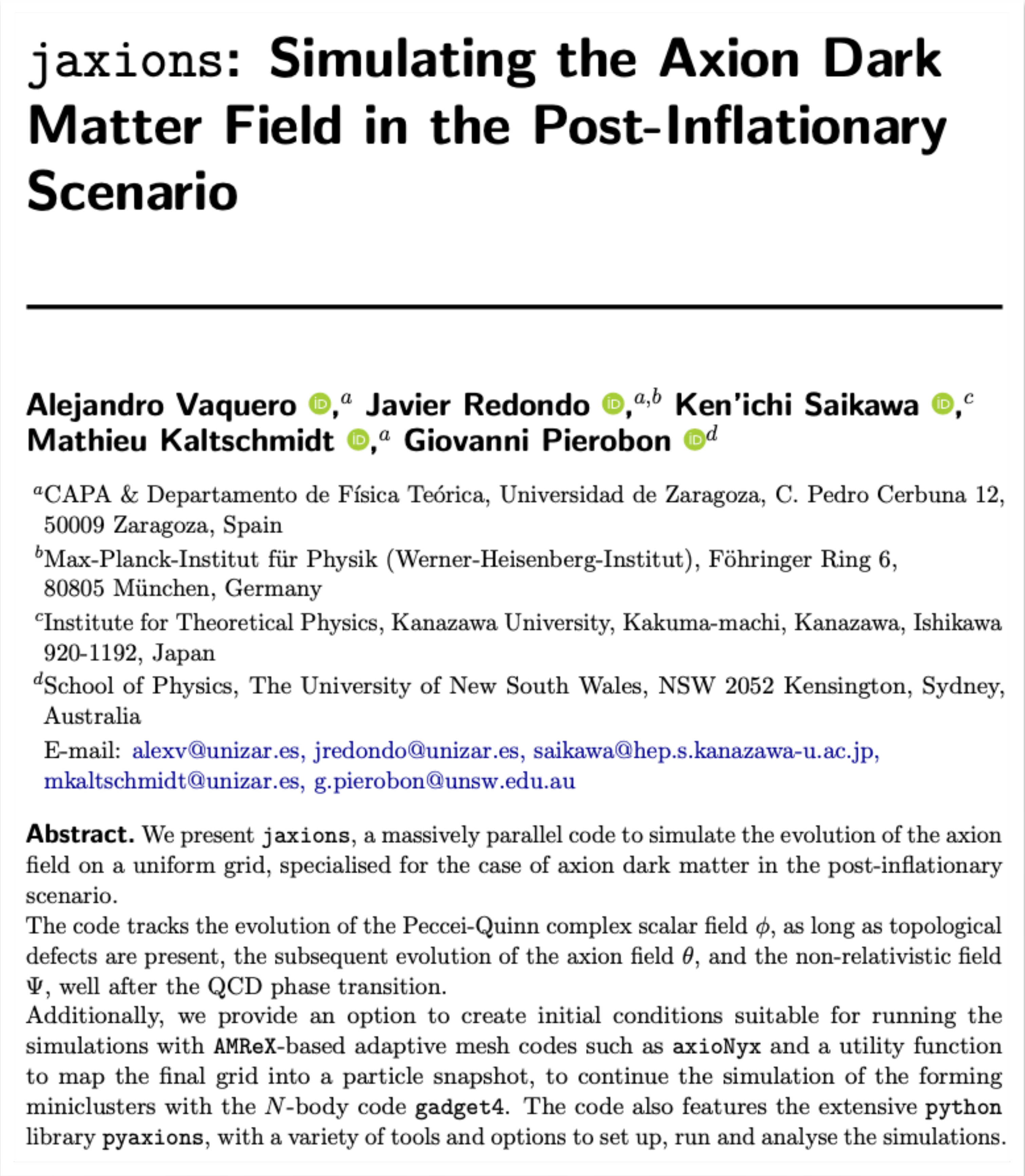
- Axion string simulations to calculate emission spectra
- String-Wall network simulations with $N_{\rm DW} = 1, N_{\rm DW} = 2$
- Generalisation to axion-like-particles
- Interface with AxionNyx and gadget-4

Details on the physics of `jaxions` are found here.

Obtain the code

To download the source code from the public repository use:

```
git clone https://github.com/veintemillas/jaxions.git
```



`jaxions`: Simulating the Axion Dark Matter Field in the Post-Inflationary Scenario

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Mathieu Kaltschmidt,[a] **Giovanni Pierobon**[d]

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[b]Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany
[c]Institute for Theoretical Physics, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan
[d]School of Physics, The University of New South Wales, NSW 2052 Kensington, Sydney, Australia

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Abstract. We present `jaxions`, a massively parallel code to simulate the evolution of the axion field on a uniform grid, specialised for the case of axion dark matter in the post-inflationary scenario.

The code tracks the evolution of the Peccei-Quinn complex scalar field ϕ, as long as topological defects are present, the subsequent evolution of the axion field θ, and the non-relativistic field Ψ, well after the QCD phase transition.

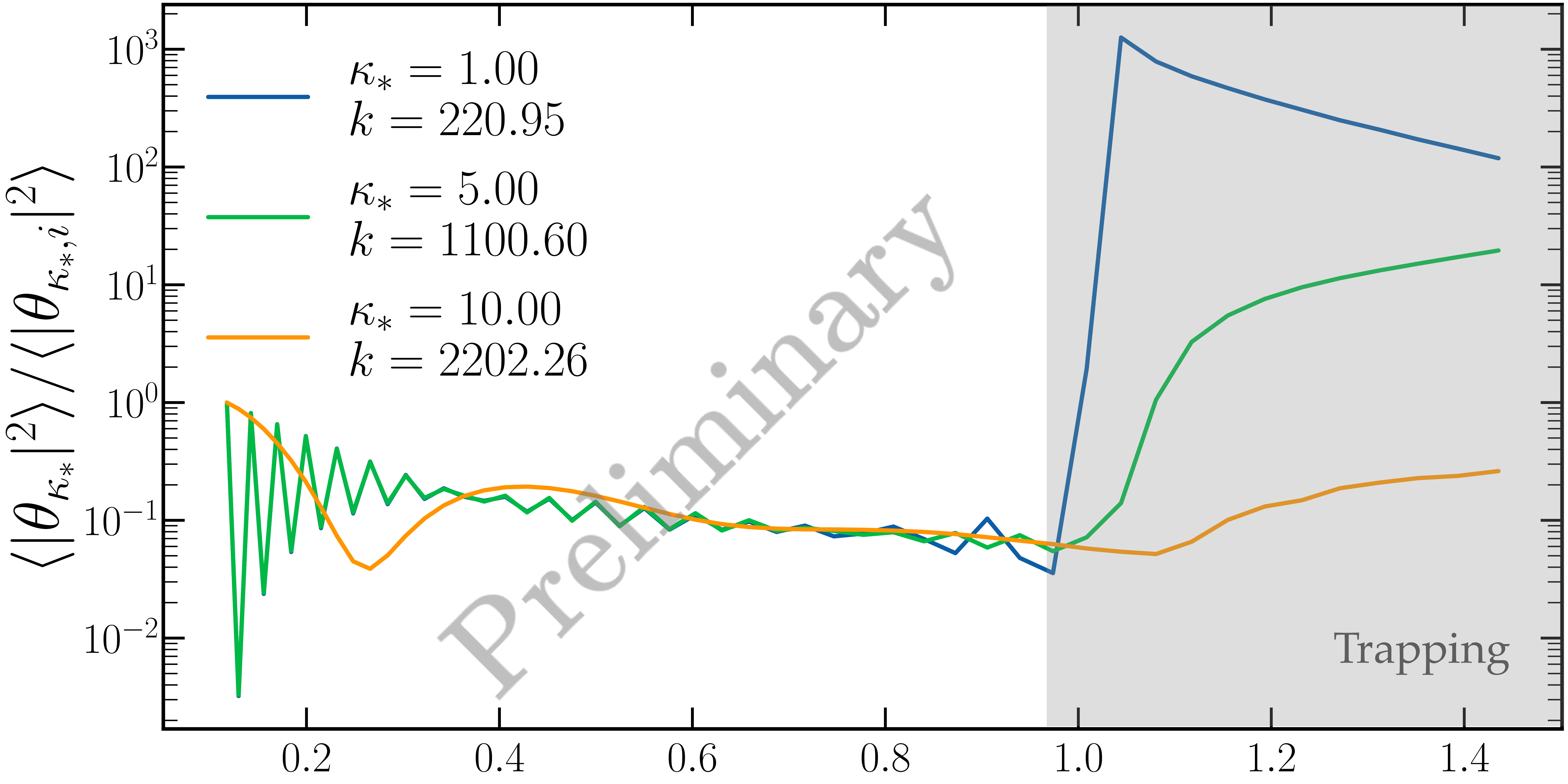
Additionally, we provide an option to create initial conditions suitable for running the simulations with `AMReX`-based adaptive mesh codes such as `axioNyx` and a utility function to map the final grid into a particle snapshot, to continue the simulation of the forming miniclusters with the N-body code `gadget4`. The code also features the extensive `python` library `pyaxions`, with a variety of tools and options to set up, run and analyse the simulations.

* https://github.com/veintemillas/jaxions

Non-Linear Mode Evolution

- Strongest amplification for modes around $\kappa_* \approx 1$

$\kappa_* = k/(R_* m_{a,*})$



$f_a = 1 \cdot 10^9$ GeV

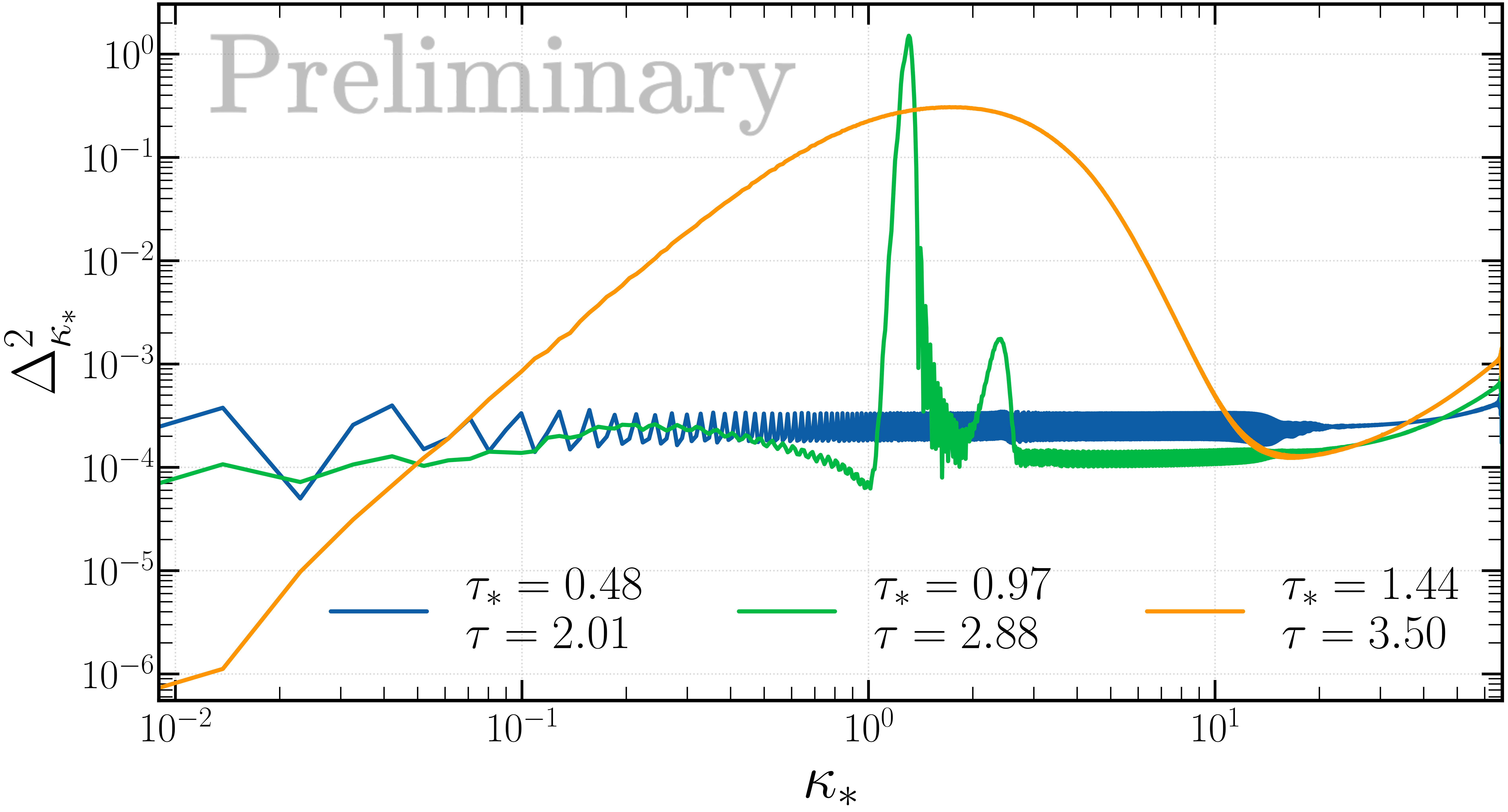
$\Theta_i \sim -2.9 \cdot 10^3$
$\sim -3.7 \cdot 10^{-3}$

$\dot{\Theta}_i / H_i \sim 4.3 \cdot 10^3$

* Note, that here Θ_1 refers to initial value of the zero mode.

Dimensionless Axion Power Spectrum

- Non-linear amplification of modes clearly visible around trapping!



Correct Dark Matter Abundance

- For every given f_a there is a degeneracy in the initial field value and velocity that produce the correct dark matter abundance.

- By solving the homogeneous EoM (in one spatial dim.) we can track the evolution of Θ_1 and $\dot{\Theta}_1/H_1$ until after the onset of oscillations, when the number density is given by

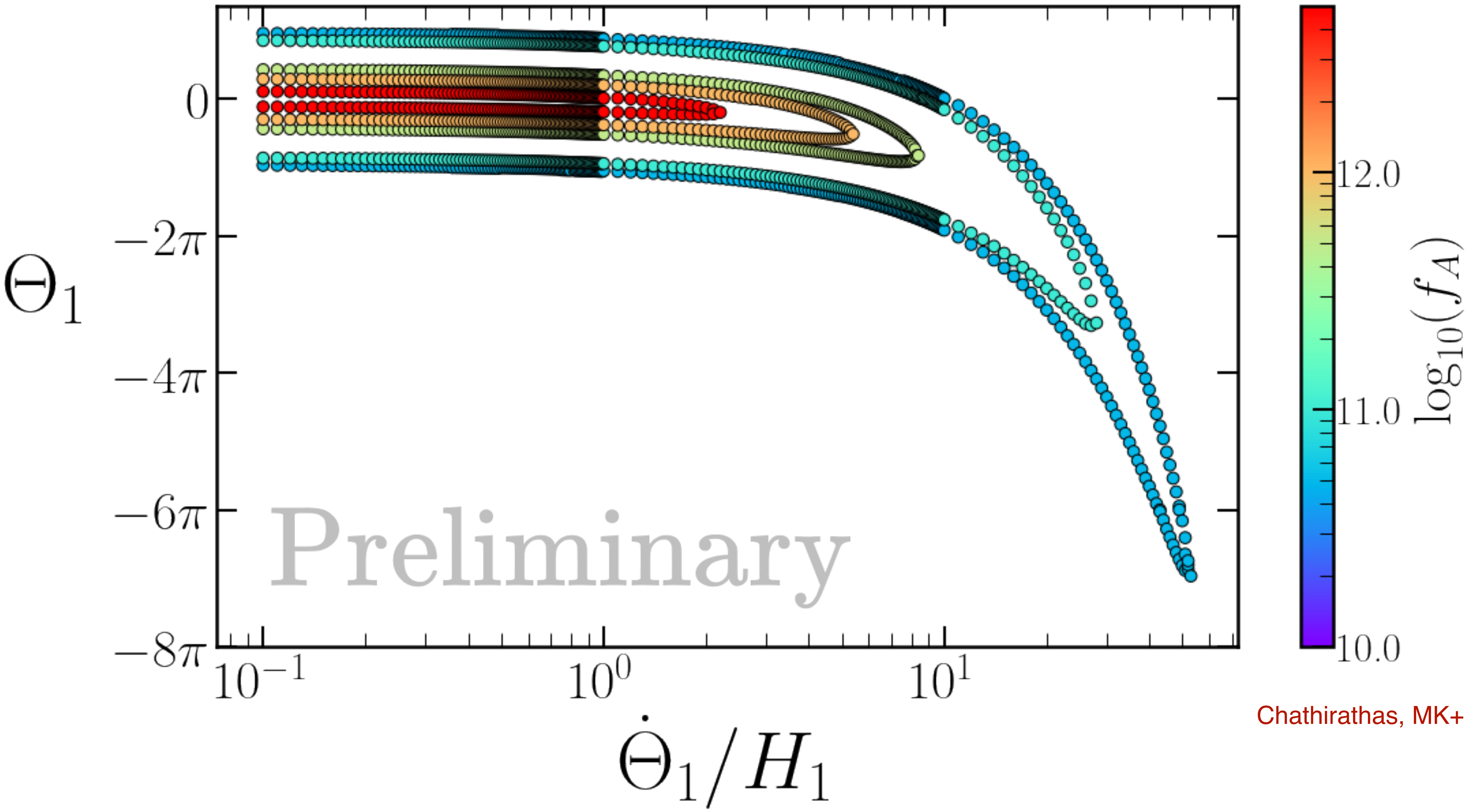
$$n_a(\tau) = \frac{H_1^2 f_A}{m_A^2(\tau)}\left[\frac{1}{2}\left(\frac{\dot{\Theta}}{H_1}\right)^2 + m_A^2(\tau) f_A^2 \left(1 - \cos\Theta\right)\right]$$

- It is straightforward to solve for $\Omega_a h^2 = \frac{m_a n_a^{\text{com}}}{\rho_{\text{crit}}}$ $\qquad n_a^{\text{com}} = n_a(\tau) R^3(\tau)$

* Note, that here Θ_1 refers to initial value of the zero mode.

Parameter Matches — Correct DM Abundance

- This reveals two regimes that correspond to the extreme cases discussed before



Chathirathas, MK+ [*to appear*]

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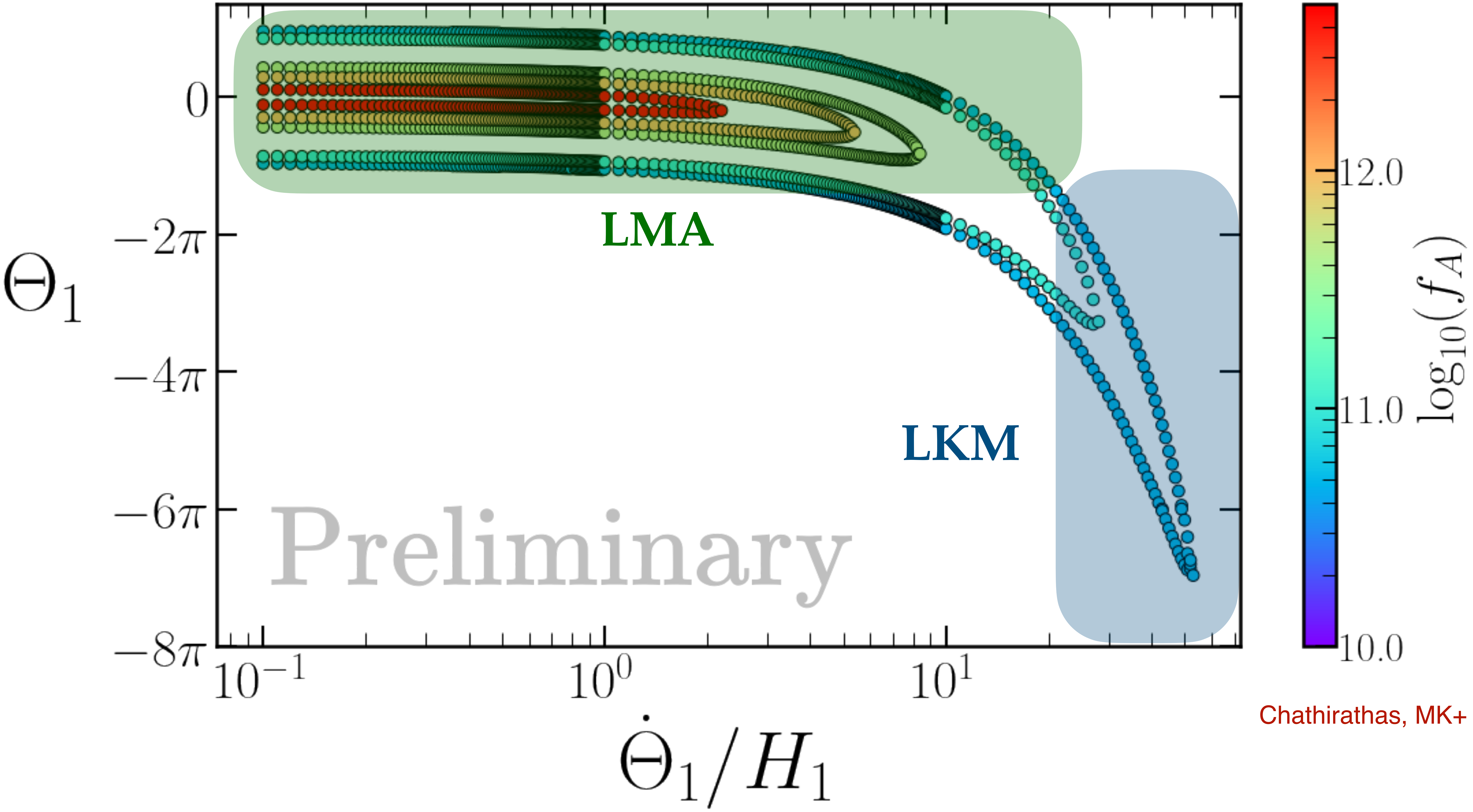
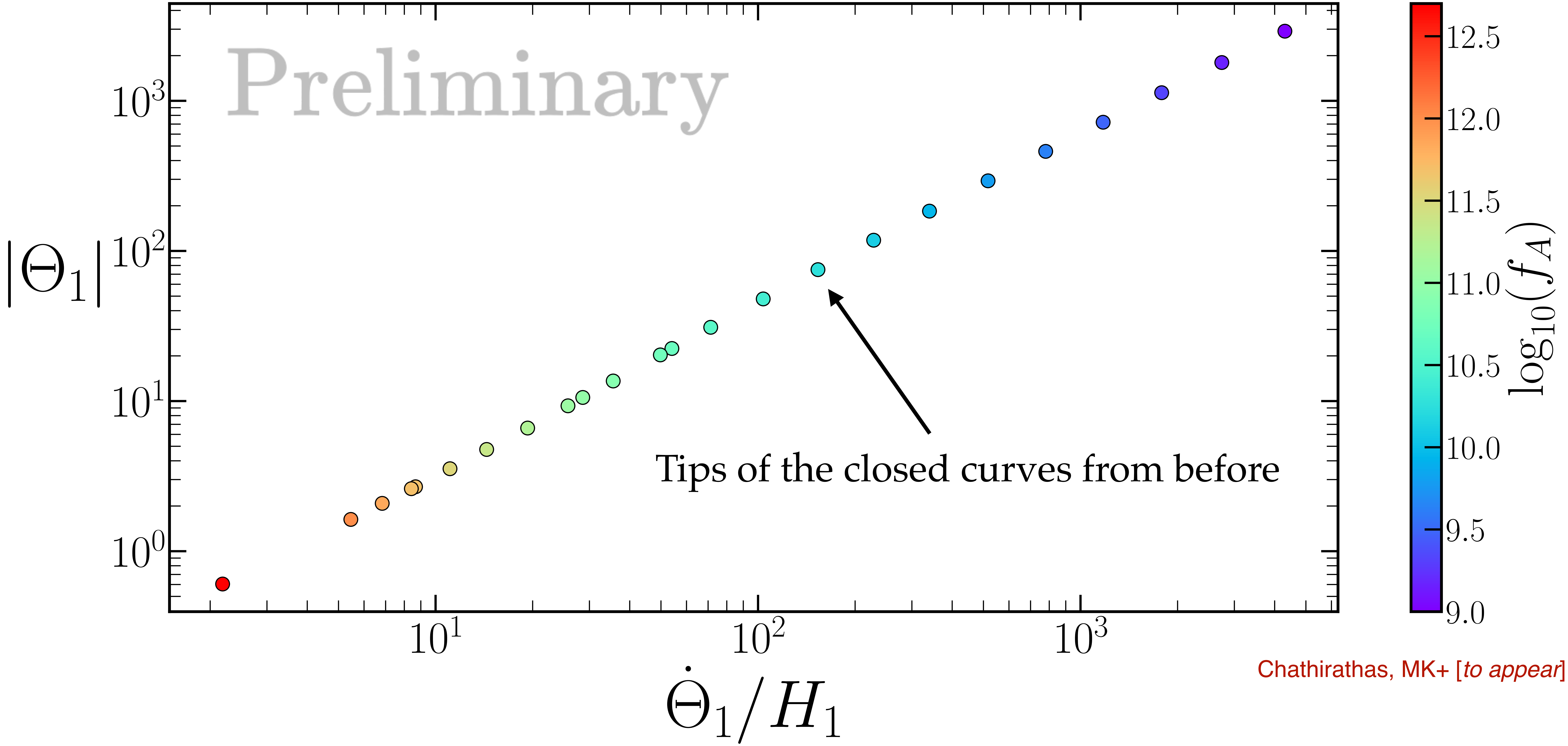


Chathirathas, MK+ [*to appear*]

Maximum Velocity

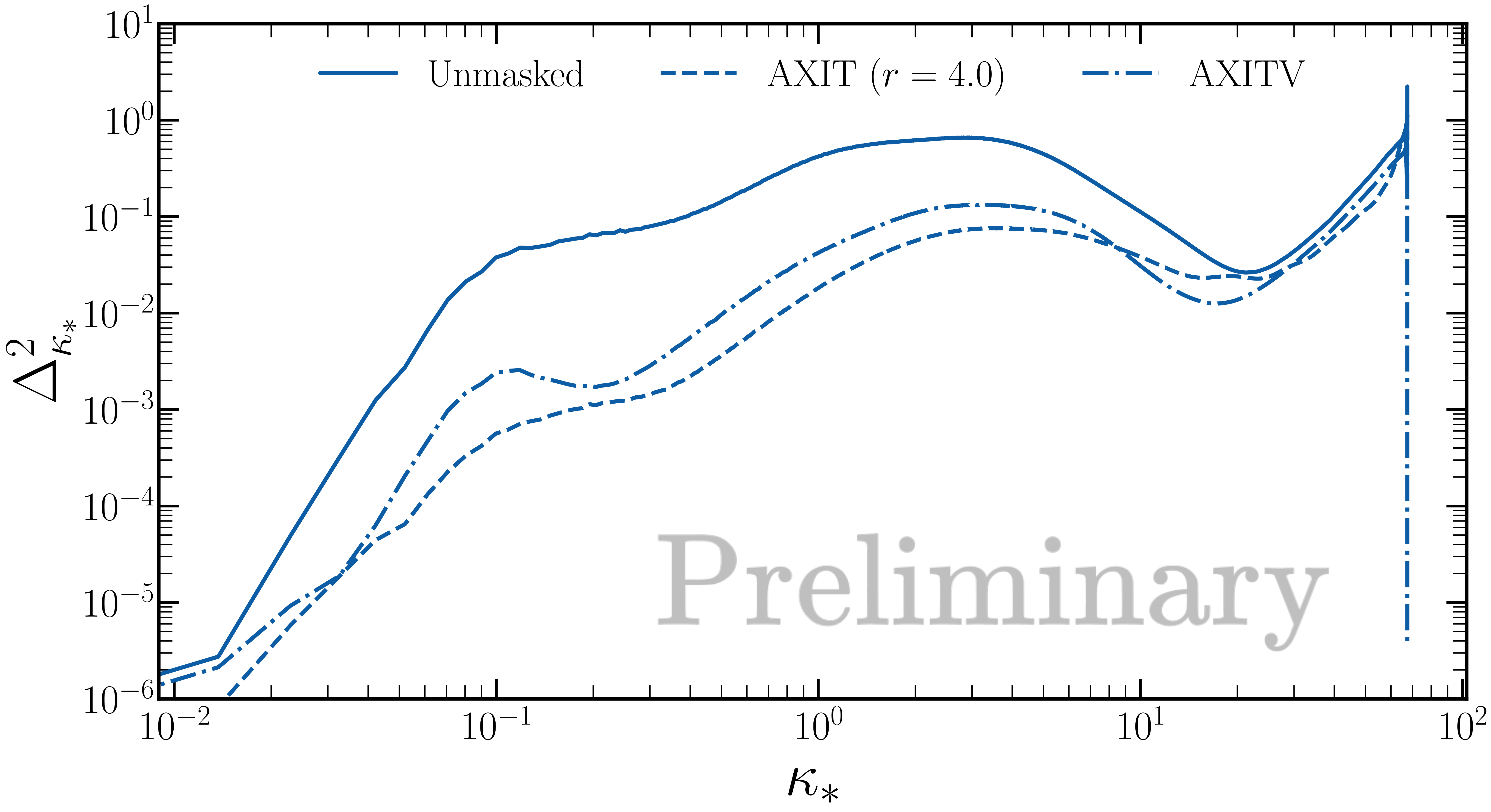
- There is a maximum velocity for a given f_a above which there is always DM overabundance



Chathirathas, MK+ [*to appear*]

Axitons and Domain Walls

- Besides the non-linear amplification of modes, some simulations suffer from the formation of **axitons** and **domain walls**. We can **mask** them, to produce cleaner power spectra.



Conclusions Part I

- In both (LMA and LKM) scenarios small density fluctuations can grow exponentially
- For the KM case, the adiabatic fluctuations are proportional to the initial velocity
- **Important:** Amplification at trapping also depends on initial angle!
- Small f_a require **large velocities**, leading to **large amplifications**, which can result in the formation of **miniclusters**.
- This happens at **smaller scales** than for the standard misalignment case.
- It might be more difficult to distinguish pre- vs. post-inflationary scenarios
- Miniclusters in the pre-inflationary scenario can have similar phenomenology, but always at **relatively large masses**.

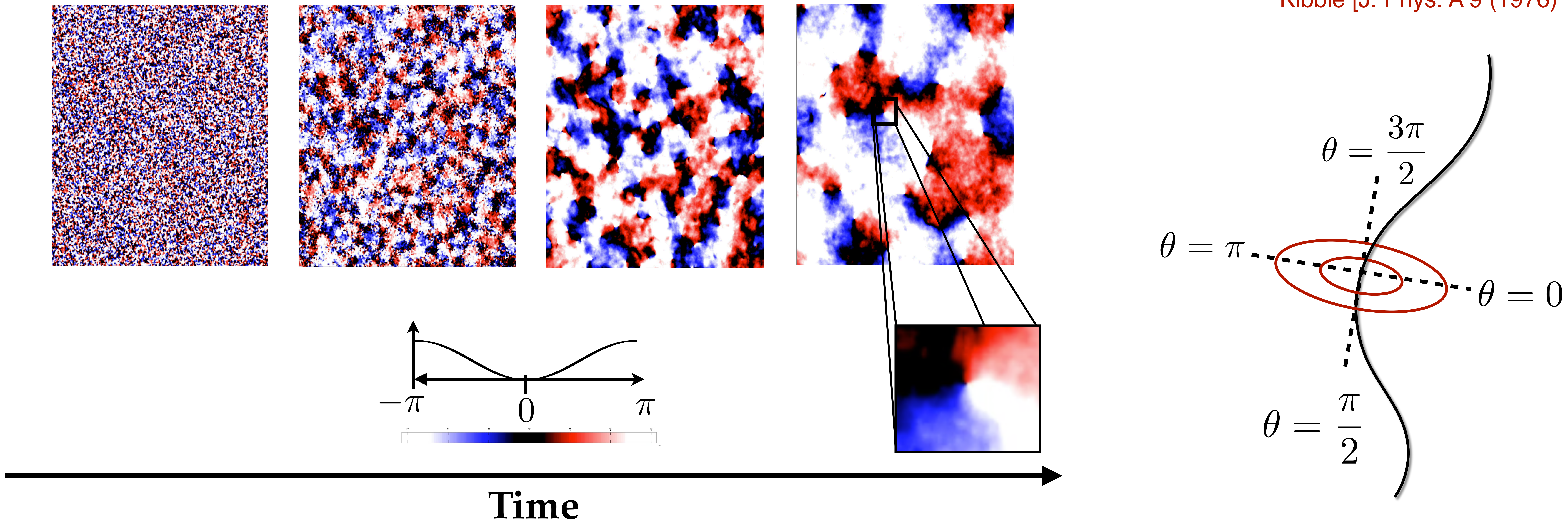
Post-Inflation!

Part II:
Global Axion Strings

String Formation

- **Kibble mechanism:** Cosmic strings form in models where a **continuous** symmetry is broken.

Kibble [J. Phys. A 9 (1976) 1387–1398]



- Phase jump can only be resolved continuously if the **complex scalar field** rises to its max. at $\phi = 0$
- Leads to a **non-zero energy density** at the core, **string tension:** $\mu \sim \mu_0 + \sim \log(R/\delta)$
- Far away from the string, the field is $\phi \sim \eta e^{i\theta}$

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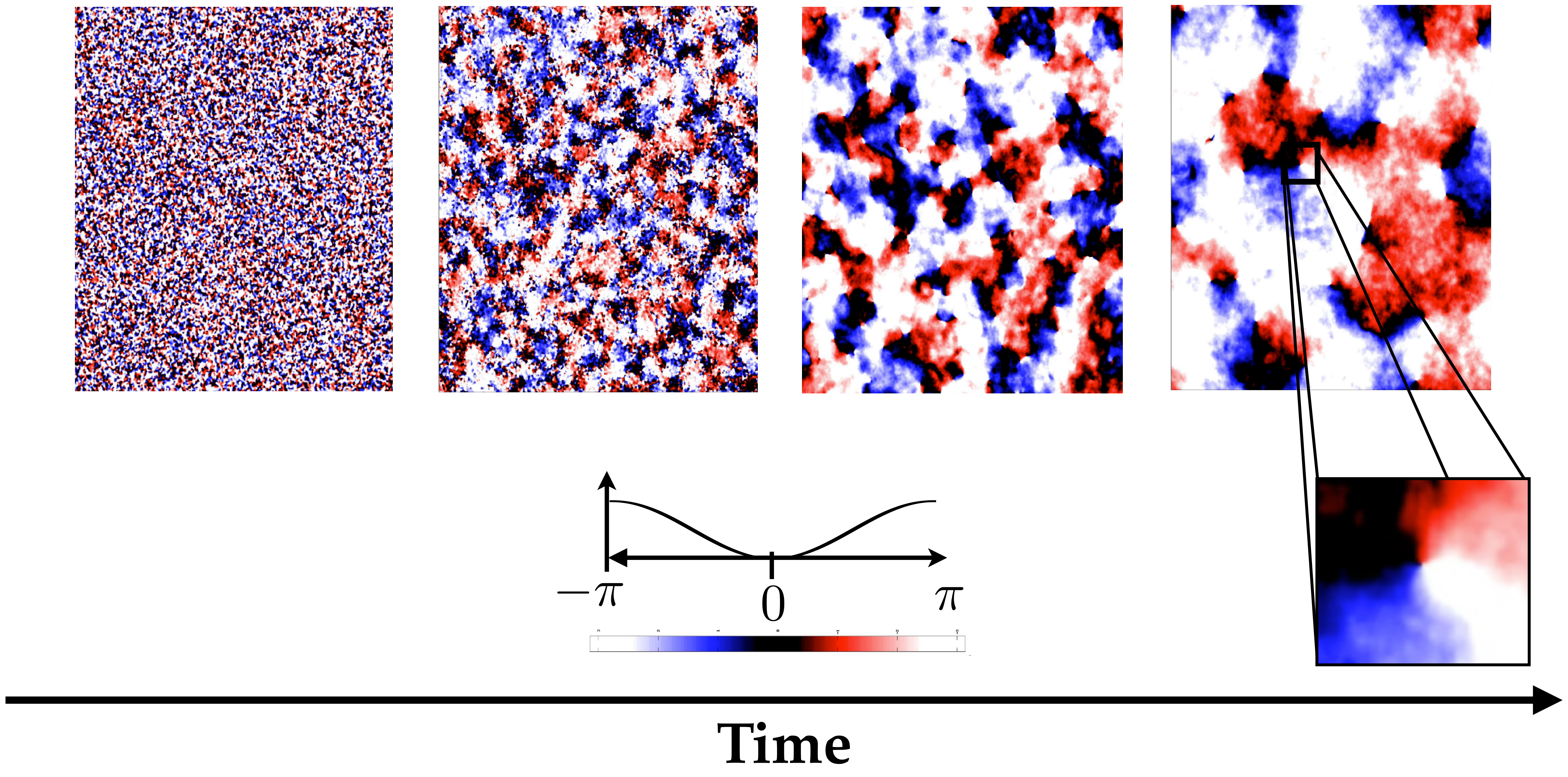

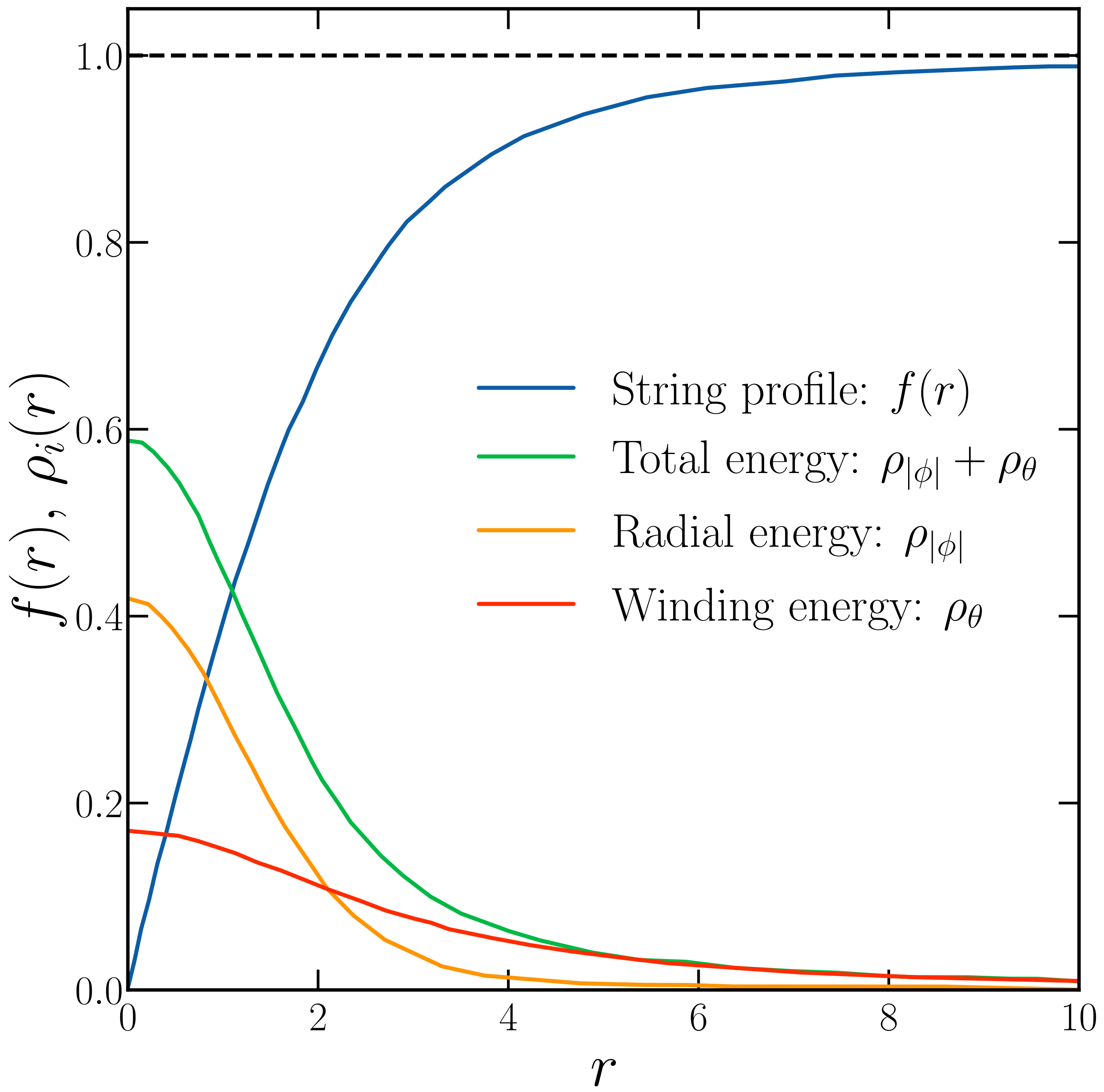

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How to simulate Axion Strings?

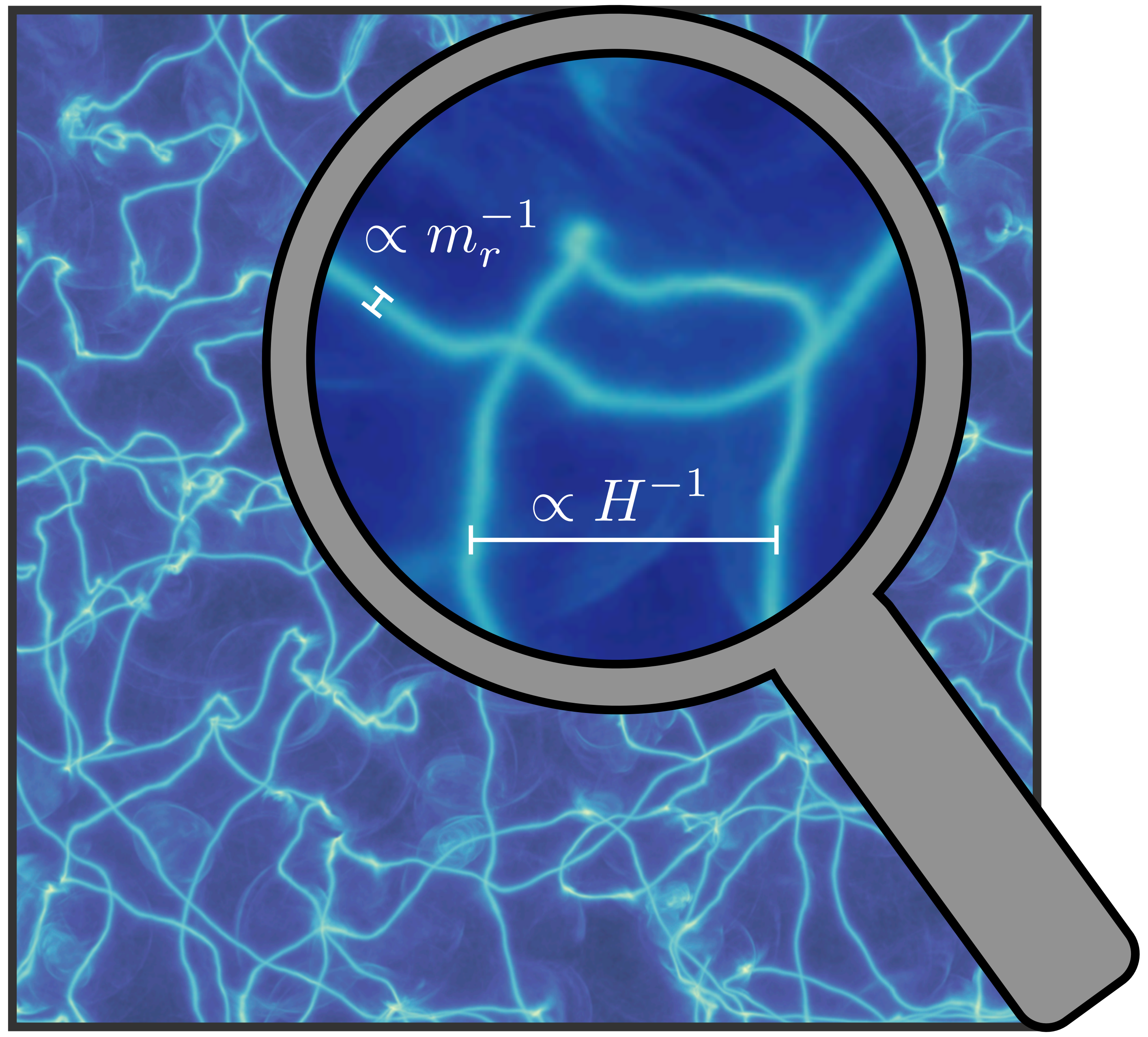
- Solve the classical EoM for a complex scalar field in comoving coordinates, discretised on a static lattice:

$$\partial_\tau^2 \phi - \nabla^2 \phi + \lambda \phi \left(|\phi|^2 - \tau^2 \right) = 0$$

- **Tricky:** Simulations require proper resolution of two very different length scales
  - String core radius

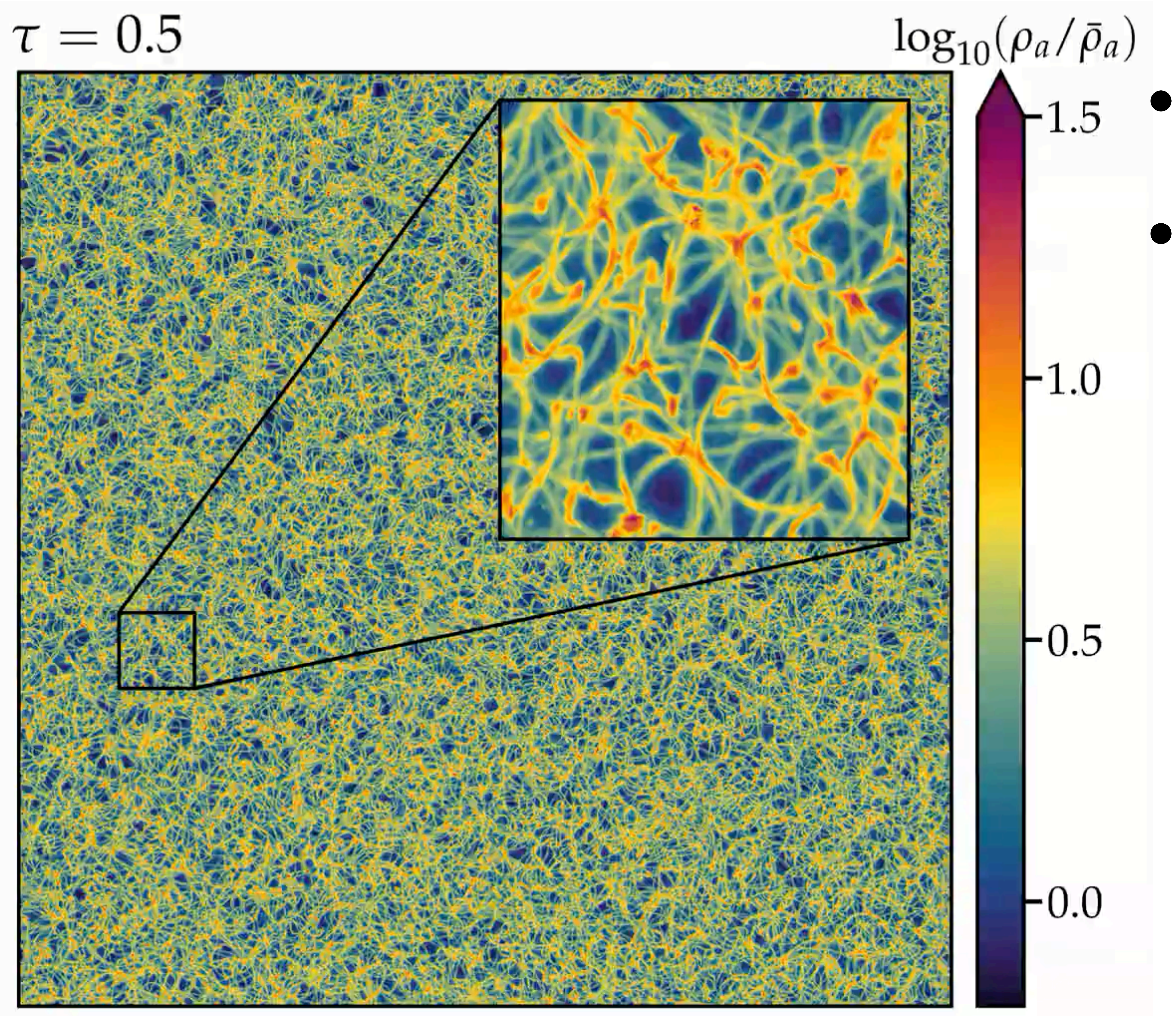
    $\propto \dfrac{1}{m_r} \propto \dfrac{1}{f_a}$, where $m_r =$ radial mass
  - String separation given by Hubble radius

    $\propto \dfrac{1}{H}$
  - Realistic value: $\dfrac{f_a}{H_{\mathrm{QCD}}} \approx 10^{30} \implies \log\left(\dfrac{m_r}{H}\right) \approx 70$



Courtesy of K. Saikawa

Cosmological Evolution in the Post-Inflationary Scenario



O’Hare+ [2110.11014]

- Network evolves to “**scaling”** solution
- Scaling maintained by radiating **relativistic, massless** axions

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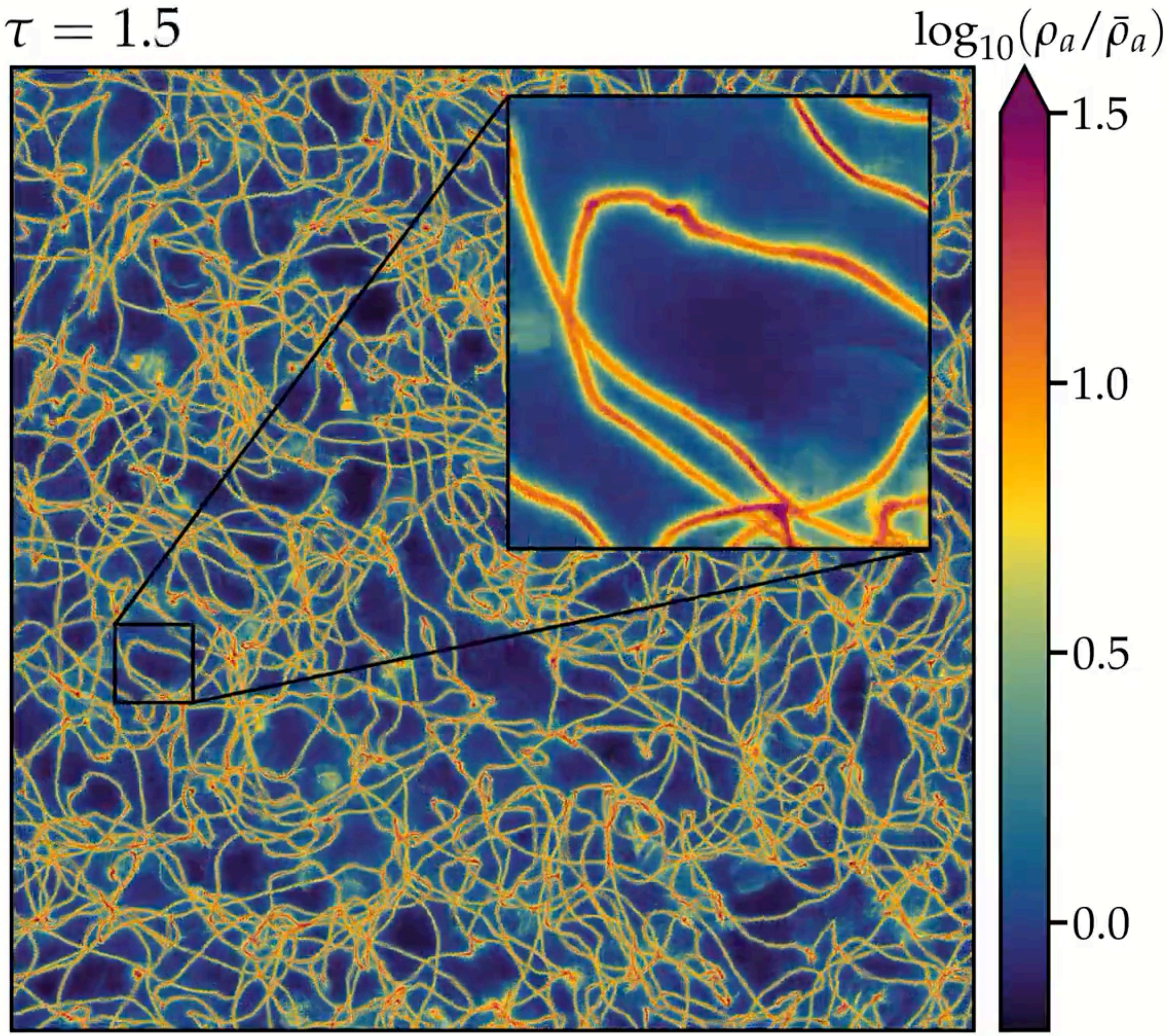

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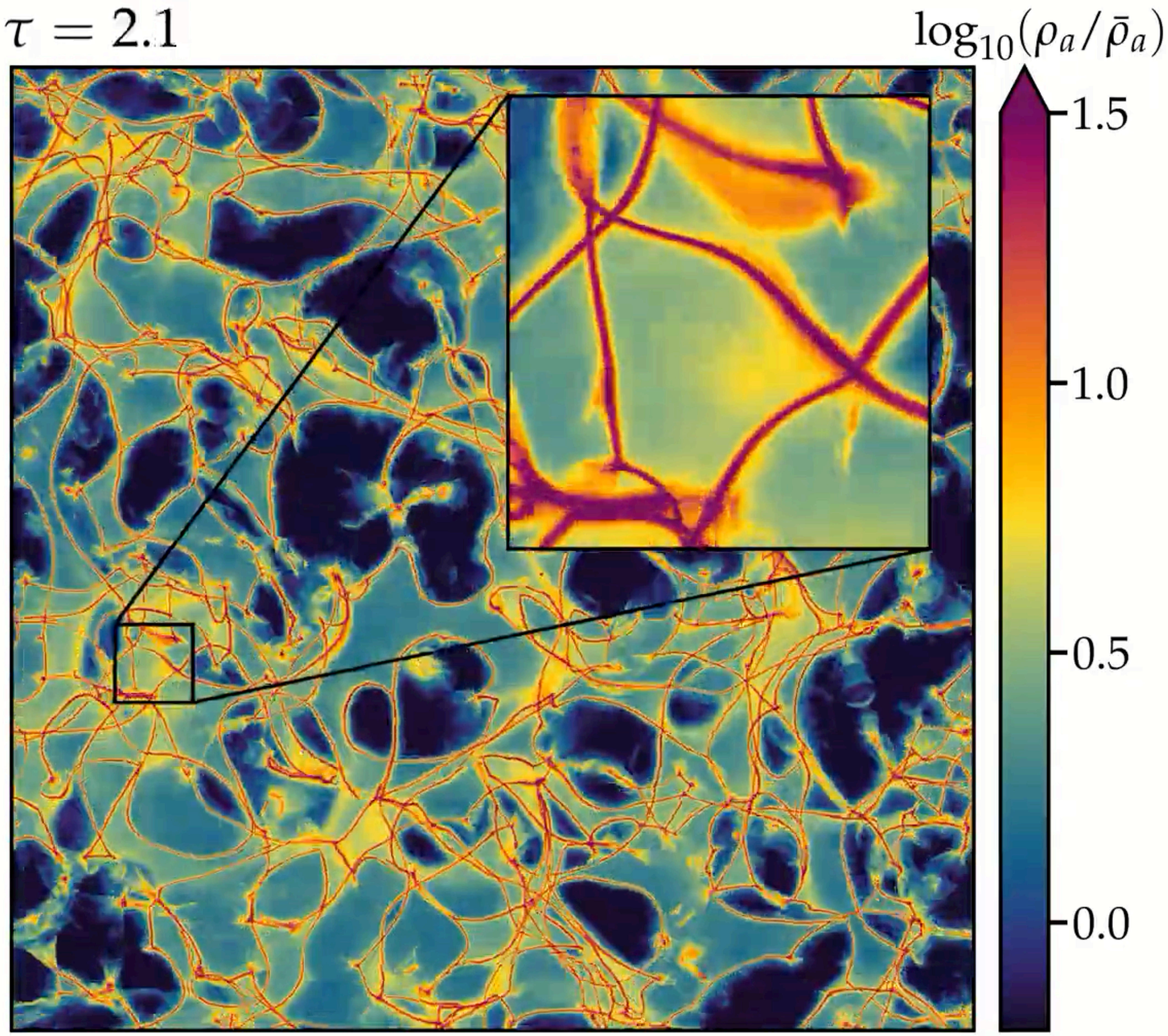
- QCD crossover at $T \sim$ GeV
- **Domain Walls*** form and network collapses
- Rapidly increasing mass renders axions **nonrelativistic**

*In general more complex dynamics if $N_{\rm DW} > 1$ (Axion Quality!)

$$V(\theta) \sim \cos(\theta) \rightarrow \cos(N_{\rm DW}\theta)$$

Cosmological Evolution in the Post-Inflationary Scenario



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Vaquero+ [1809.09241]
Pierobon+ [2307.09941]
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…

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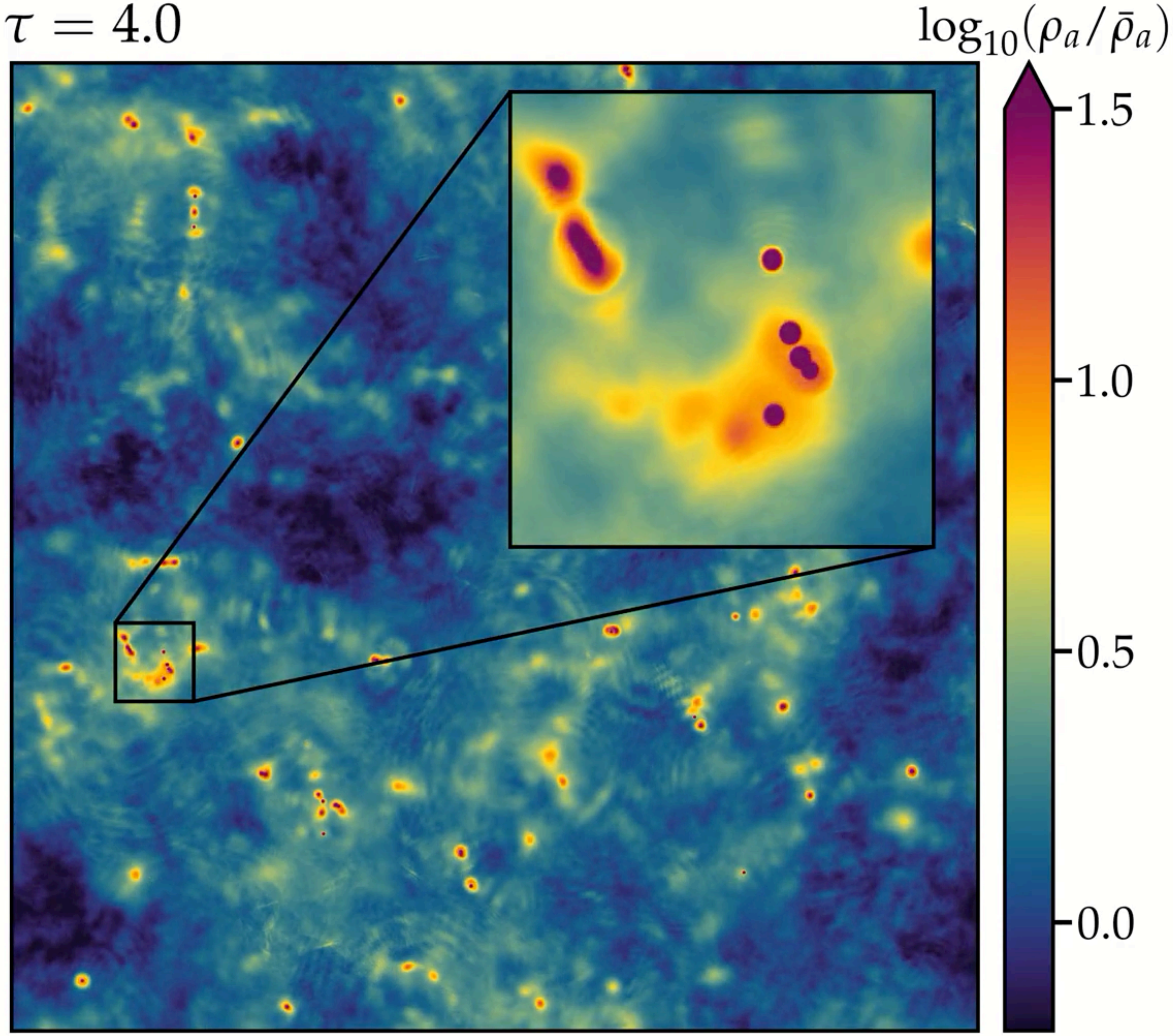

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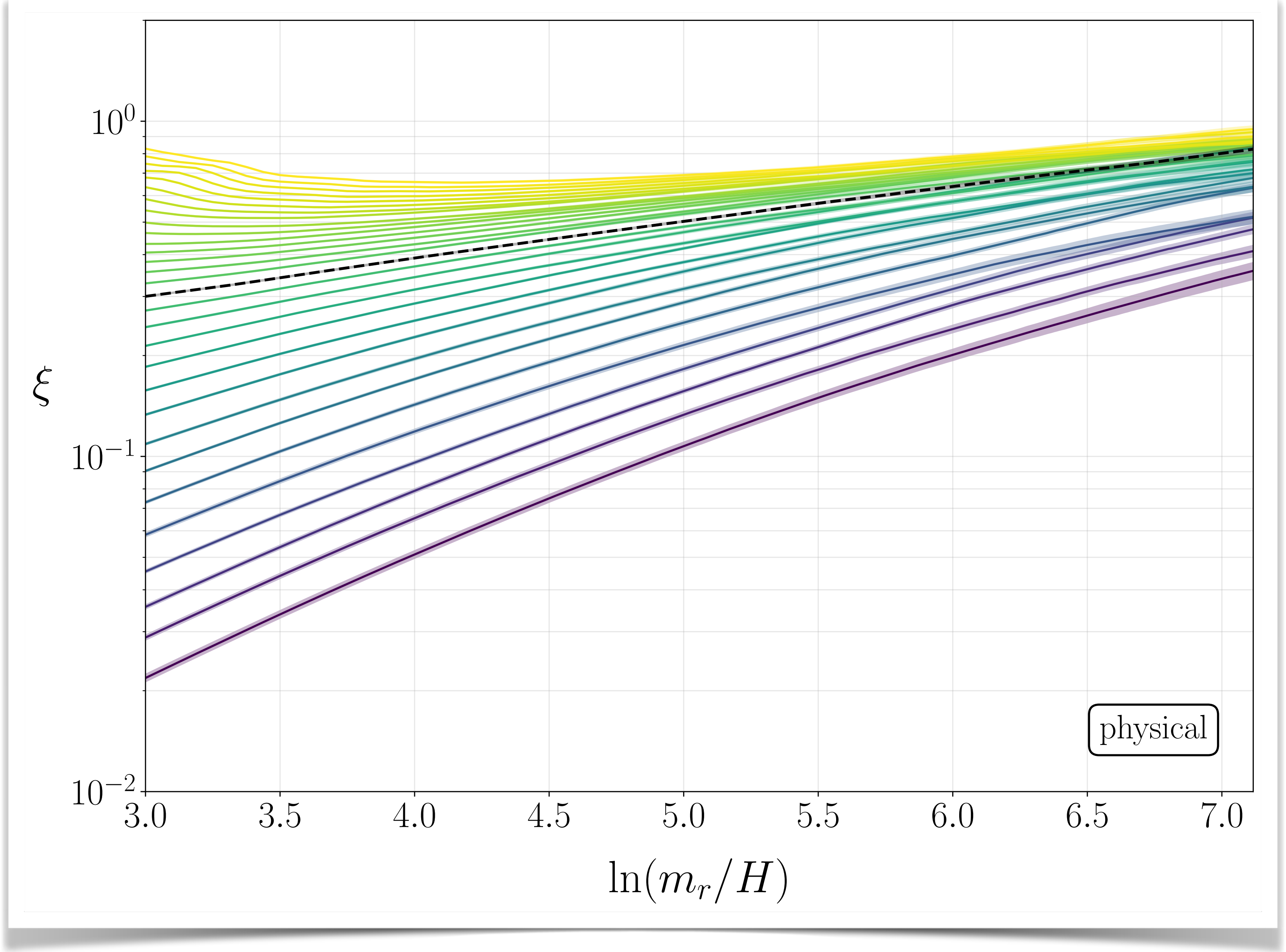
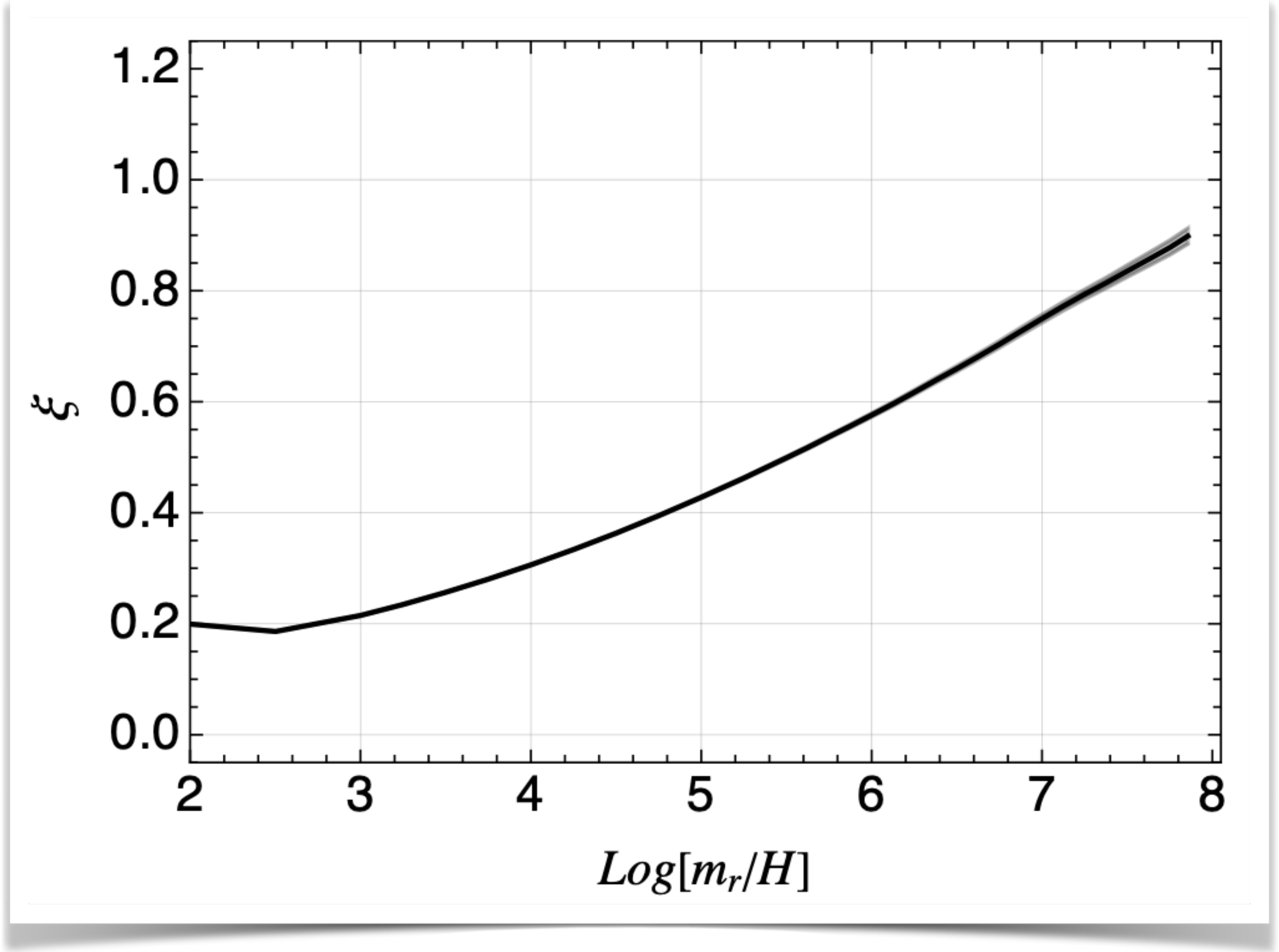
Logarithmic Growth of String Density

log

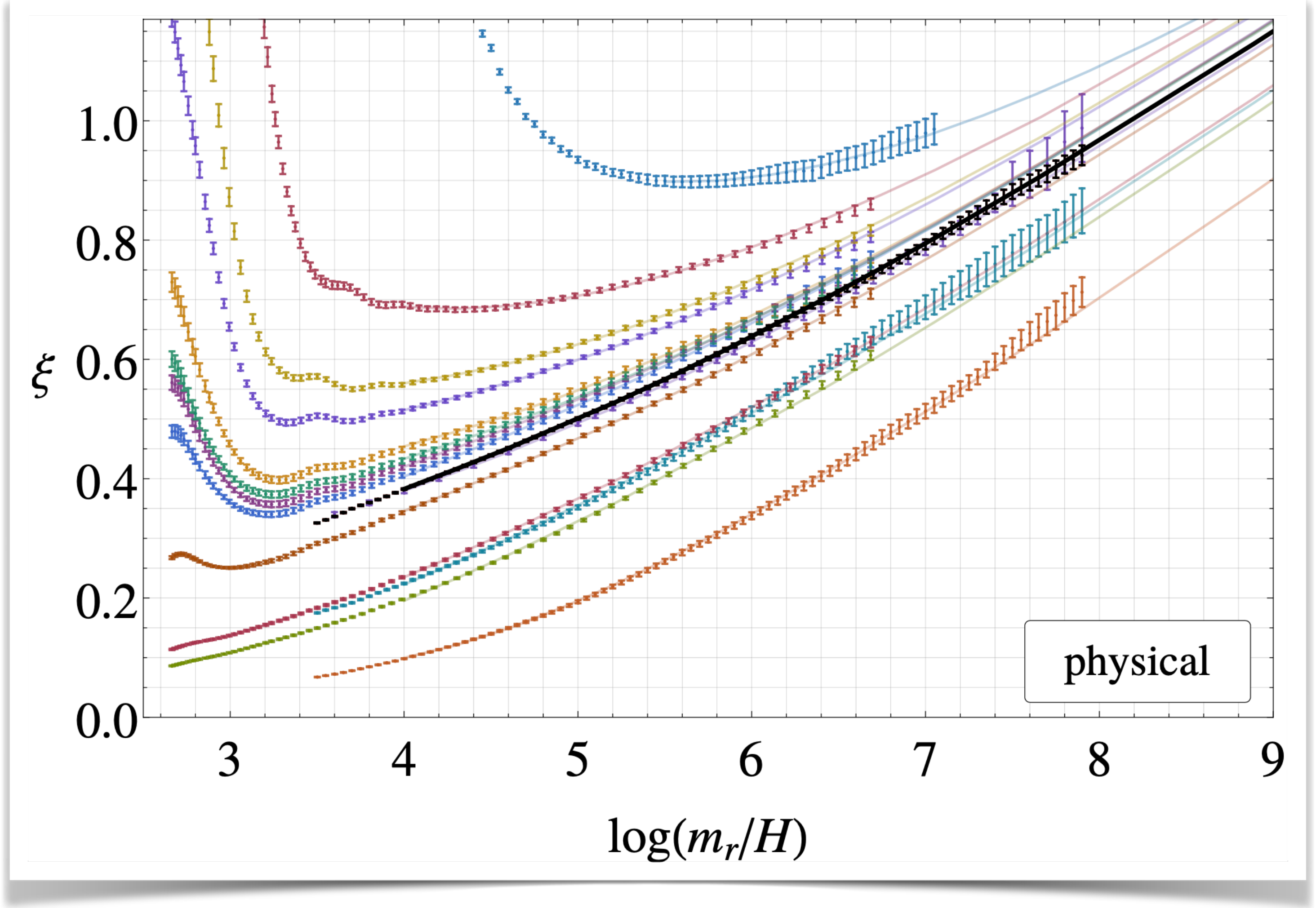
- Energy density: $\rho \sim 8\pi\xi \log(m_r/H) H^2 f_a^2$
- String density: $\xi = \dfrac{l_{\text{string}} t^2}{\mathcal{V}} = \dfrac{\rho_{\text{string}} t^2}{\mu}$
- Most recent simulations observe increase in ξ



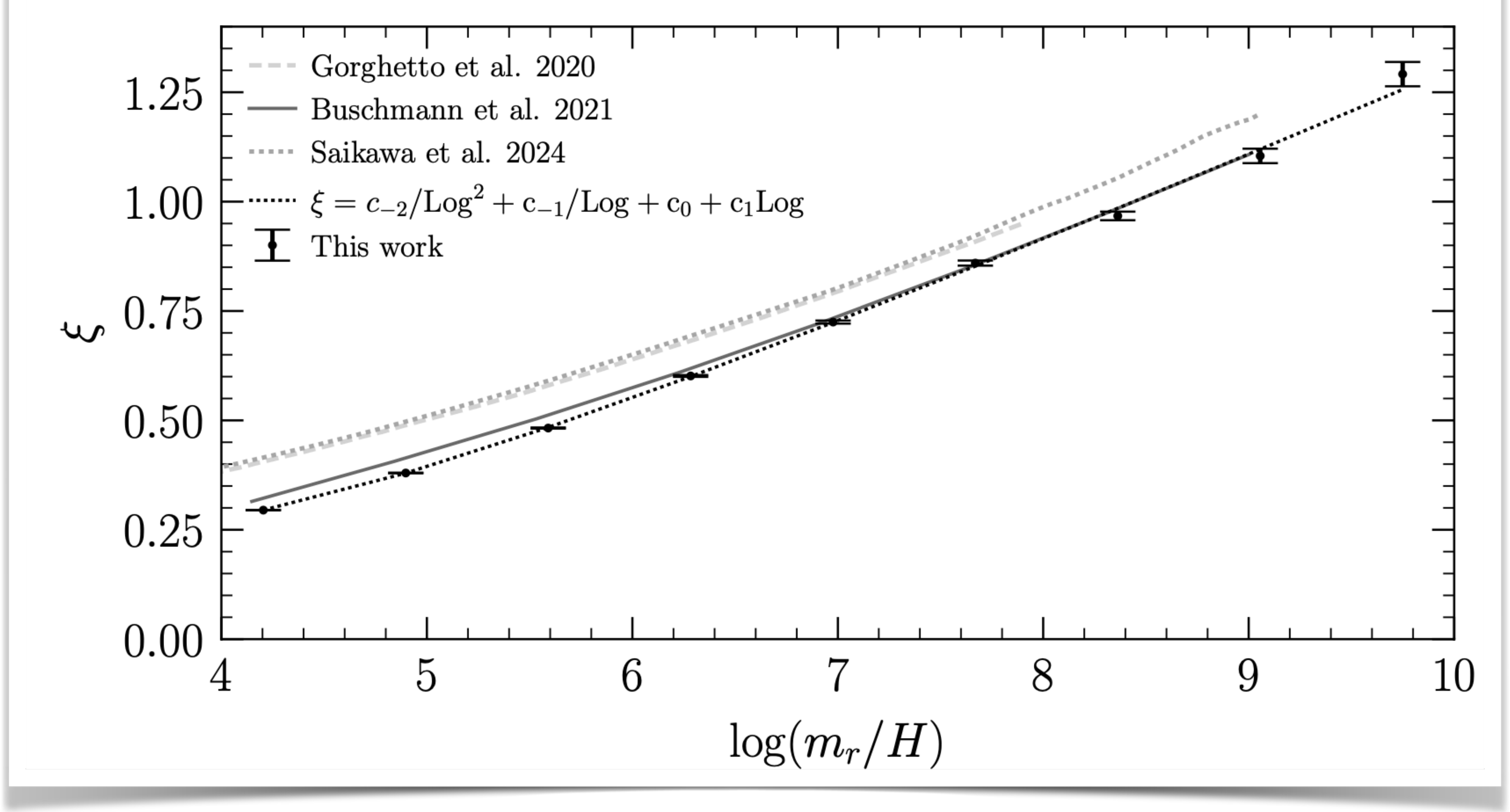
Saikawa, MK+ [2401.17253]



Kim+ [2402.00741v3]



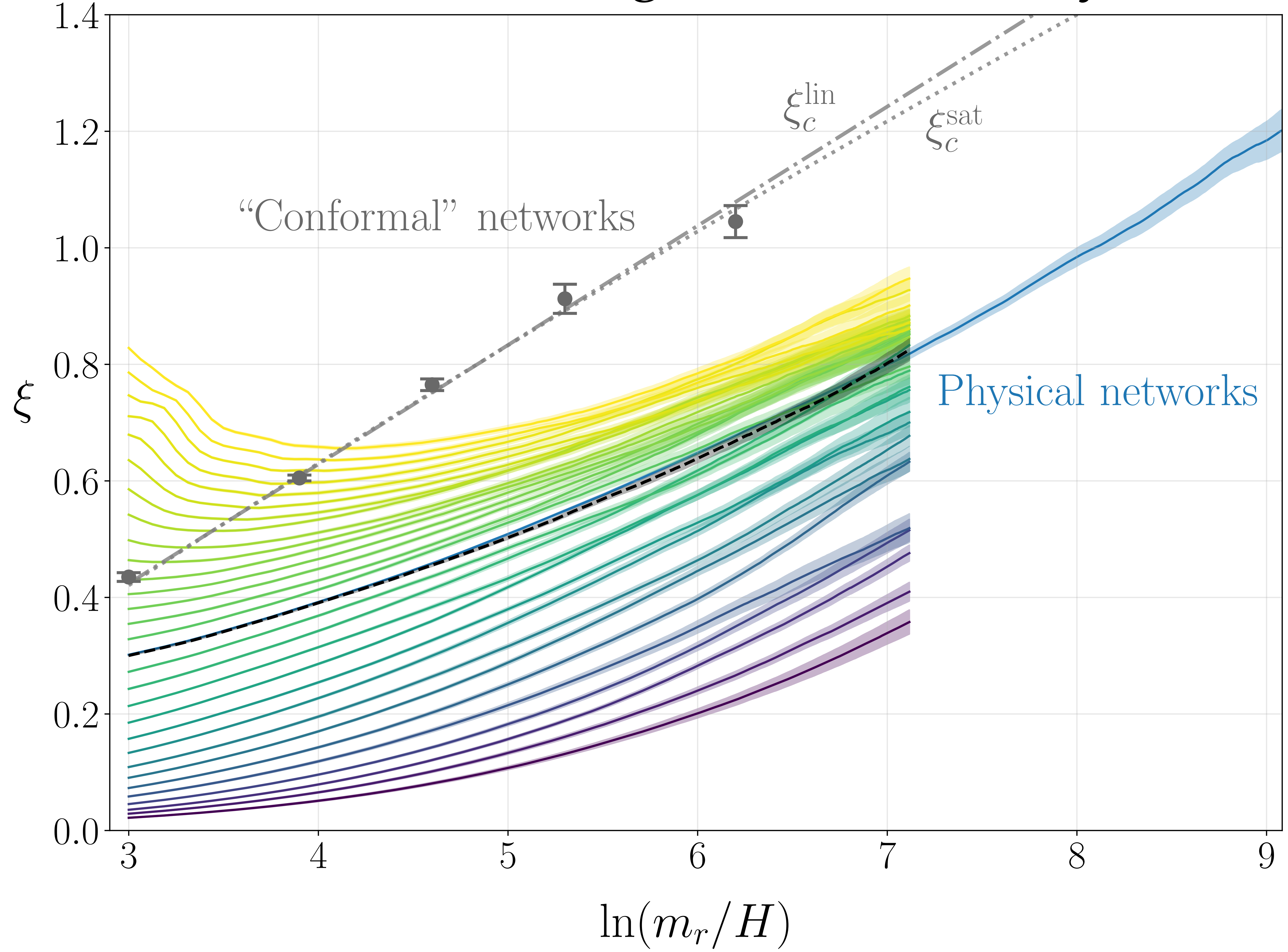
Gorghetto+ [2007.04990]



Benabou+ [2412.08699]

Evolution of String Density

- Model evolution of string network density with semi-analytic model



$$\frac{d\xi}{dt} = \boxed{\frac{C(x)}{t}}\boxed{(\xi_c(\ell(t))} - \xi(t))$$

Fleury & Moore [1509.00026]
Gorghetto+ [1806.04677; 2007.04990]
Kawasaki+ [1806.05566]
Hindmarsh+ [1908.03522]
Buschmann+ [2108.05368; 2412.08699]
Kim+ [2402.00741v3]

- **Logarithmic growth** and **attractor** behaviour compatible with previous findings but behaviour at large $\log(m_r/H)$ still uncertain (**lin**ear growth vs. **sat**uration)

Axion Radiation from Strings

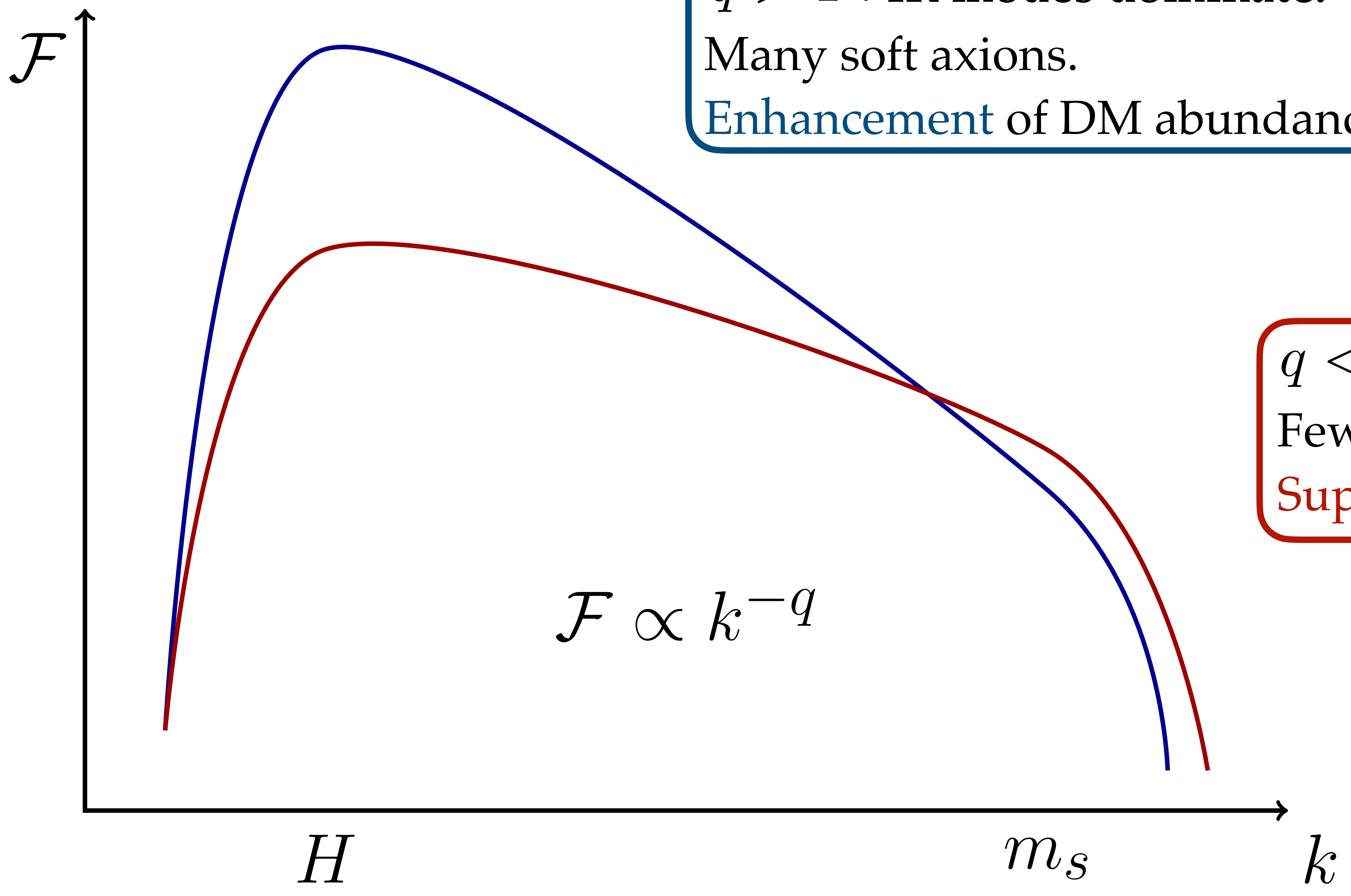
- Differential energy transfer rate:

$$\mathcal{F}\left(\frac{k}{RH}, \frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t}\left(R^4 \frac{\partial \rho_a}{\partial k}\right)$$

(R: scale factor)

- Slope is important! Gorghetto+ [1806.04677, 2007.04990], Buschmann+ [2108.05368, 2412.08699], Saikawa, MK+ [2401.17253]

$q > 1$: **IR modes dominate.**
Many soft axions.
Enhancement of DM abundance.



$$\frac{n_a}{f_a^2 H} = \frac{1}{f_a^2 H} \int \mathrm{d}k \frac{1}{\omega} \frac{\partial \rho_a}{\partial k}$$
$$= \int^{\tau} \frac{\mathrm{d}\tau'}{\tau'} \int \frac{\mathrm{d}x'}{x'} \mathcal{F}$$

$q < 1$: **UV modes dominate.**
Fewer hard axions.
Suppression of DM abundance.

Axion Radiation from Strings

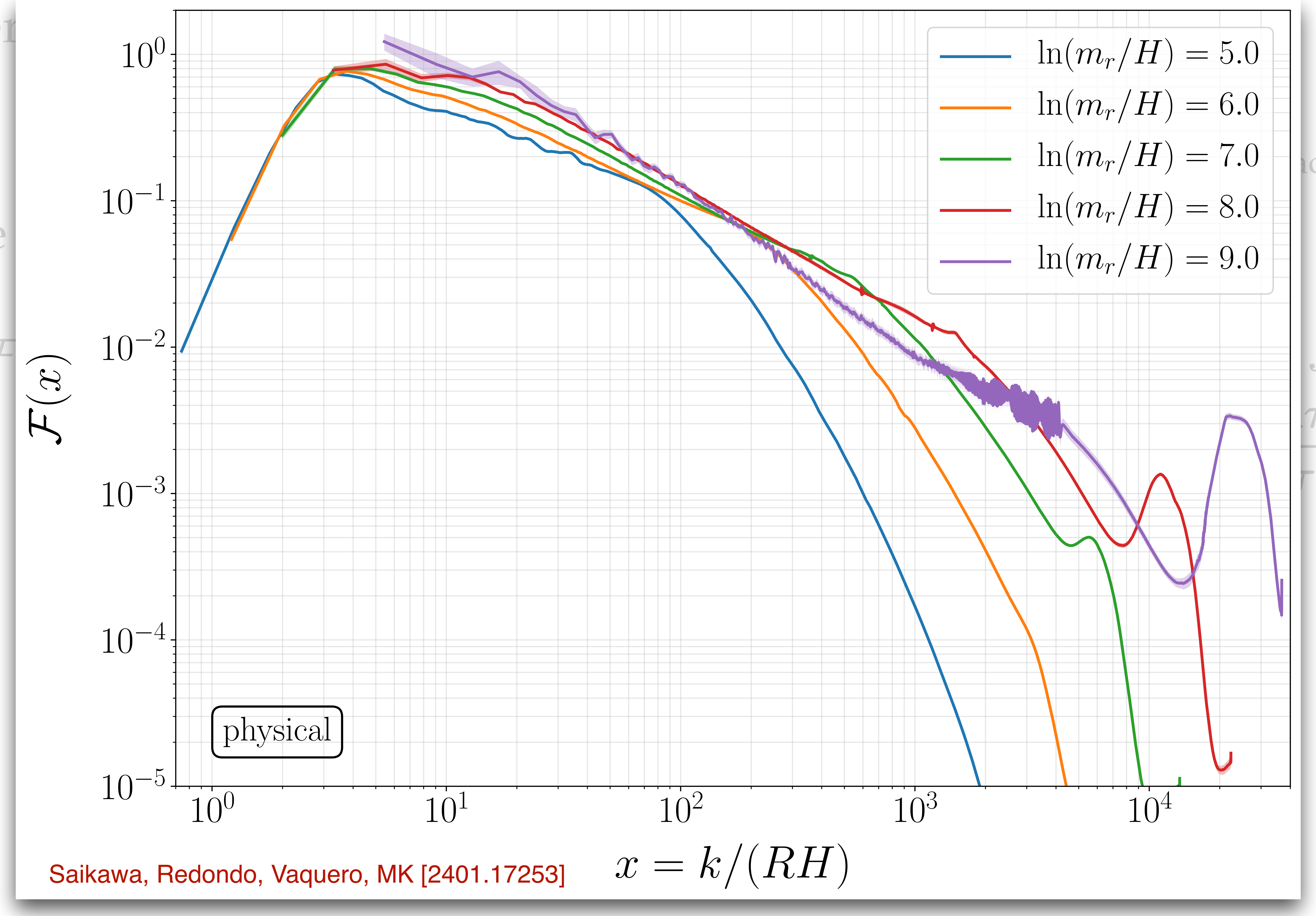
- Differ

actor)

- Slope

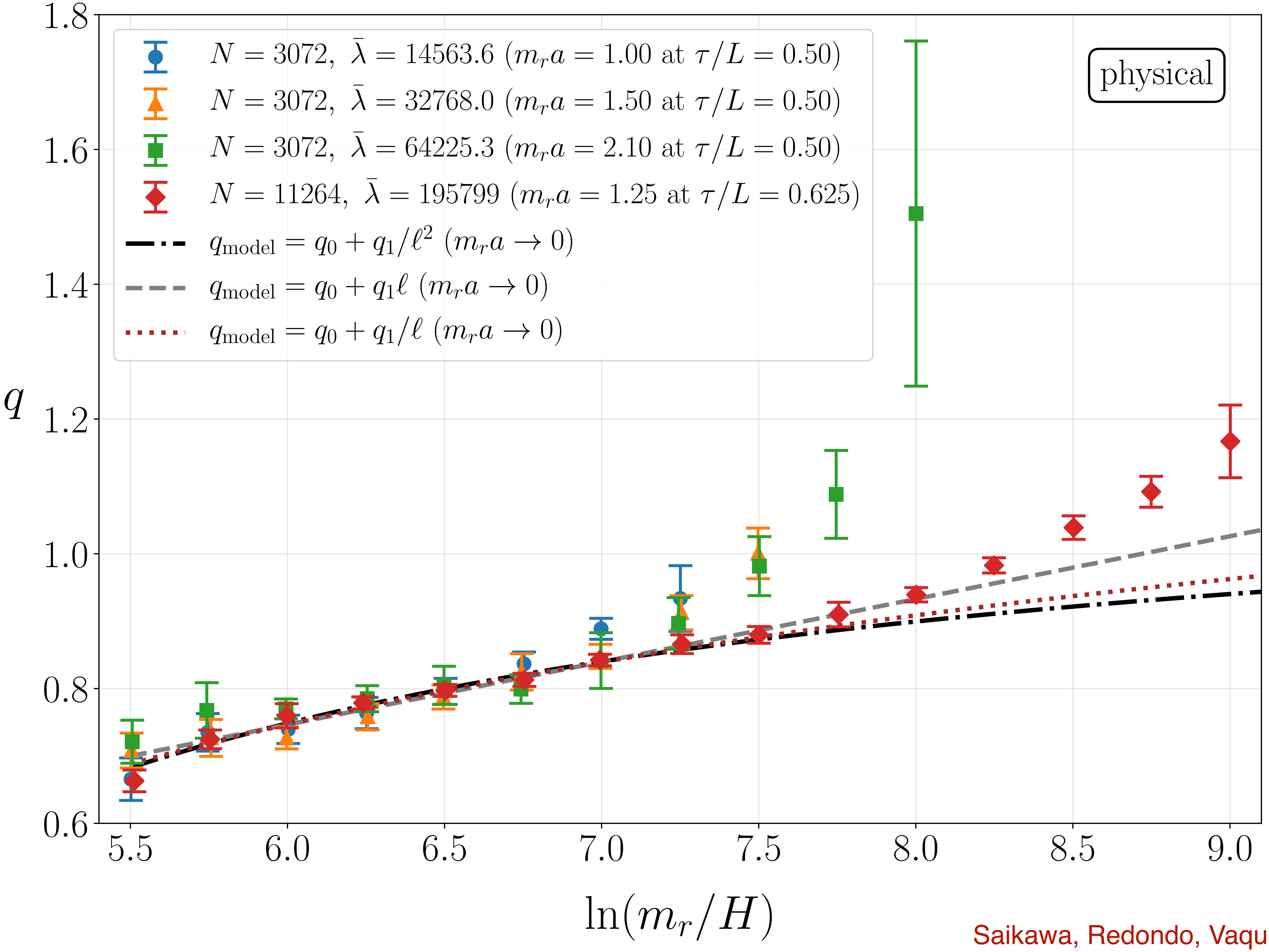
$\mathcal{F}$ $\int \mathrm{d}k \frac{1}{\omega}\frac{\partial \rho_a}{\partial k}$

$\int \frac{\mathrm{d}x'}{x'}\mathcal{F}$



Saikawa, Redondo, Vaquero, MK [2401.17253]

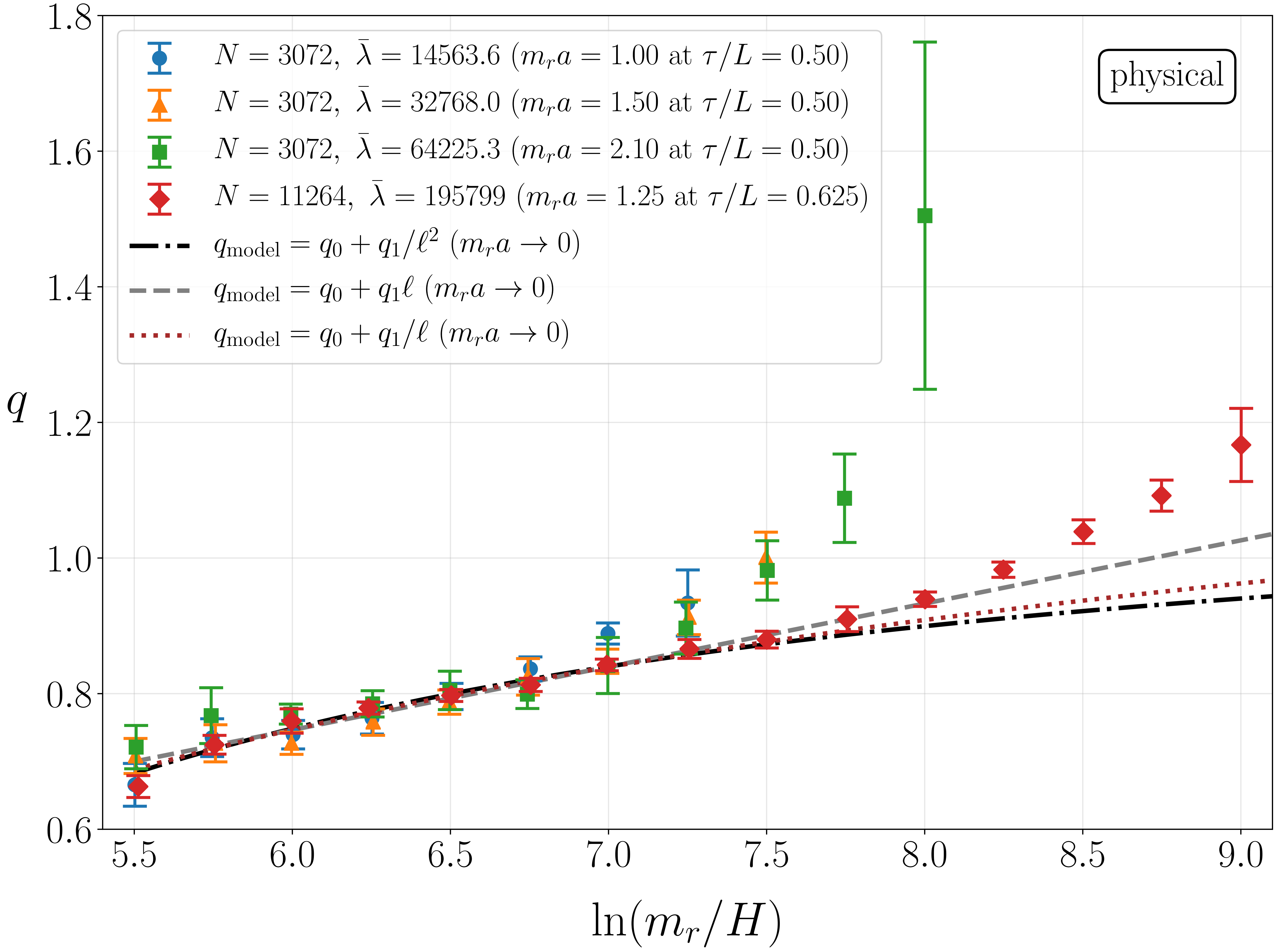
Evolution of Spectral Index



Saikawa, Redondo, Vaquero, MK [2401.17253]

What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



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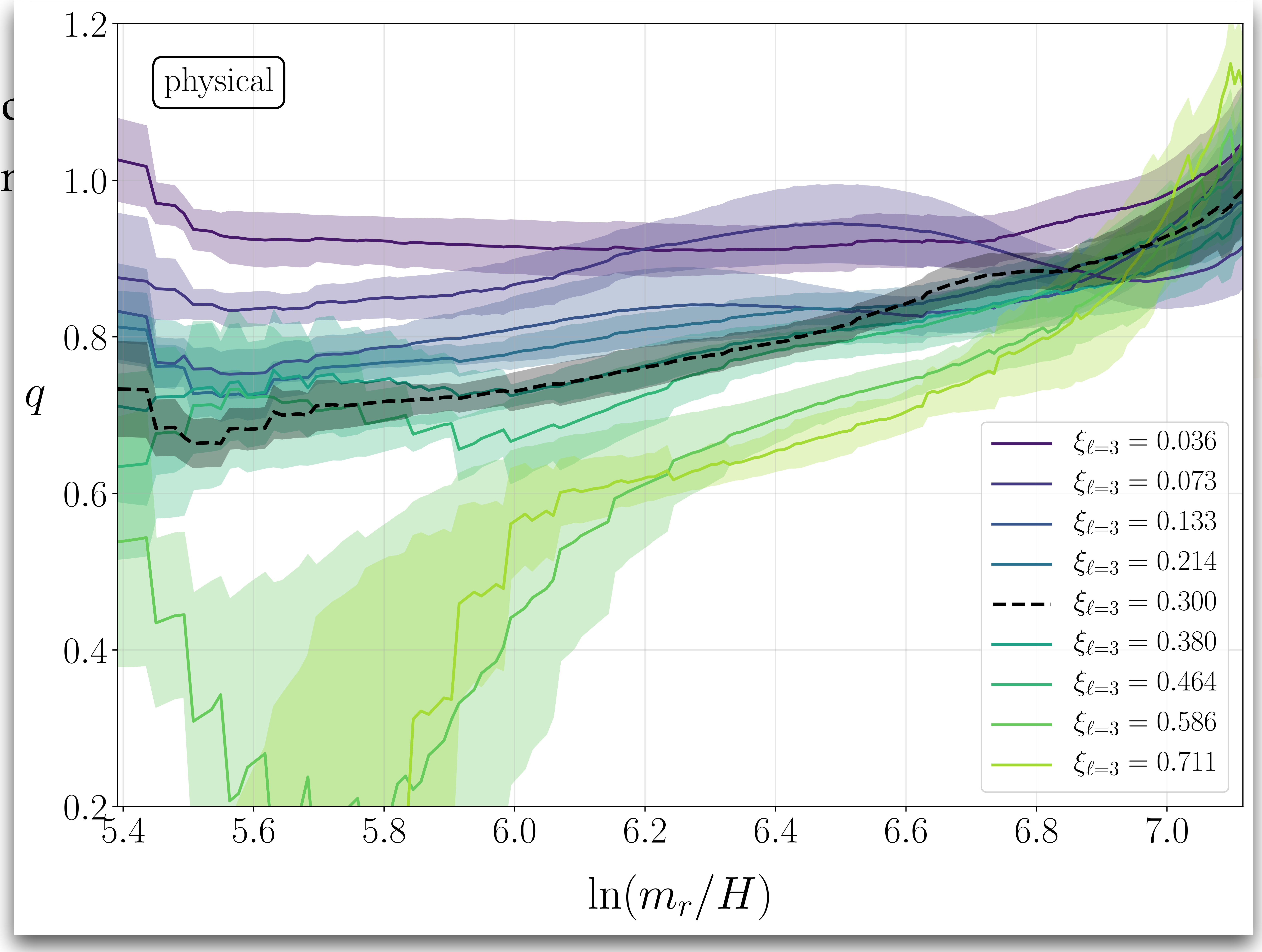

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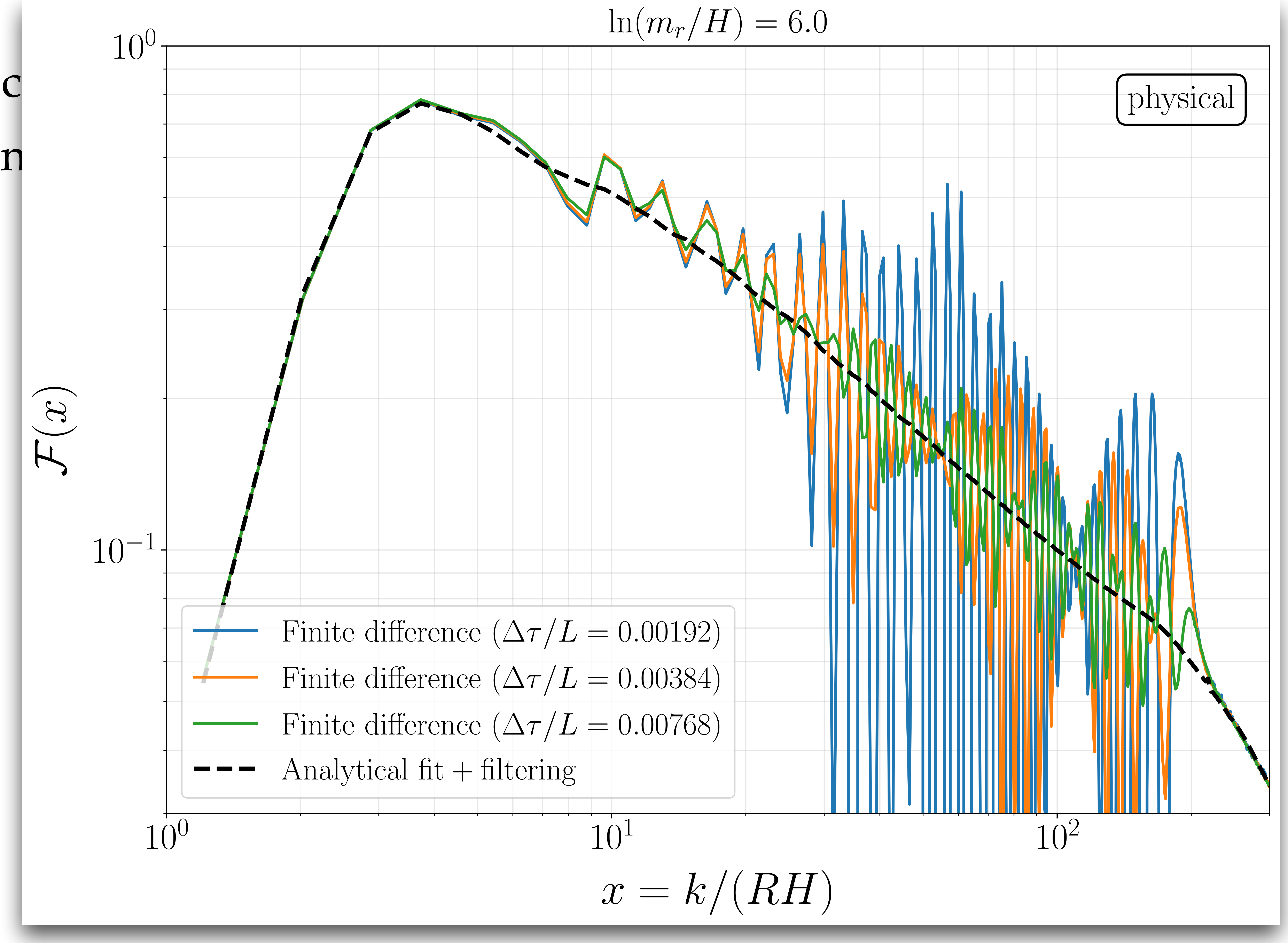

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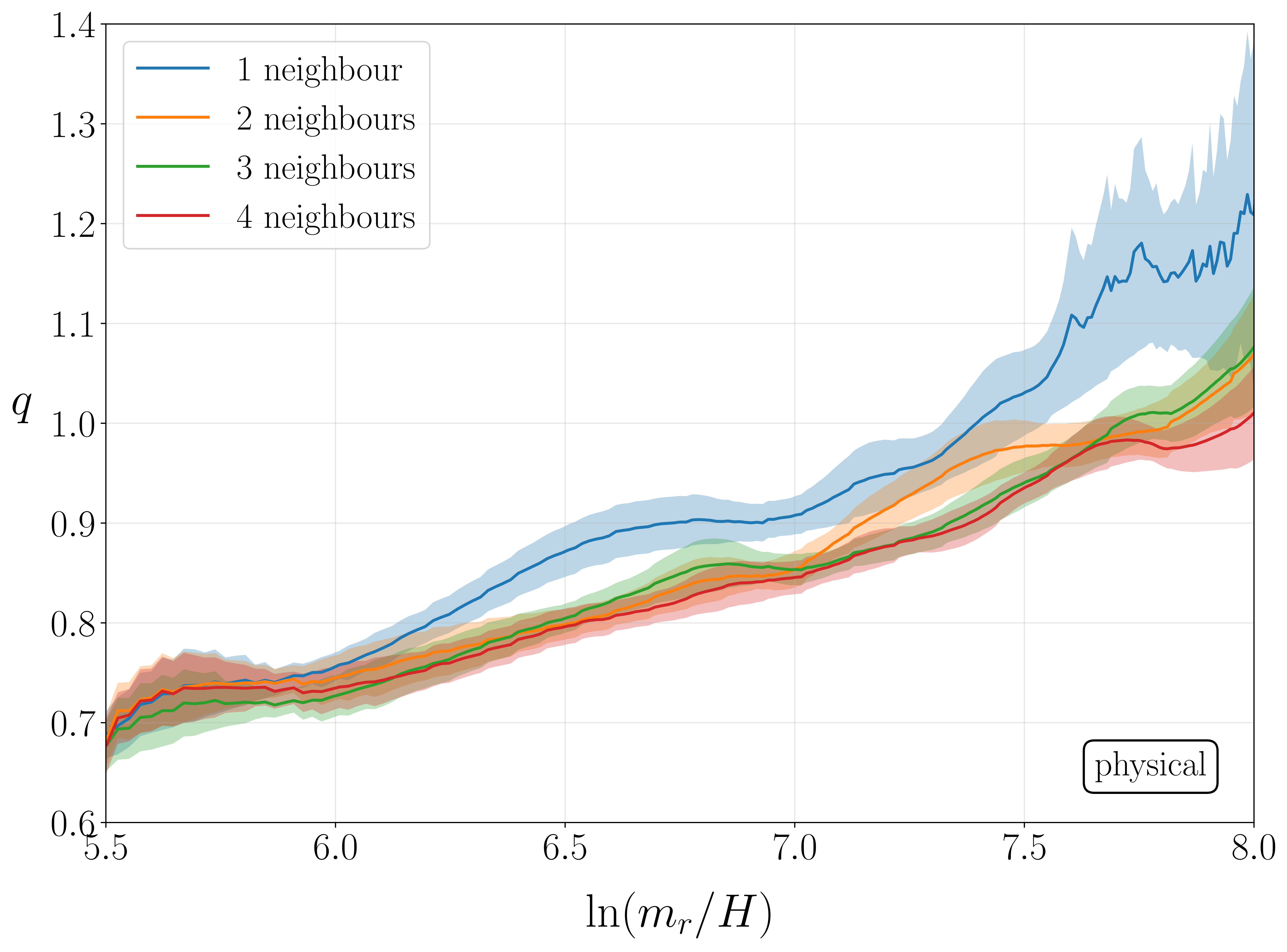
    - **Laplacian**
    - Resolution of the string core $m_r a$



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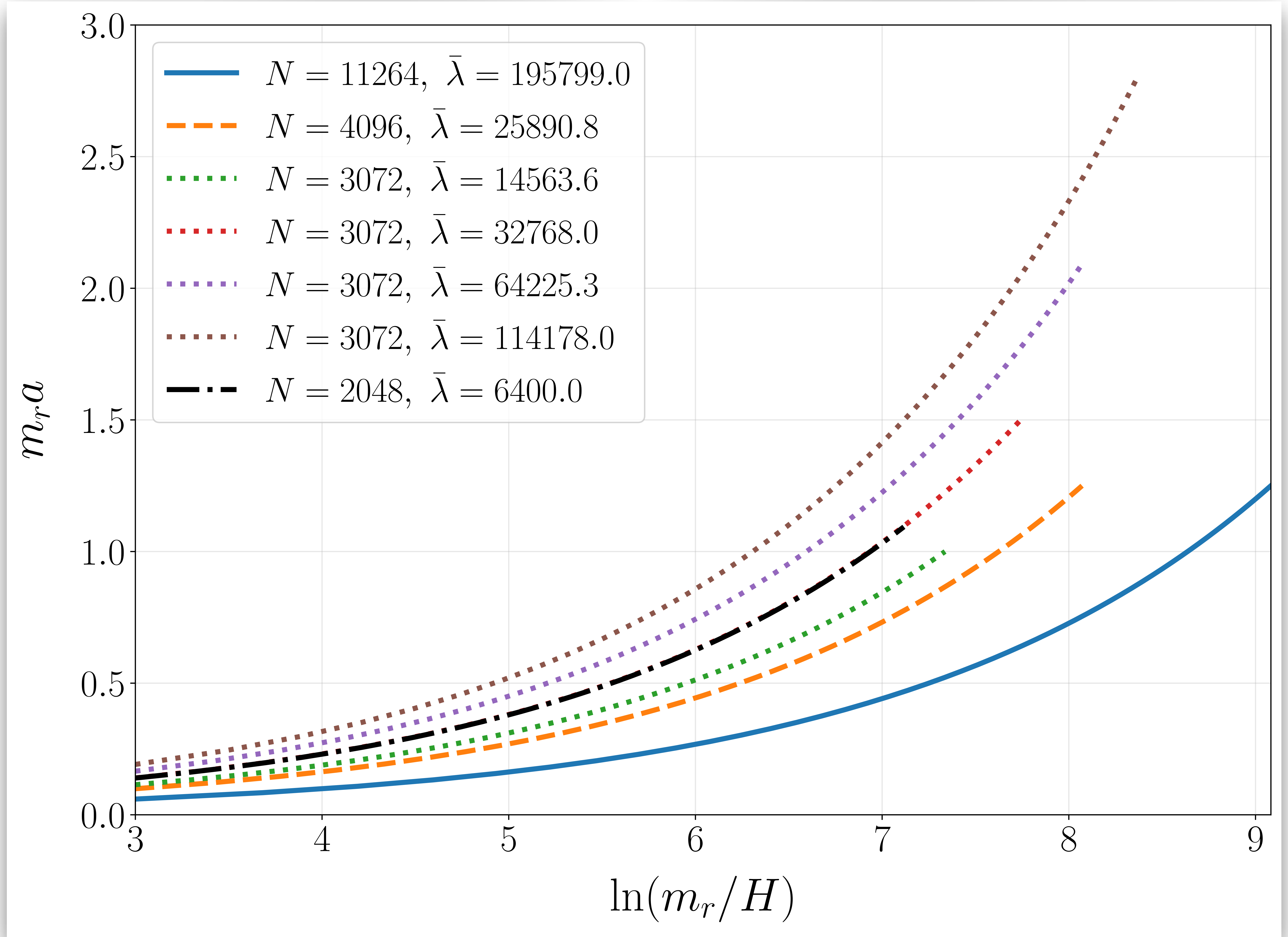
    - **Resolution of the string core $m_r a$**

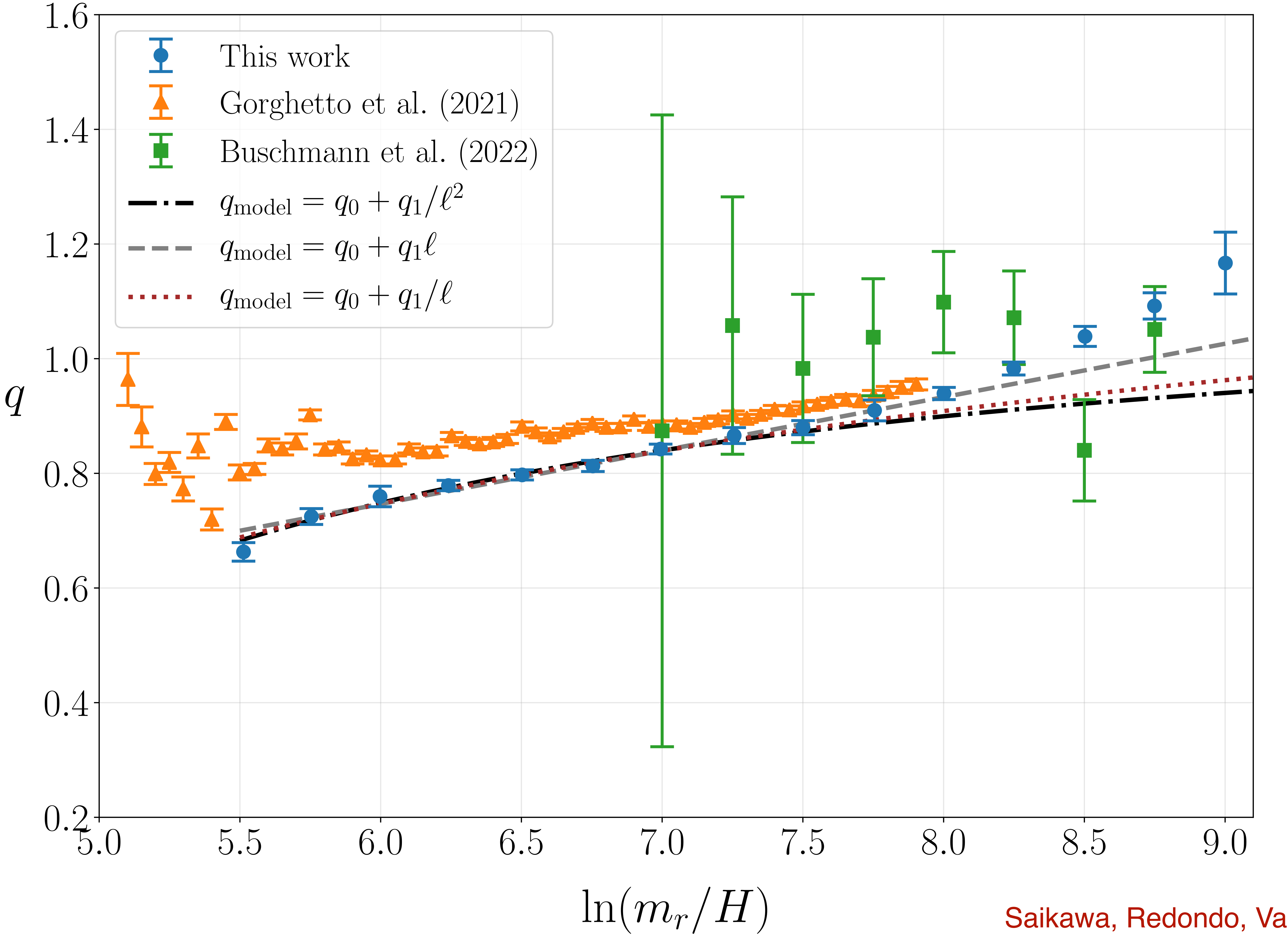


$\ln(m_r/H)$

Saikawa, Redondo, Vaquero, MK [2401.17253]

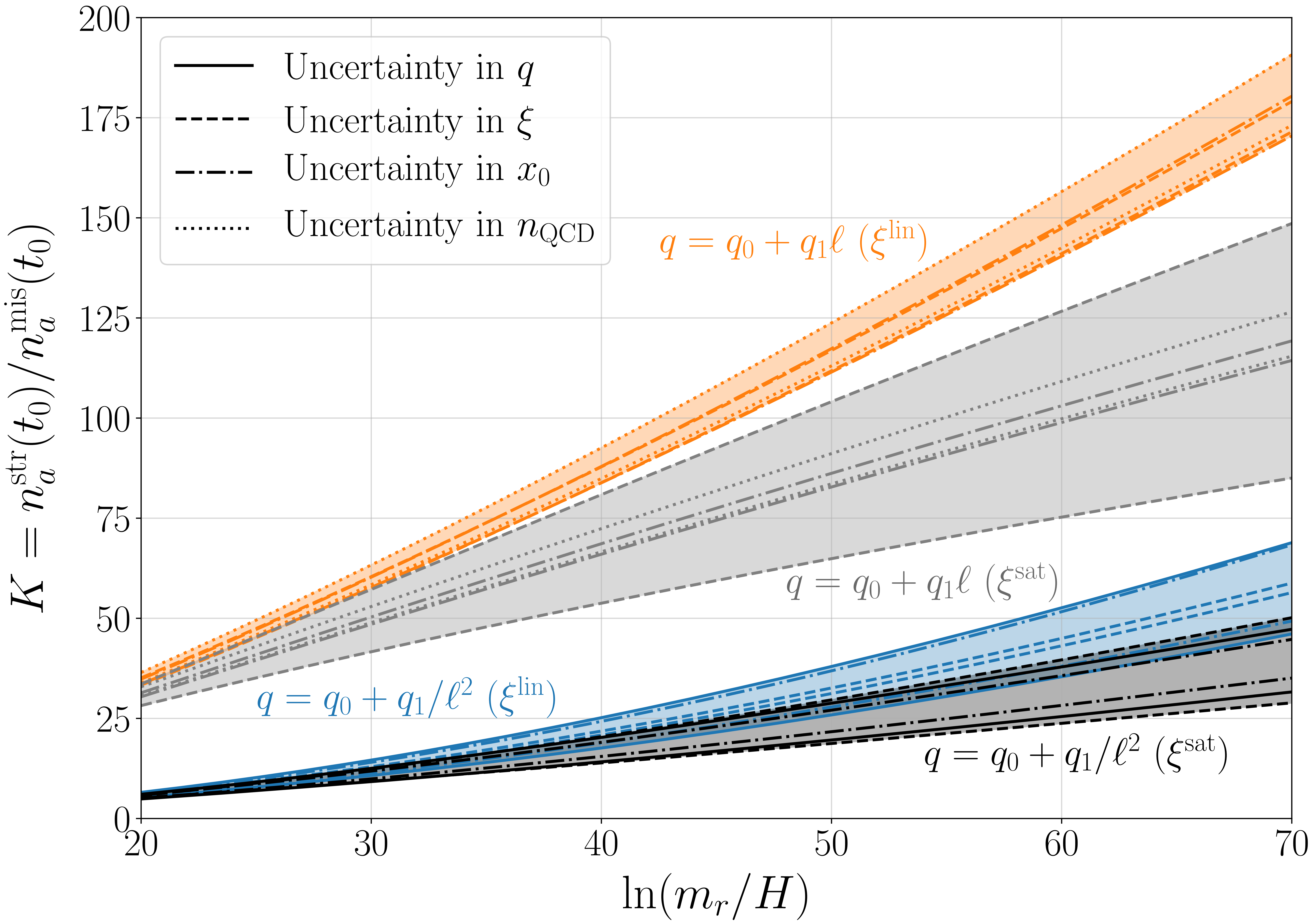
Late Time Behaviour of Spectral Index

$$q = q_{\mathrm{model}}(\ell) + q_{\mathrm{disc}}(\ell, m_r a)$$



Saikawa, Redondo, Vaquero, MK [2401.17253]

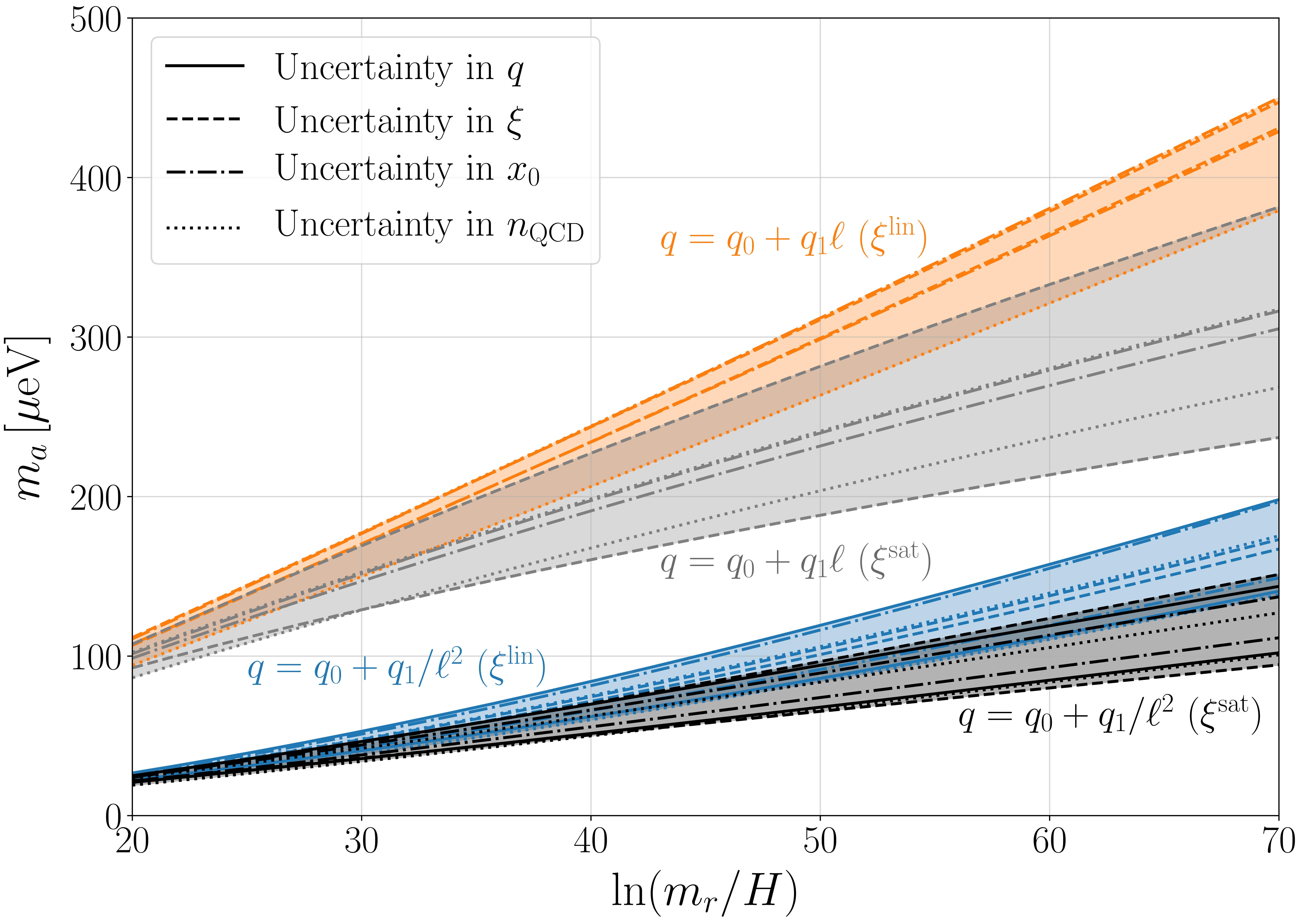
Axions from Strings vs. Misalignment



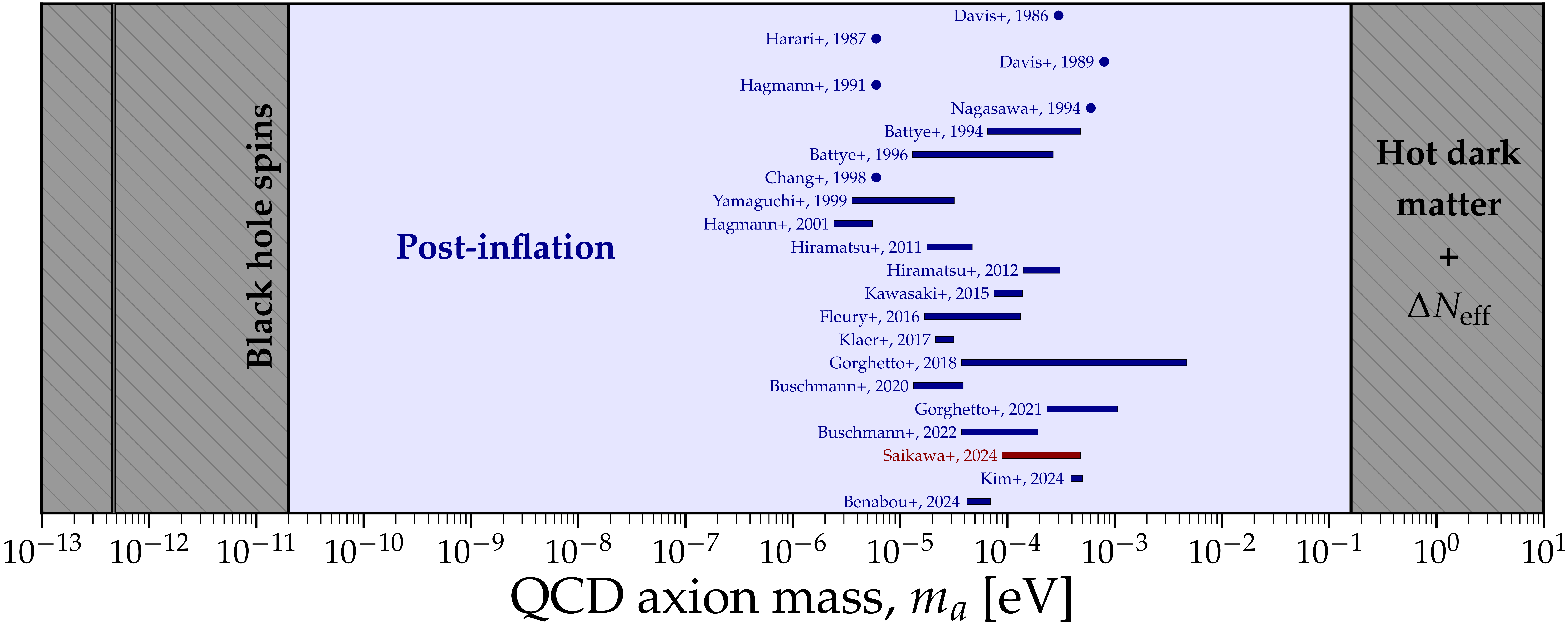
Known for "standard" angle-averaged misalignment:

$$\Omega_a h^2 = K \Omega_a^{\rm mis} h^2$$

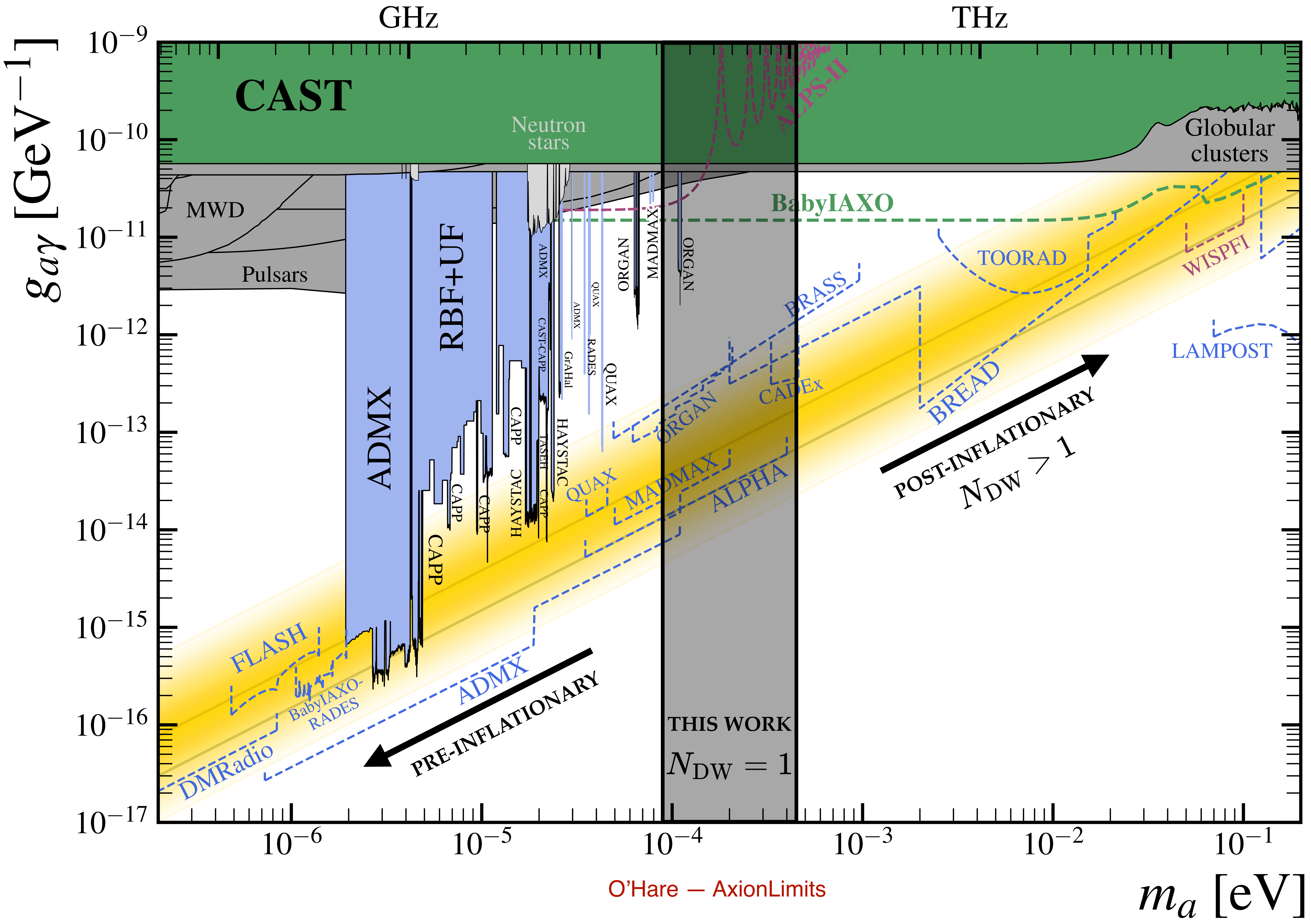
Axion Dark Matter Mass Prediction



Axion DM Mass Predictions from String Simulations



MK+ [2502.02398]



O'Hare — AxionLimits

How can we refine these predictions?

— Theory —

Based on 251X.XXXX (also soon!) with J. Redondo & I. Rybak

Global String Dynamics

- Couple **Kalb-Ramond** field to **Nambu-Goto string**:

$$S = -\mu_0 \int \sqrt{-\gamma d^2\sigma} \quad + \frac{1}{6} \int H^2 \sqrt{-g} d^4x \quad - g \int B_{\mu\nu} d\sigma^{\mu\nu}$$

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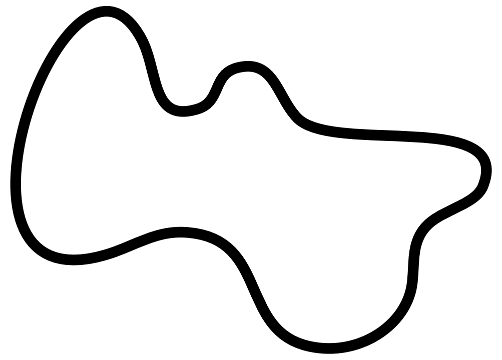
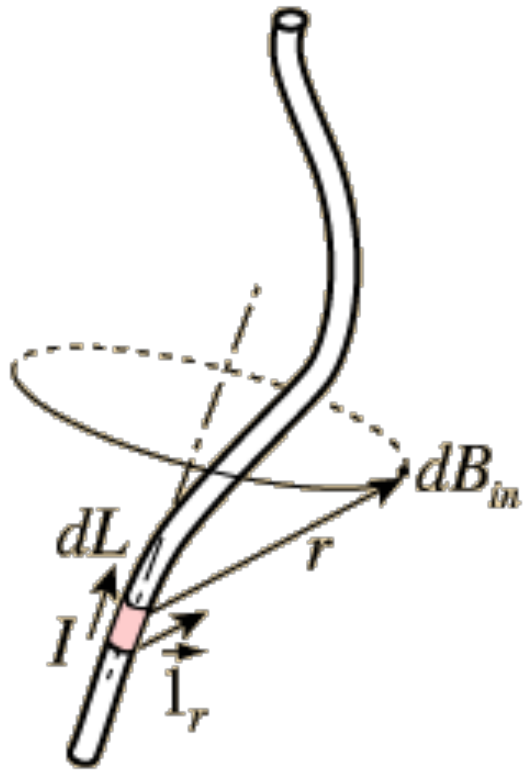


Axion:

$$\epsilon_{\alpha\beta\gamma\delta} H_{\beta\gamma\delta} \propto \partial_\alpha \theta$$

Coupling



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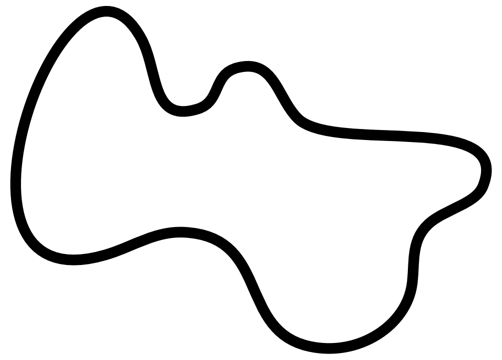

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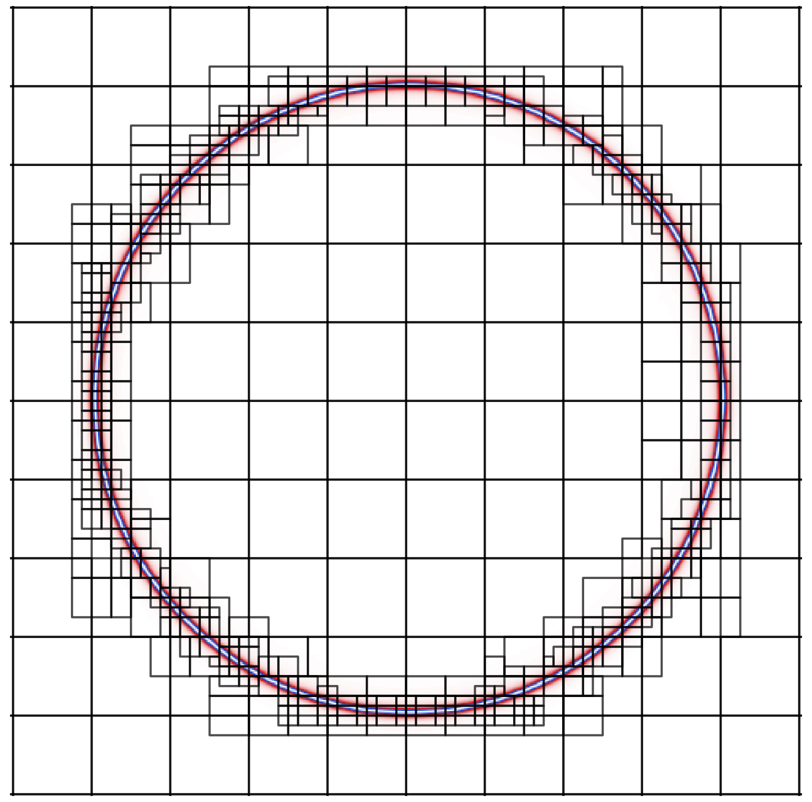


- KR field renormalises the NG string tension $\mu_0 \to \mu_0(1+\log)$

- **Example:** Circular Loop $\mathbf{X} = \Big\{r(\tau)\cos(2\pi\sigma), r(\tau)\sin(2\pi\sigma), 0\Big\}$

$$\ddot{r} + (1-\dot{r}^2)\left[2\dot{r}\frac{\dot{a}}{a} + \frac{1}{r}\right] = \frac{2g}{\mu_0}\left(1-\dot{r}^2\right)^{3/2}\frac{H_{012}}{a^2}$$

with

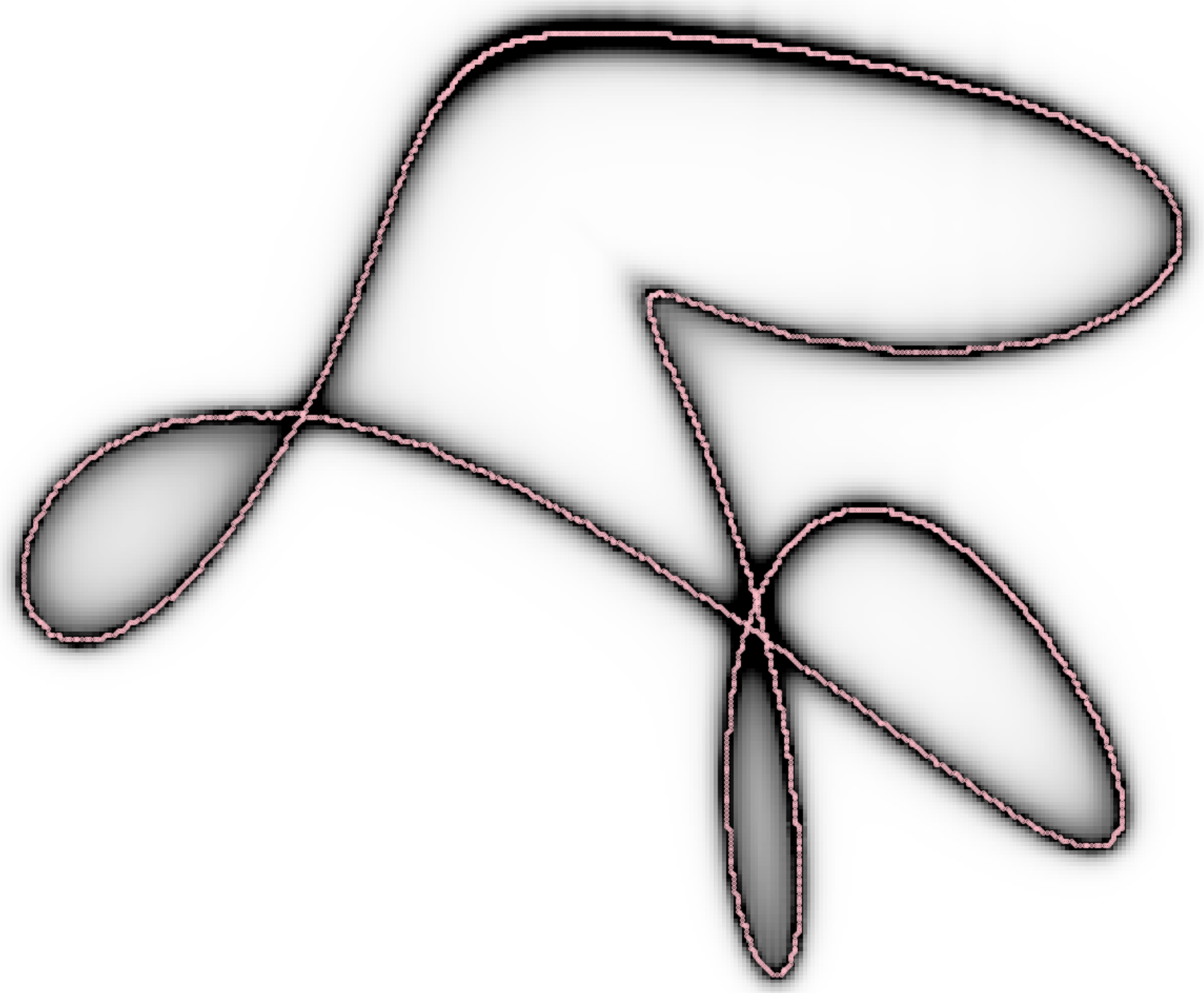
$$\lim_{\tilde{\tau}\to\tau} H_{012} = \frac{ga^2}{4\pi(1-\dot{r}^2)^{3/2}}\left[\ddot{r} + (1-\dot{r}^2)\left(2\dot{r}\frac{\dot{a}}{a} + \frac{1}{r}\right)\right]\log\left[\frac{\tau-\tau_{\mathrm{f}}}{\tau-\tau_{\mathrm{i}}}\right]$$

LHS: Nambu-Goto

RHS: Axion

Expectation: Approach NG limit for large log

Constructing the Axion Field around Strings

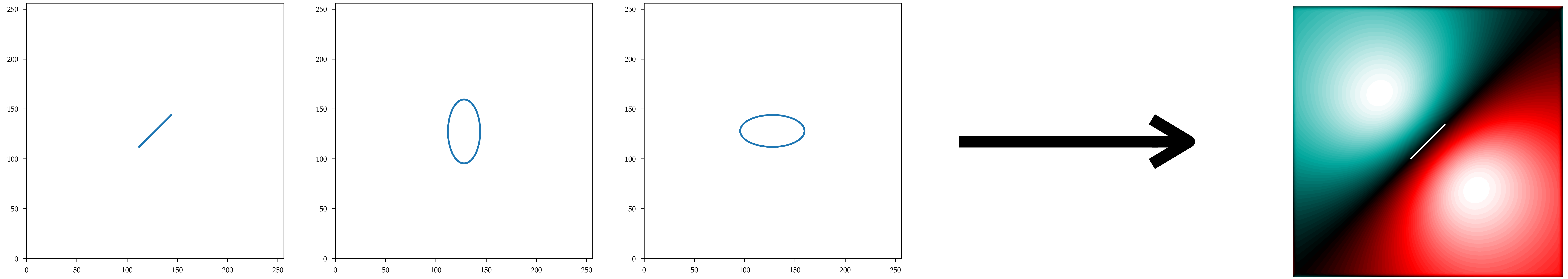


- Contribution of a short, straight section of the string:

$$\nabla\theta = K \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$

- Calculate links to construct the axion field in the full plane:

$$\theta_{\mathbf{x}+\mathbf{d}x} - \theta_{\mathbf{x}} = \int_x^{x+dx} d^3\mathbf{x} \cdot \nabla\theta = -\frac{1}{2} \int_x^{x+dx} d^3\mathbf{x} \cdot \int d\sigma \frac{(\mathbf{x} - \mathbf{X}(\sigma)) \times \mathbf{X}'}{|\mathbf{x} - \mathbf{X}(\sigma)|^3}$$



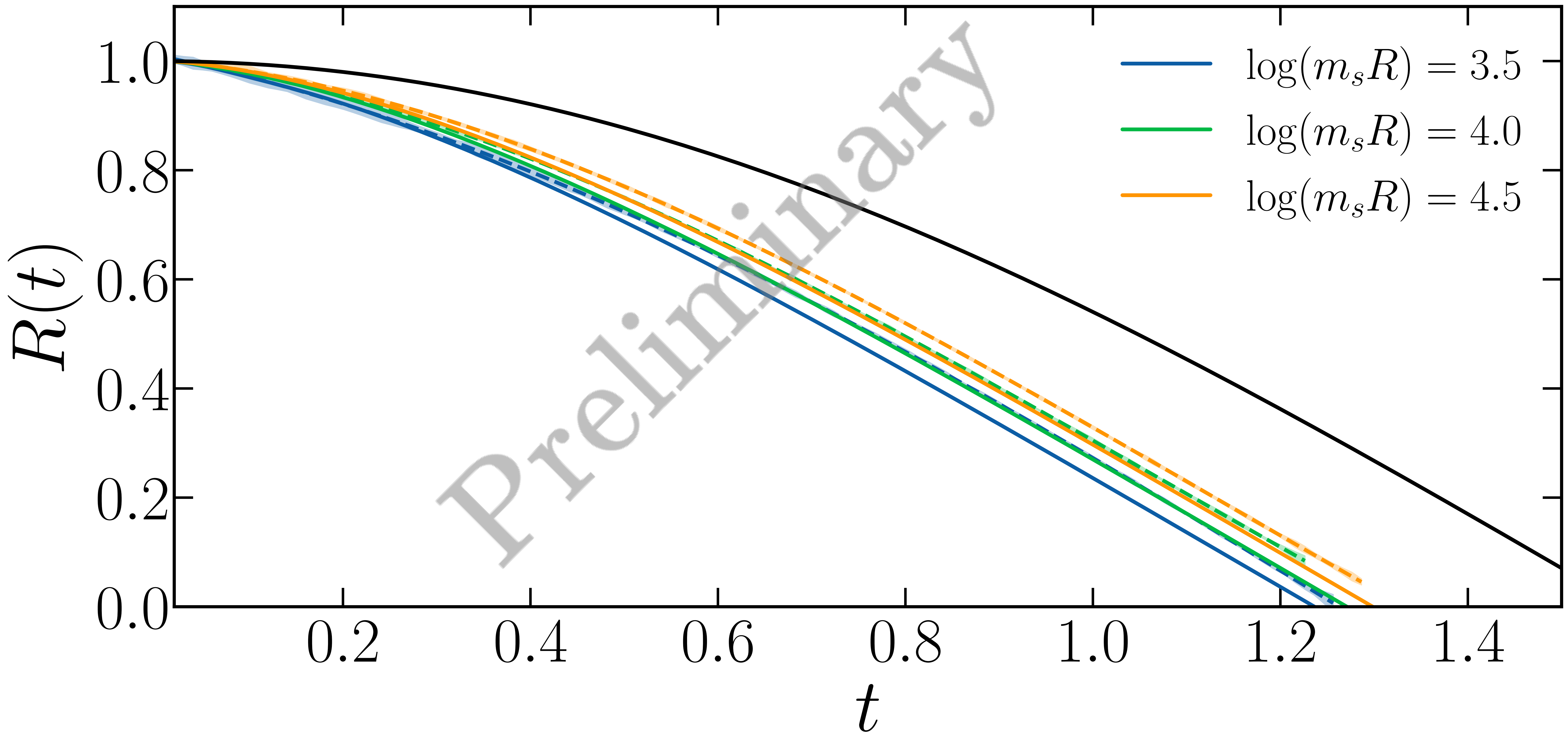
Theory vs. Simulations

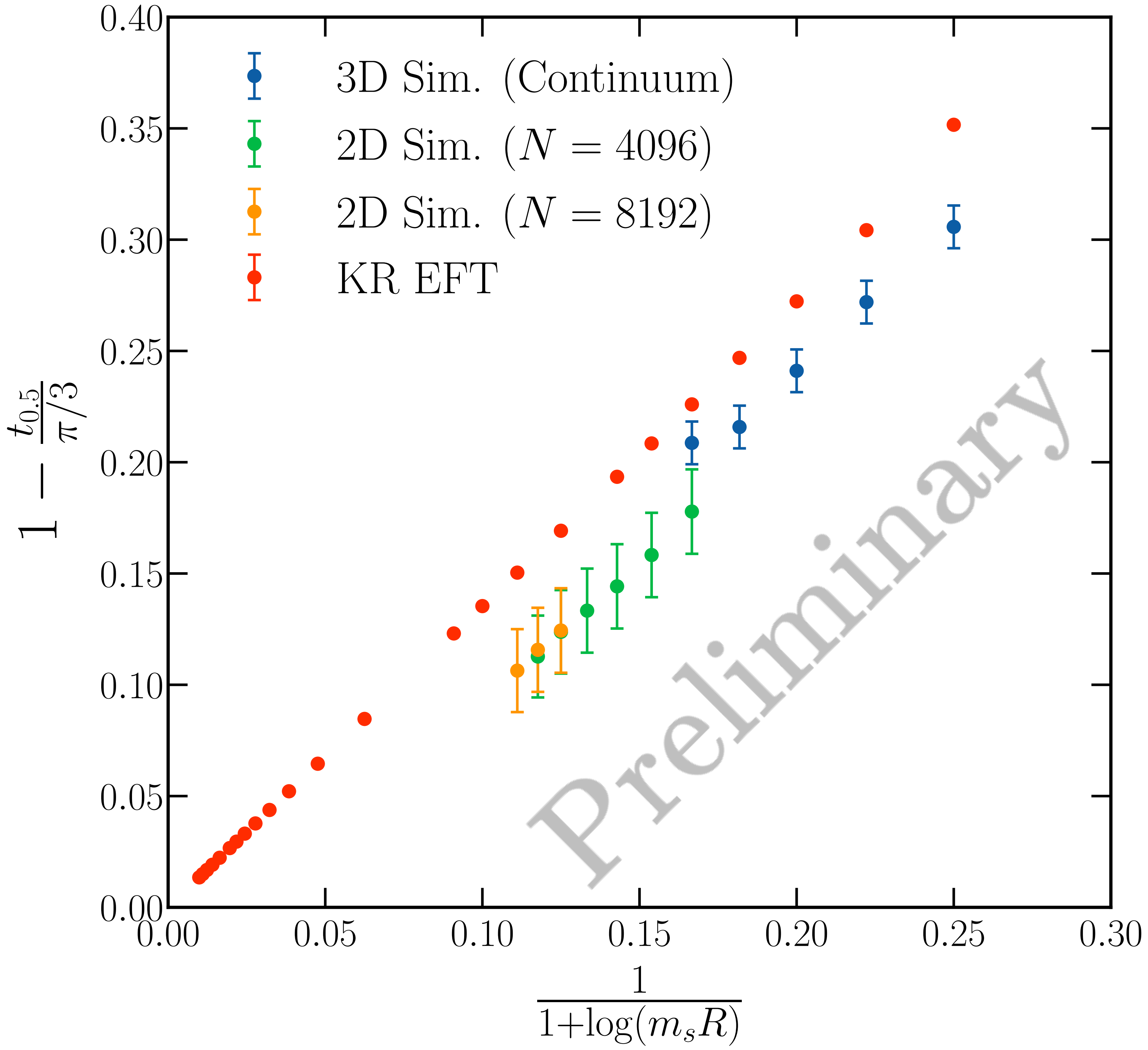
- We see correct trends, **but** still unaccounted differences (*work in progress ..*)

$$\ddot{r} + \left(1-\dot{r}^2\right)\left[2\dot{r}\frac{\dot{a}}{a}+\frac{1}{r}\right] = \frac{4\pi}{\mu(\tau)}\frac{\left(1-\dot{r}^2\right)^{3/2}}{ga^2}\left(H_{012}-\lim_{\tilde{\tau}\to\tau}H_{012}\right)$$

large log suppresses KR field

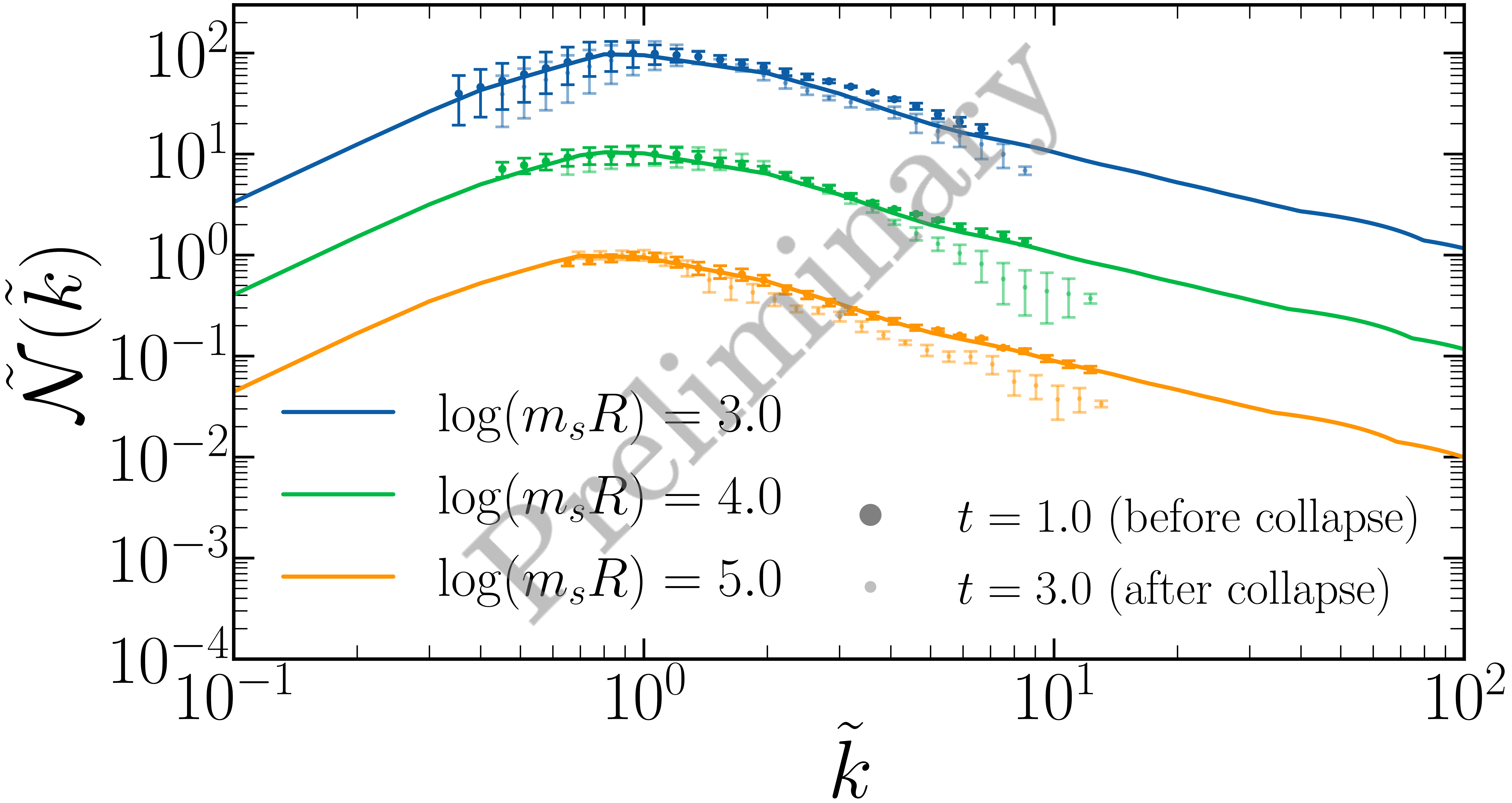
renormalised KR field has no log





Circular Loop Spectrum

- Dimensionless number spectrum from simulations (*points*):



$$\mathcal{N}(k) = \frac{1}{H f_a^2} \frac{\partial \rho(k)}{\partial (k/R)}$$

$$\tilde{\mathcal{N}} = \mathcal{N}/\mathcal{N}|_{\text{peak}}$$

+ shift for better visibility

$$\tilde{k} = kL/(kL)|_{\text{peak}}$$

- **But:** Quantitatively different to effective theory (*solid lines*):

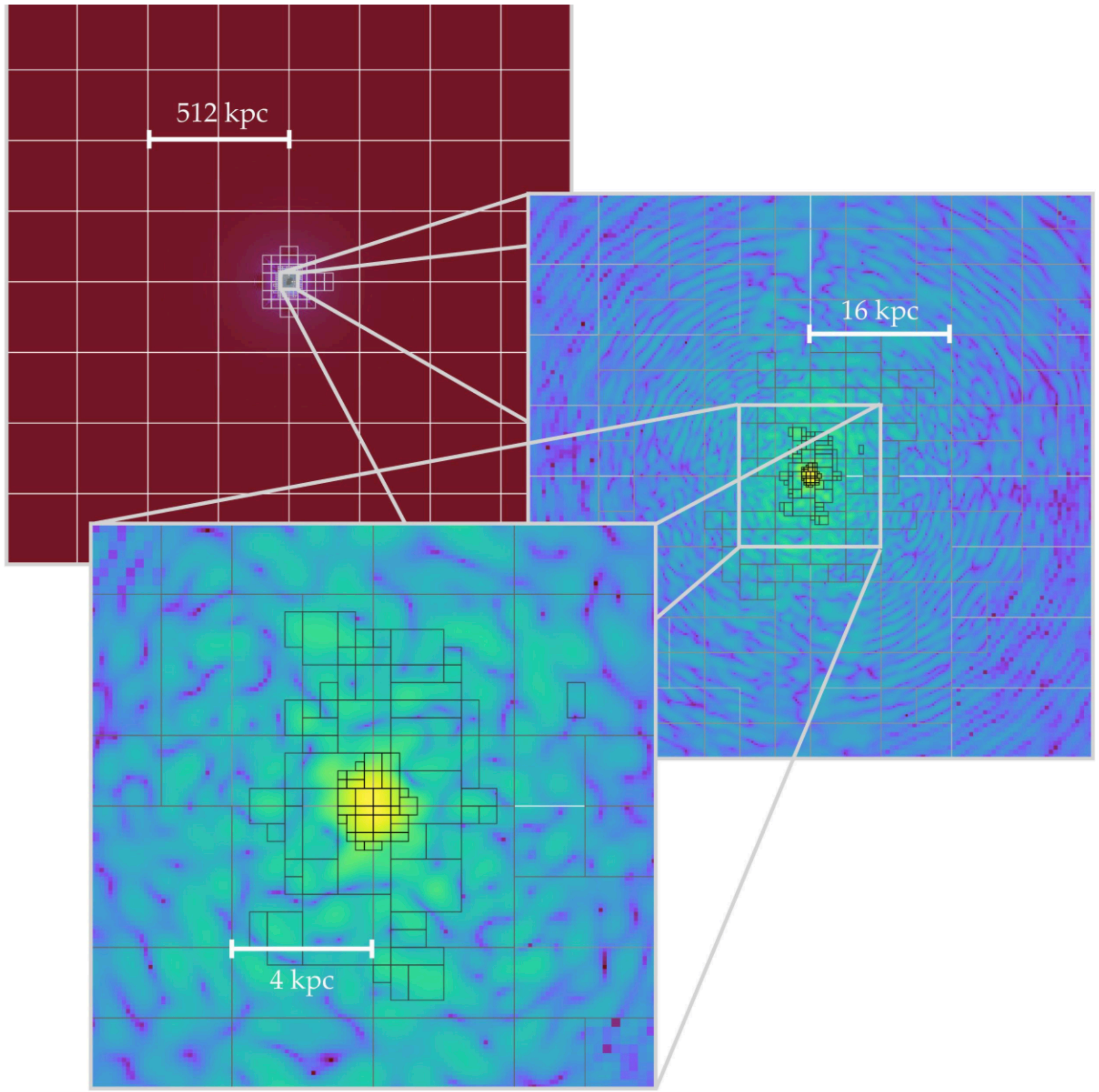
$$\frac{dI}{d\omega} = \frac{1}{R_0} \frac{\omega^2 (f_a R_0)^2}{8\pi} \int d\cos\theta \cos^2\theta \left| \int_0^{2\pi} d\sigma \int_0^{t_c} dt g(t) \dot{g}(t) e^{i\omega(t - \sin\theta \cos\sigma g(t))} \right|^2$$

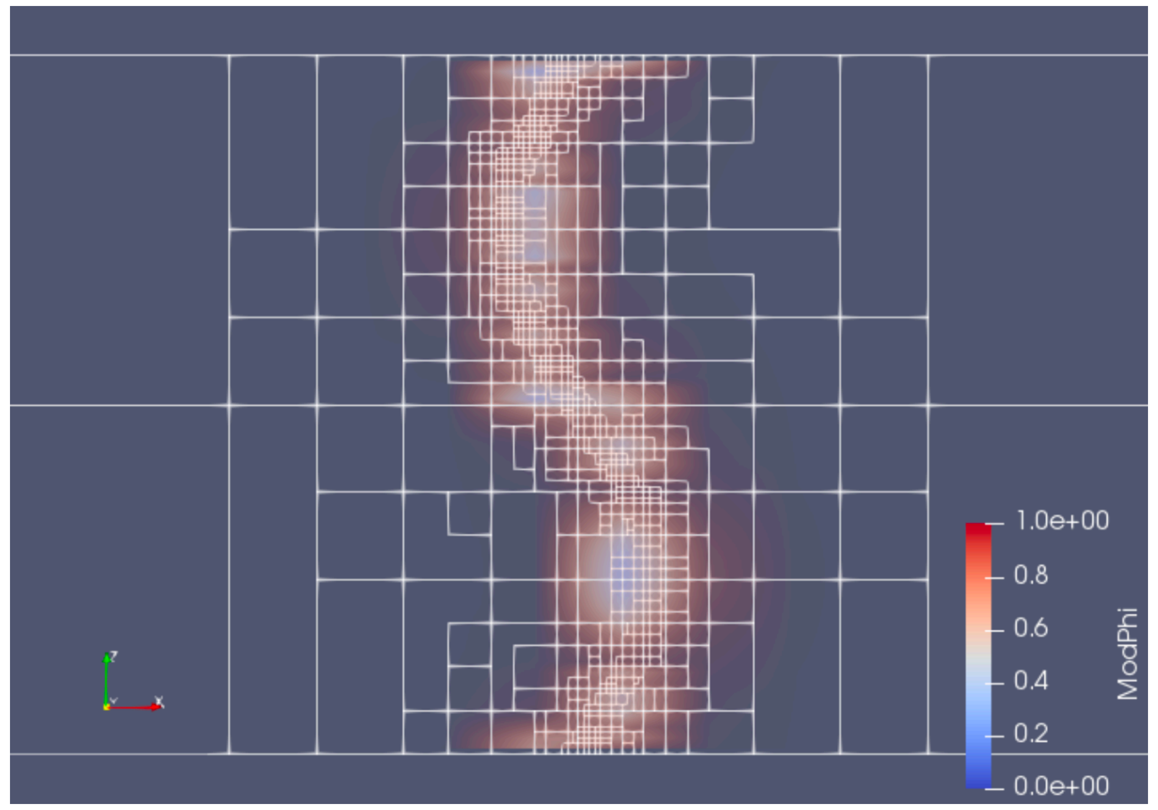
How can we refine these predictions?

— Numerical Setup —

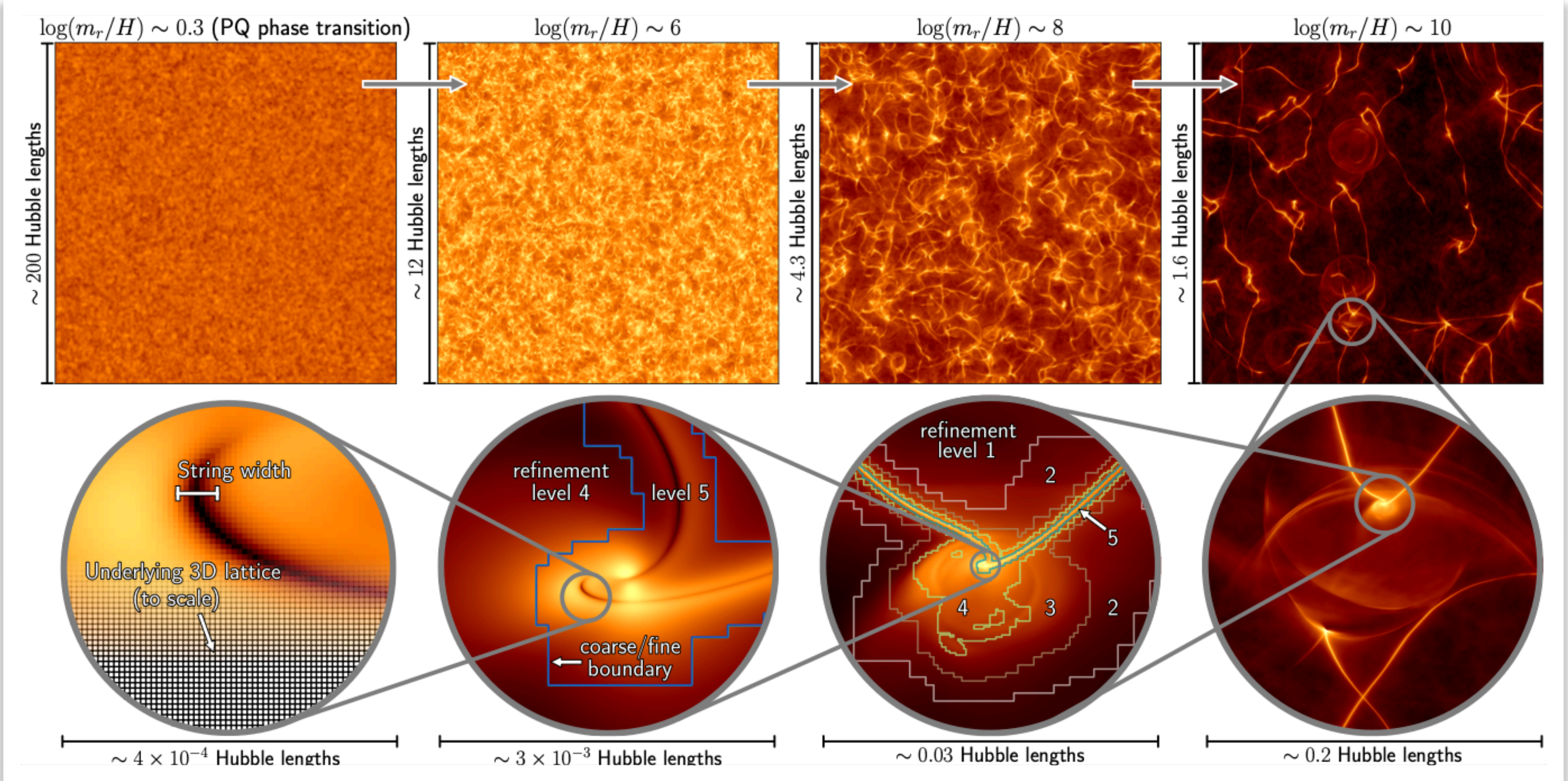
Adaptive Mesh Refinement (AMR)

- **Idea:** Focus computational power on specific parts of the grid
- Nowadays widely used in cosmological simulation codes, numerical relativity **and** in axion string simulations
- Current codes mostly based on AMReX





Drew & Shellard [1910.01718]
“GRChombo”



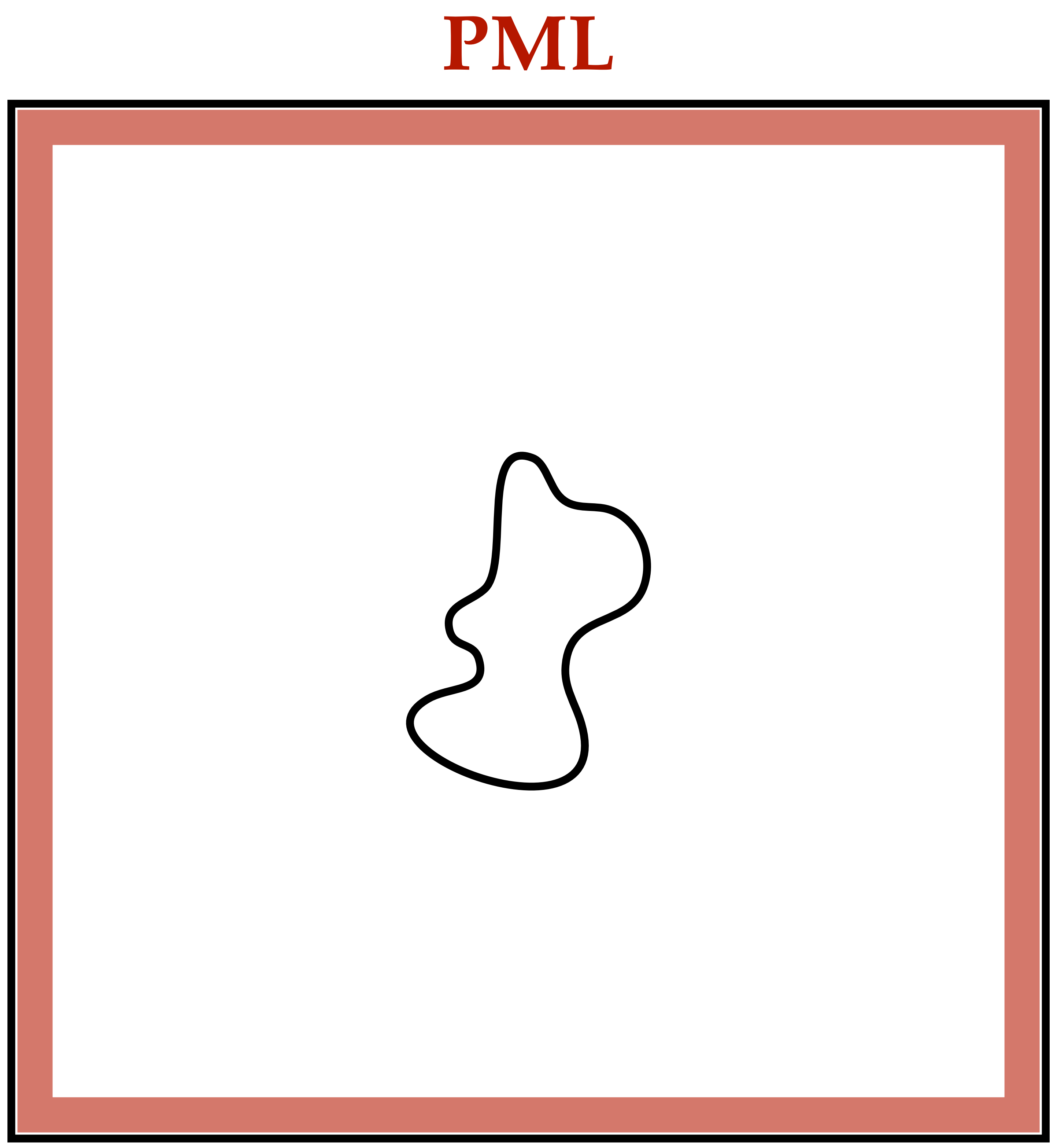
Buschmann+ [2412.08699]
“sledgehamr”

Schwabe+ [2007.08256]
“axioNyx”

Perfectly Matched Layers

- **Problem:** Simulations usually use periodic boundaries
- PMLs common tools in numerical wave problems
- **Idea:** Modify EoMs to effectively damp the outgoing radiation, preventing it from reentering the box:

$$\frac{\partial \Phi_1}{\partial t} = \Pi_1 - \sigma(\mathbf{x})\Phi_1$$
$$\frac{\partial \Phi_2}{\partial t} = \Pi_2 - \sigma(\mathbf{x})\Phi_2$$
$$\frac{\partial \Pi_1}{\partial t} = \mathcal{L}[\Phi_1] - V'(\Psi_1, \Phi_2) - \frac{2\Pi_1}{\eta} - \sigma(\mathbf{x})\Pi_1$$
$$\frac{\partial \Pi_2}{\partial t} = \mathcal{L}[\Phi_2] - V'(\Psi_1, \Phi_2) - \frac{2\Pi_2}{\eta} - \sigma(\mathbf{x})\Pi_2$$



$$\sigma(\mathbf{x}) = \sigma_{\max}\left(\frac{d_{\mathrm{PML}} - d(\mathbf{x})}{d_{\mathrm{PML}}}\right)^2$$

Perfectly Matched Layers

- Extract radiation spectrum by measuring outgoing flux through surface around string

$$P(t) = \oint_{\partial B} T^{0i}(\mathbf{x}, t) \cdot \hat{\mathbf{n}}_i$$

- Massless (*axion*) and massive (*saxion*) channel via decomposition of energy-momentum tensor

Massive

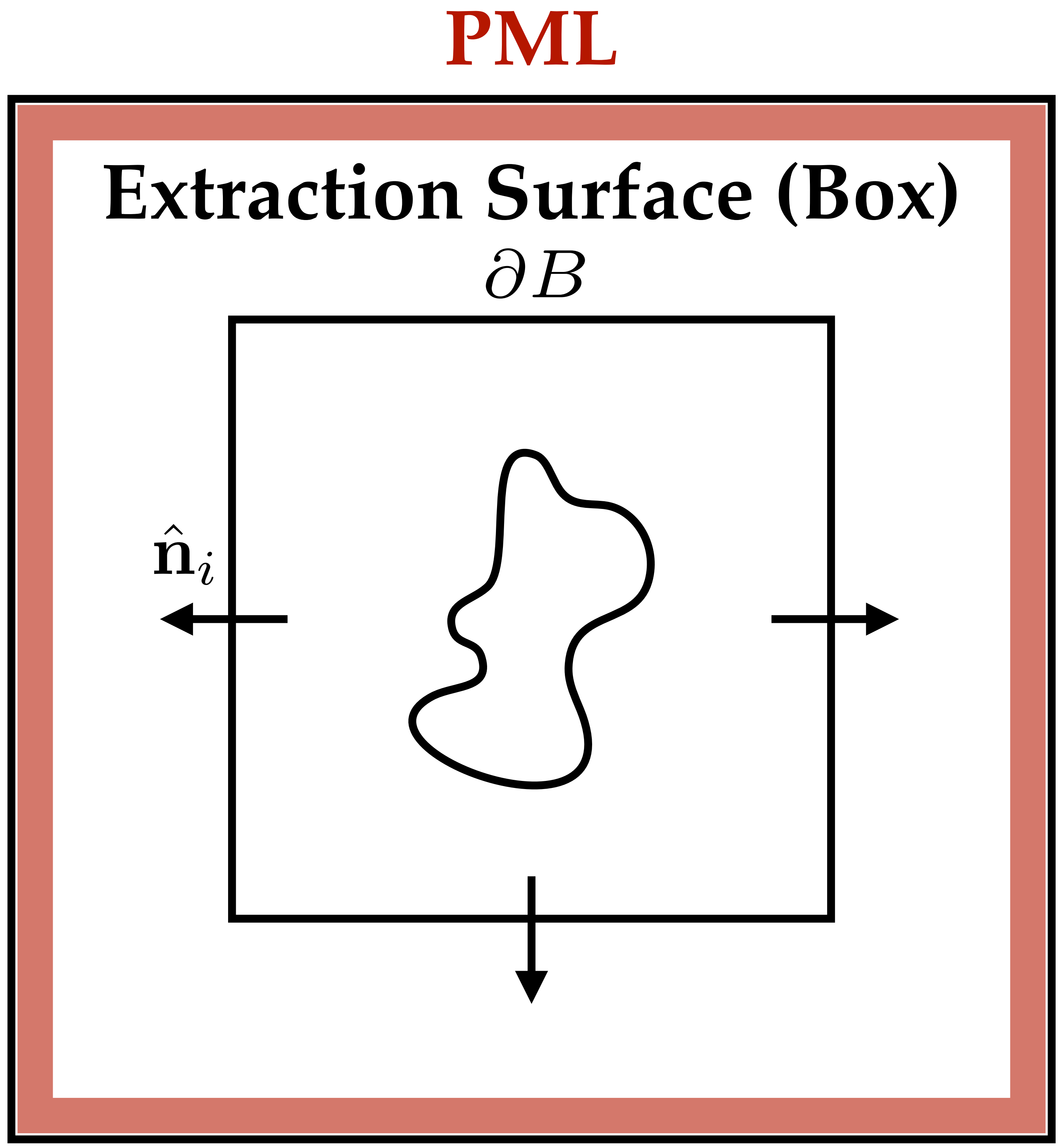
$$\Pi_\phi = \frac{\phi_1 \dot{\phi}_1 + \phi_2 \dot{\phi}_2}{\phi}$$

$$\mathcal{D}_i \phi = \frac{\phi_1 \nabla_i \phi_1 + \phi_2 \nabla_i \phi_2}{\phi}$$

Massless

$$\Pi_\theta = \frac{\phi_1 \dot{\phi}_2 - \phi_2 \dot{\phi}_1}{\phi}$$

$$\mathcal{D}_i \theta = \frac{\phi_1 \nabla_i \phi_2 - \phi_2 \nabla_i \phi_1}{\phi}$$



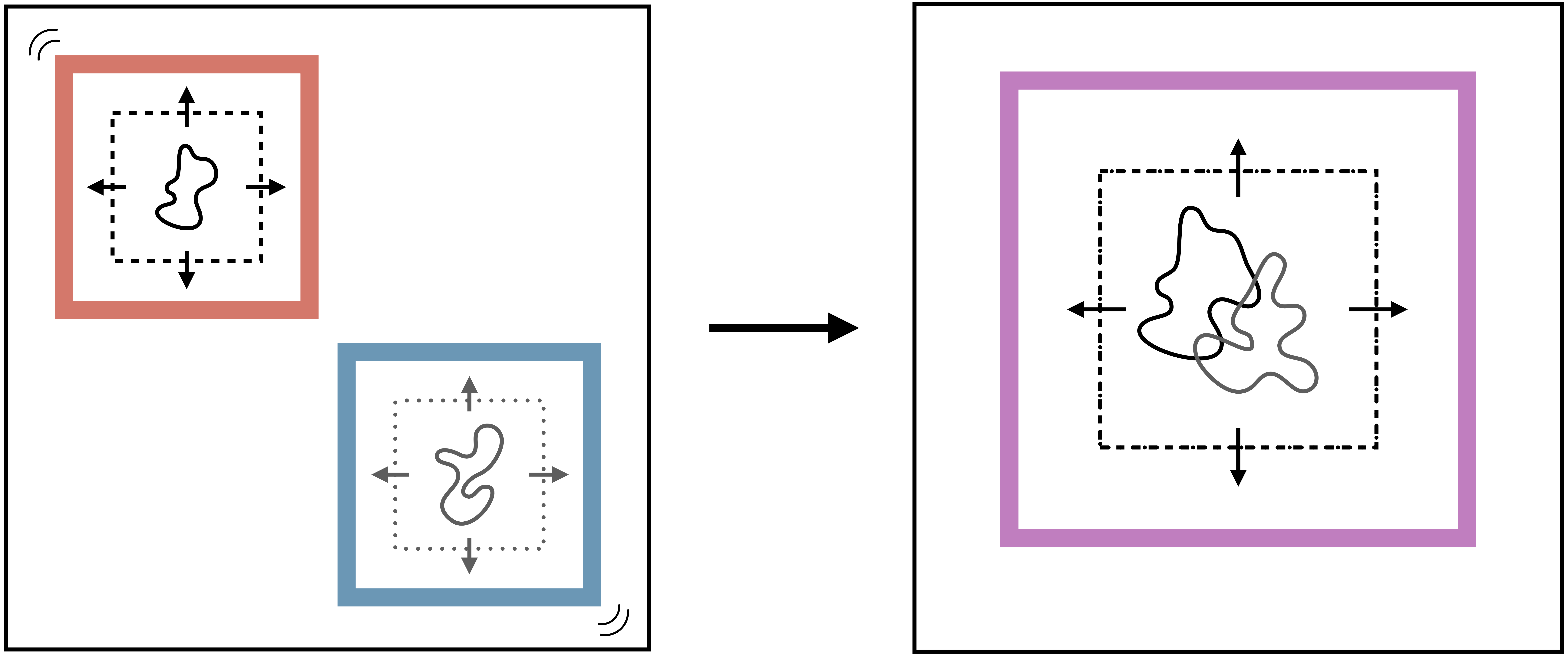
- Energy-momentum tensor components:

$$T^{00} = \Pi_\phi^2 + (\mathcal{D}\phi)^2 + \Pi_\theta^2 + (\mathcal{D}\theta)^2 + \frac{\lambda}{4}(\phi^2 - 1)^2, \qquad T^{0i} = 2(\Pi_\phi \mathcal{D}_i \phi + \Pi_\theta \mathcal{D}_i \theta)$$

e.g. Drew+ [1910.01718]

In an ideal world . . .

- Make this setup **dynamical** and handle complex cases such as collisions etc.



- Currently work in progress with Josh Foster at UW Madison

Conclusions Part II

- Understanding of global string dynamics is very important for a precise prediction of the **axion dark matter mass** in the post-inflationary scenario.

- Our most powerful network simulations predict $95\mu\mathrm{eV} \lesssim m_a \lesssim 450\mu\mathrm{eV}$

- There are several **systematic effects** that could bias the result, that could explain discrepancies in the literature:

  - **Initial conditions, axion field oscillations** & most importantly **discretisation effects**

- Further improvements in the d**ynamical range** and **spectrum measurements** are needed to make the required extrapolation trustworthy. This can be achieved for example with **Adaptive Mesh Refinement** & **Perfectly Matched Layers**.

- Additionally, **effective Kalb-Ramond theory for global strings** can reveal insights about the different constituents of the string network, but we are still facing some issues in the comparison of theory and simulations (stay tuned for updates!).

Conclusions Part II

- Understanding of global string dynamics is very important for a precise prediction of the **axion dark matter mass** in the post-inflationary scenario.
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Initial conditions, axion field oscillation [illegible], discretisation effects

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Thank you!

Any Questions?

Contact me: mkaltschmidt@unizar.es

Backup Slides

General

Axion Quality & Domain Wall Problem

cf. Lu+ [2312.07650v2]

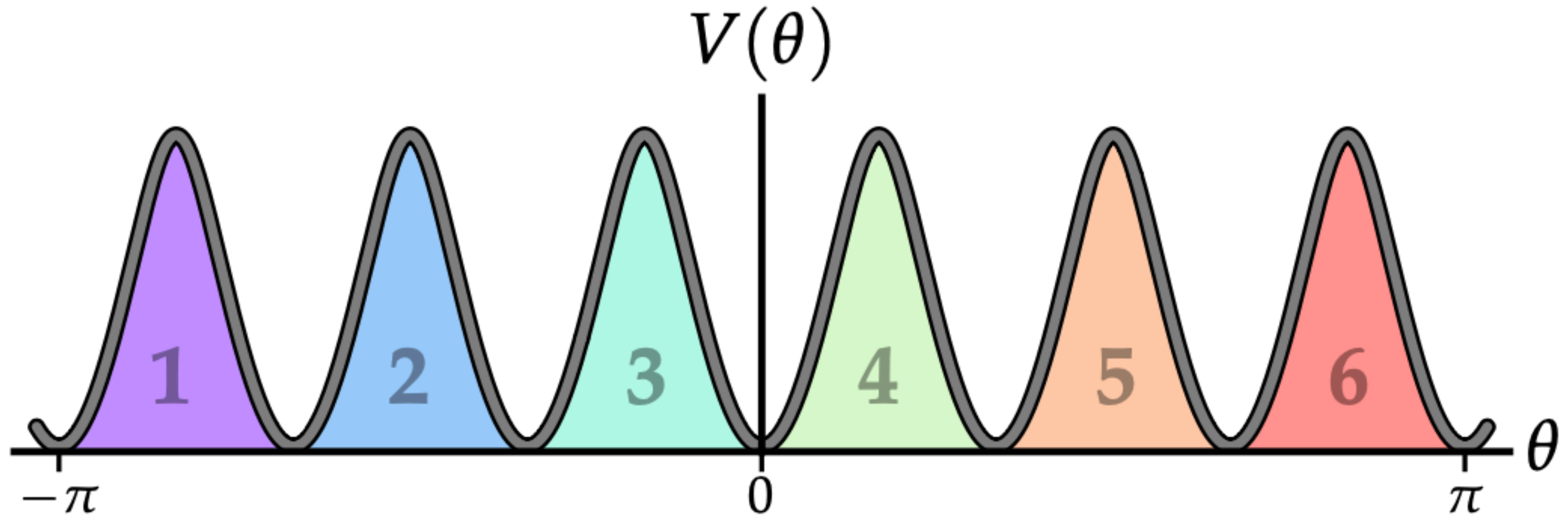
- $U(1)_{\mathrm{PQ}}$ as global symmetry is at best **approximate**.
- Tiny explicit breaking would spoil axion solution to strong CP problem!

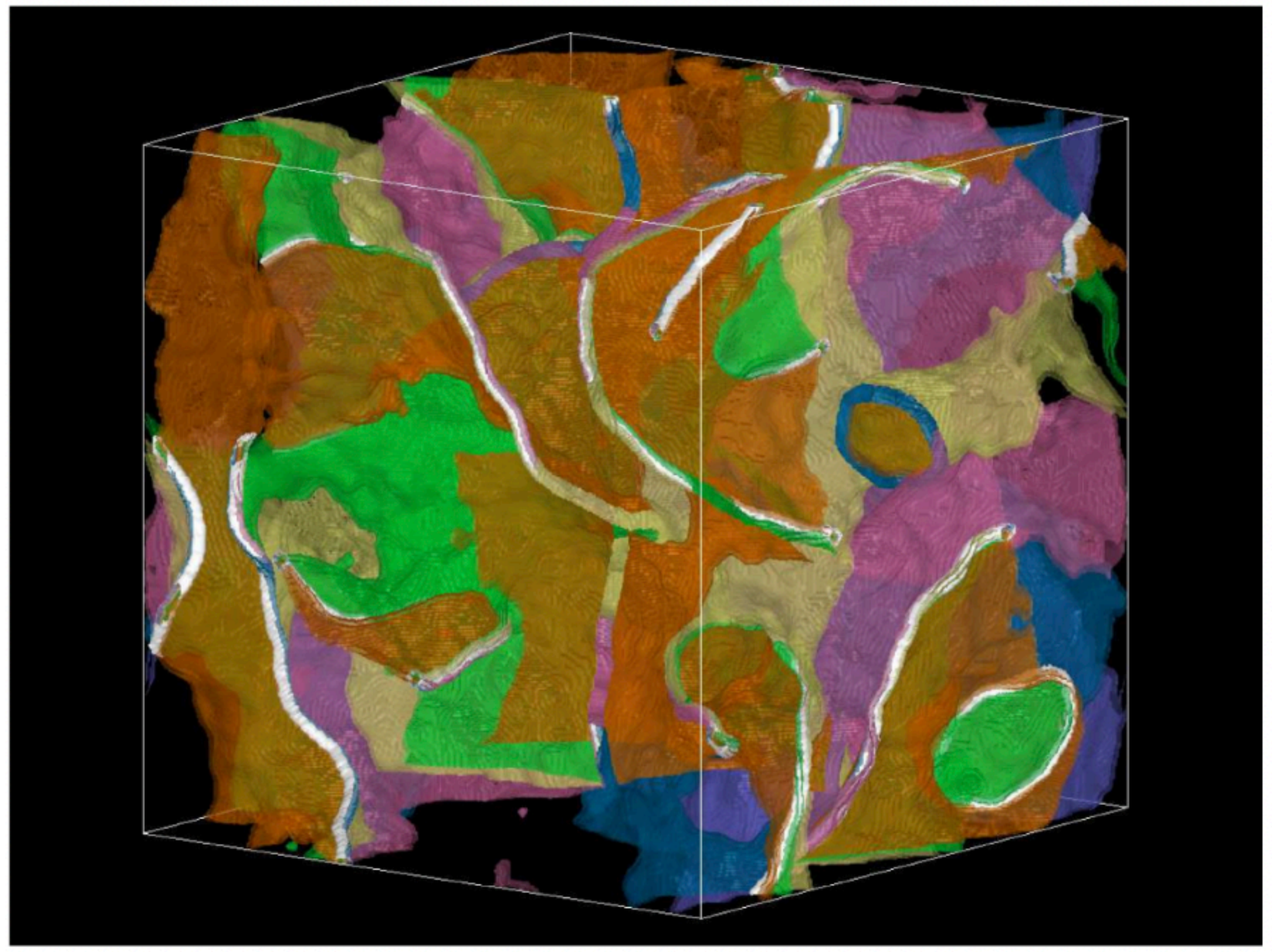
Shift of minimum away from 0!

$$\mathcal{L} \sim \frac{c}{M_{\mathrm{Pl}}^{n-4}} \phi^n \longrightarrow V(a) \sim M_{\mathrm{Pl}}^4 \left(\frac{f_a}{M_{\mathrm{Pl}}} \right)^n \cos\left(n \frac{a}{f_a} + \phi \right)$$

- Impose additional discrete $\mathbb{Z}_n$ symmetry: $N_{\mathrm{DW}} > 1$
- Phenomenologically difficult! Domain wall network gets stuck and overwhelms the cosmic energy density.

$$V(\theta) \approx -\chi(T) \cos(N_{\mathrm{DW}} \theta)$$





(e) $N_{\mathrm{DW}} = 6$

Hiramatsu+ [1207.3166]

More details on JCAP 10 (2024) 043

Departamento de Física Teórica Universidad Zaragoza

CAPA Centro de Astropartículas y Física de Altas Energías Universidad Zaragoza

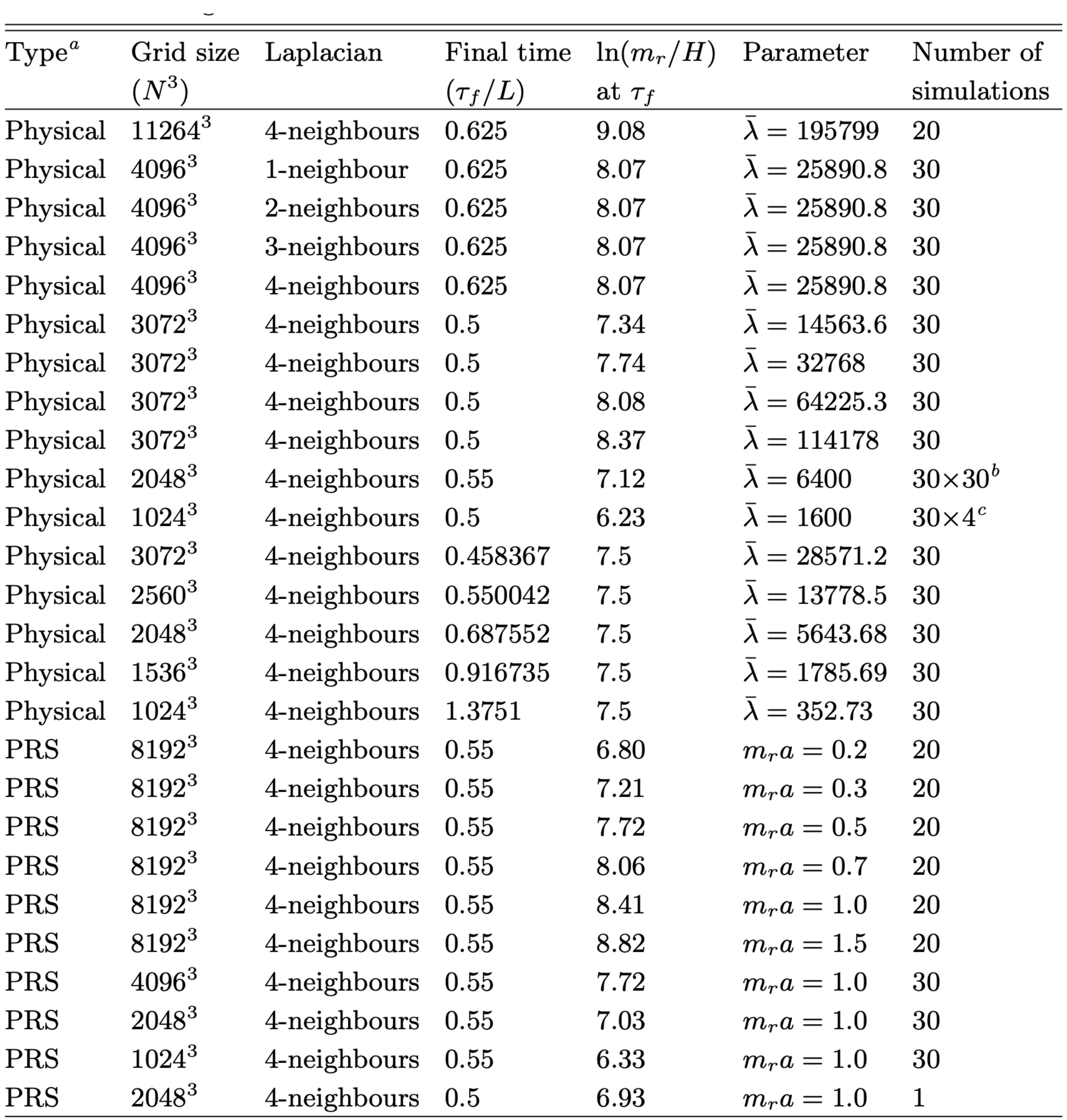
GOBIERNO DE ARAGON

WISCONSIN UNIVERSITY OF WISCONSIN–MADISON

WIPAC Wisconsin IceCube Particle Astrophysics Center

Simulation Overview

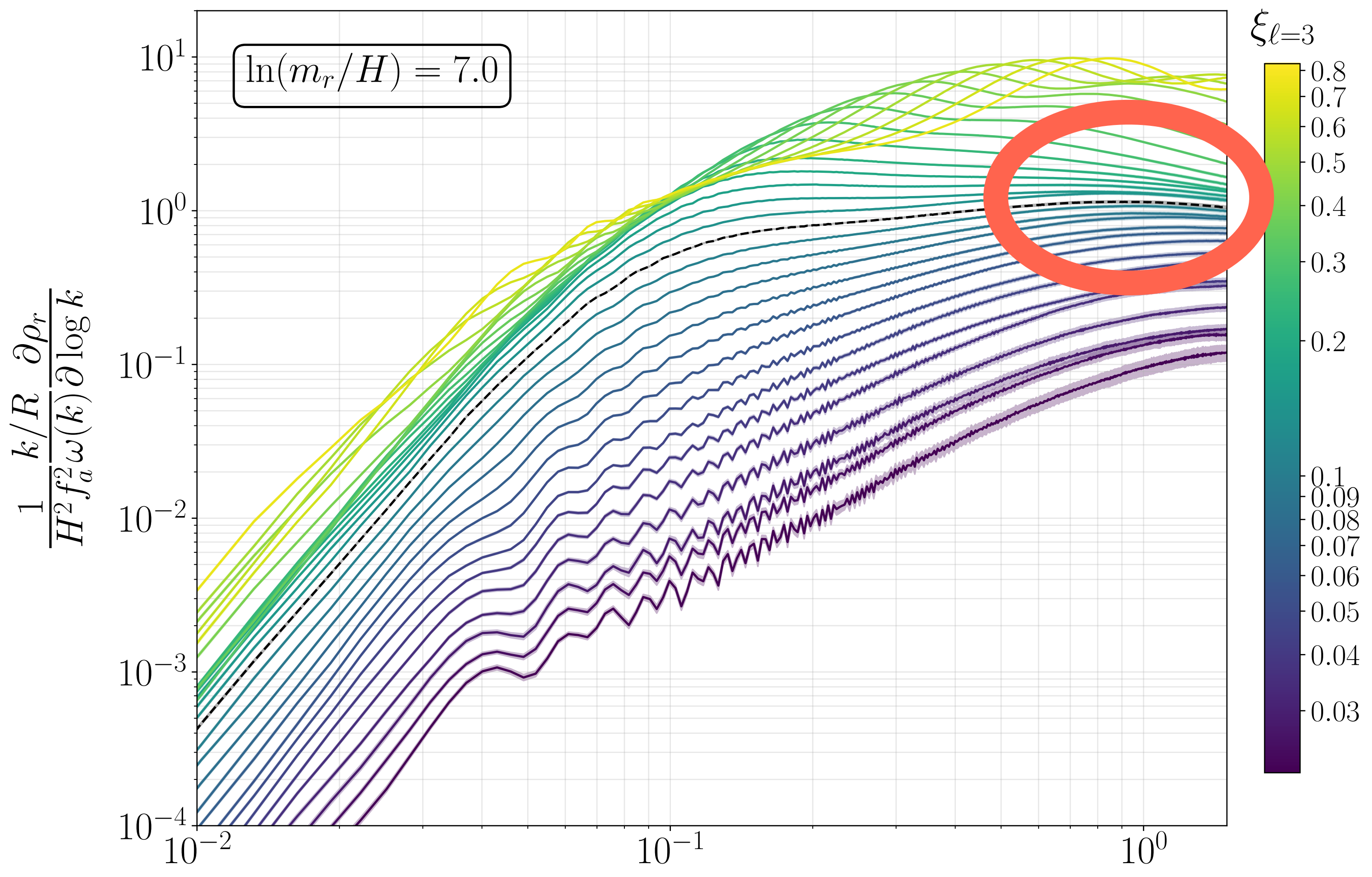
- More than 1500 simulations performed at
  - RAVEN and COBRA supercomputers at Max Planck Computing and Data Facility (MPCDF)
  - SQUID supercomputer at Cybermedia Center, Osaka University
- Box sizes of up to 11.264^3 (256 CPU nodes)

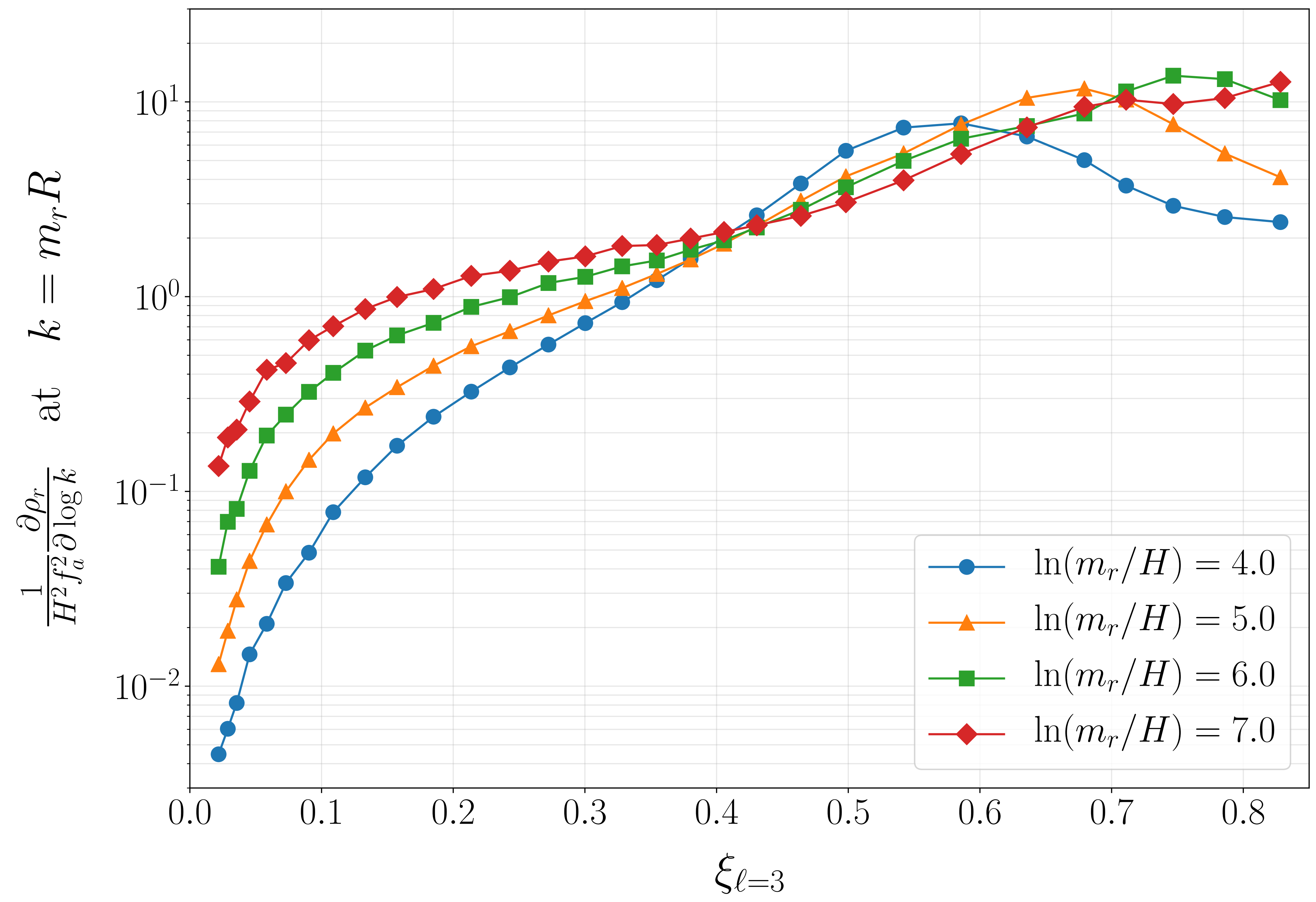


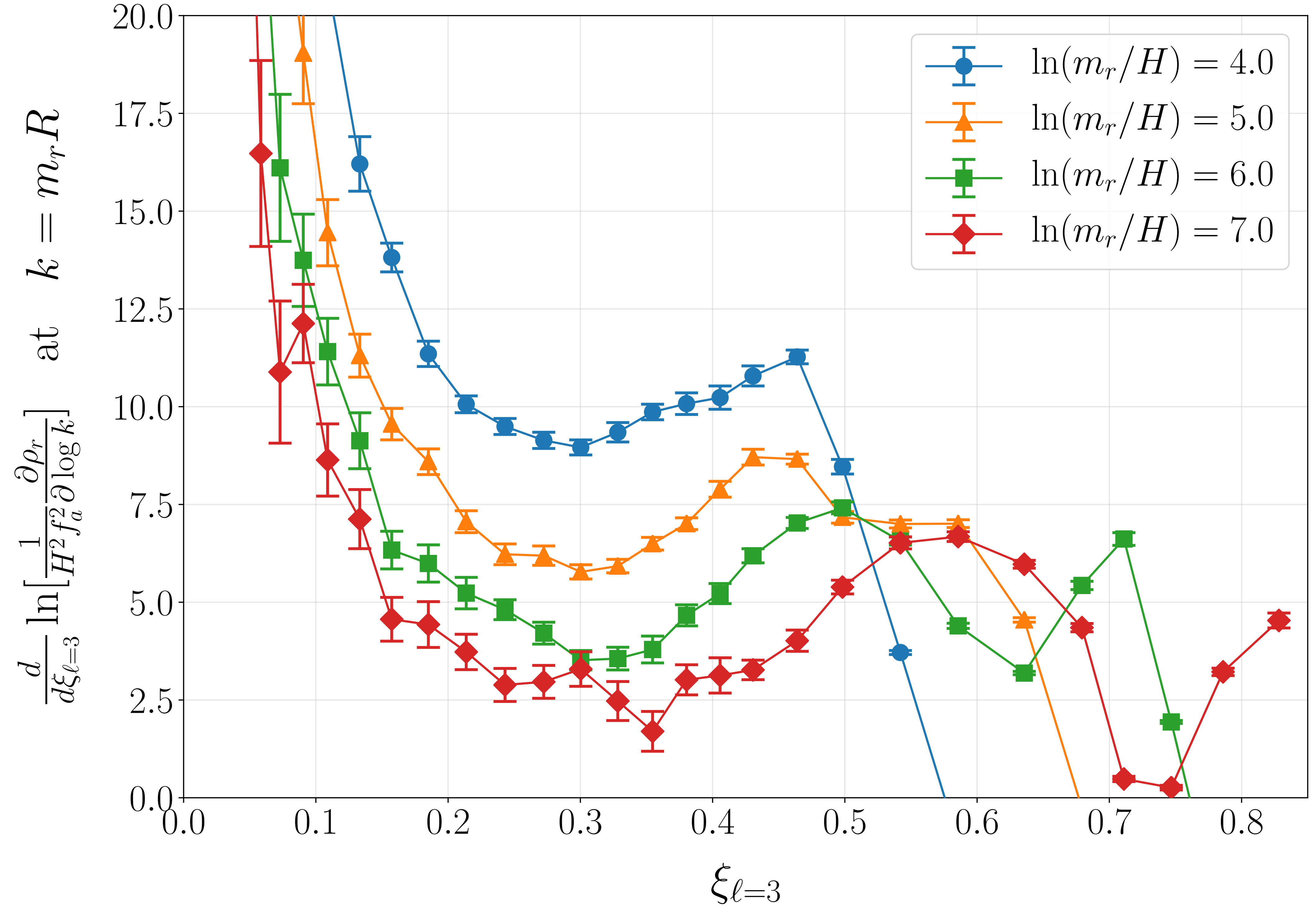
Type[a]	Grid size (N^3)	Laplacian	Final time (τ_f/L)	$\ln(m_r/H)$ at τ_f	Parameter	Number of simulations
Physical	11264^3	4-neighbours	0.625	9.08	$\bar{\lambda} = 195799$	20
Physical	4096^3	1-neighbour	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	2-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	3-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	4-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	3072^3	4-neighbours	0.5	7.34	$\bar{\lambda} = 14563.6$	30
Physical	3072^3	4-neighbours	0.5	7.74	$\bar{\lambda} = 32768$	30
Physical	3072^3	4-neighbours	0.5	8.08	$\bar{\lambda} = 64225.3$	30
Physical	3072^3	4-neighbours	0.5	8.37	$\bar{\lambda} = 114178$	30
Physical	2048^3	4-neighbours	0.55	7.12	$\bar{\lambda} = 6400$	30×30[b]
Physical	1024^3	4-neighbours	0.5	6.23	$\bar{\lambda} = 1600$	30×4[c]
Physical	3072^3	4-neighbours	0.458367	7.5	$\bar{\lambda} = 28571.2$	30
Physical	2560^3	4-neighbours	0.550042	7.5	$\bar{\lambda} = 13778.5$	30
Physical	2048^3	4-neighbours	0.687552	7.5	$\bar{\lambda} = 5643.68$	30
Physical	1536^3	4-neighbours	0.916735	7.5	$\bar{\lambda} = 1785.69$	30
Physical	1024^3	4-neighbours	1.3751	7.5	$\bar{\lambda} = 352.73$	30
PRS	8192^3	4-neighbours	0.55	6.80	$m_r a = 0.2$	20
PRS	8192^3	4-neighbours	0.55	7.21	$m_r a = 0.3$	20
PRS	8192^3	4-neighbours	0.55	7.72	$m_r a = 0.5$	20
PRS	8192^3	4-neighbours	0.55	8.06	$m_r a = 0.7$	20
PRS	8192^3	4-neighbours	0.55	8.41	$m_r a = 1.0$	20
PRS	8192^3	4-neighbours	0.55	8.82	$m_r a = 1.5$	20
PRS	4096^3	4-neighbours	0.55	7.72	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.55	7.03	$m_r a = 1.0$	30
PRS	1024^3	4-neighbours	0.55	6.33	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.5	6.93	$m_r a = 1.0$	1

Finding the Attractor

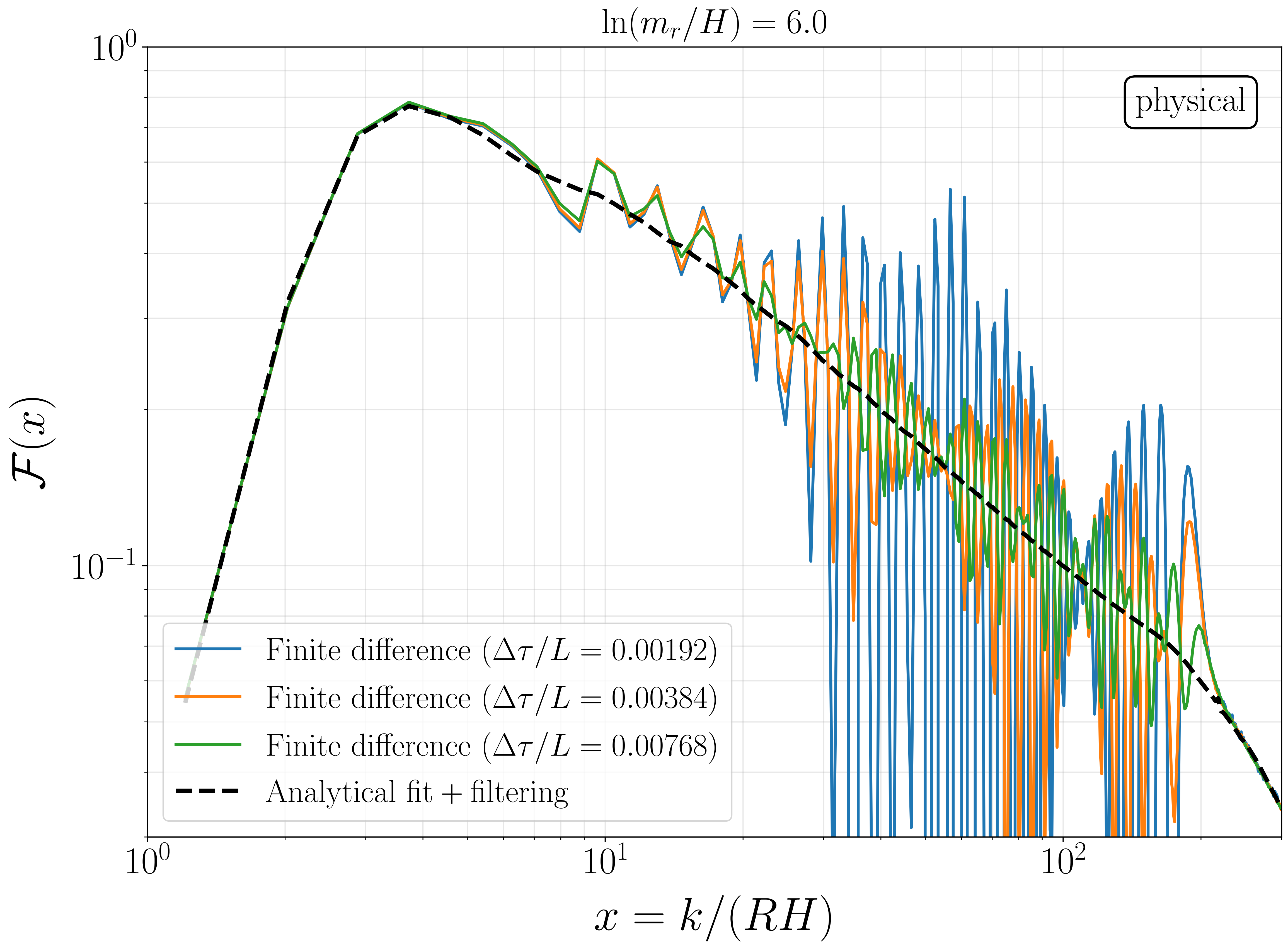
- Convergence behaviour is best observed in radial field spectra.
- Saxions are produced at $k/R \approx m_r$.
- Look for point that is least sensitive to ICs.





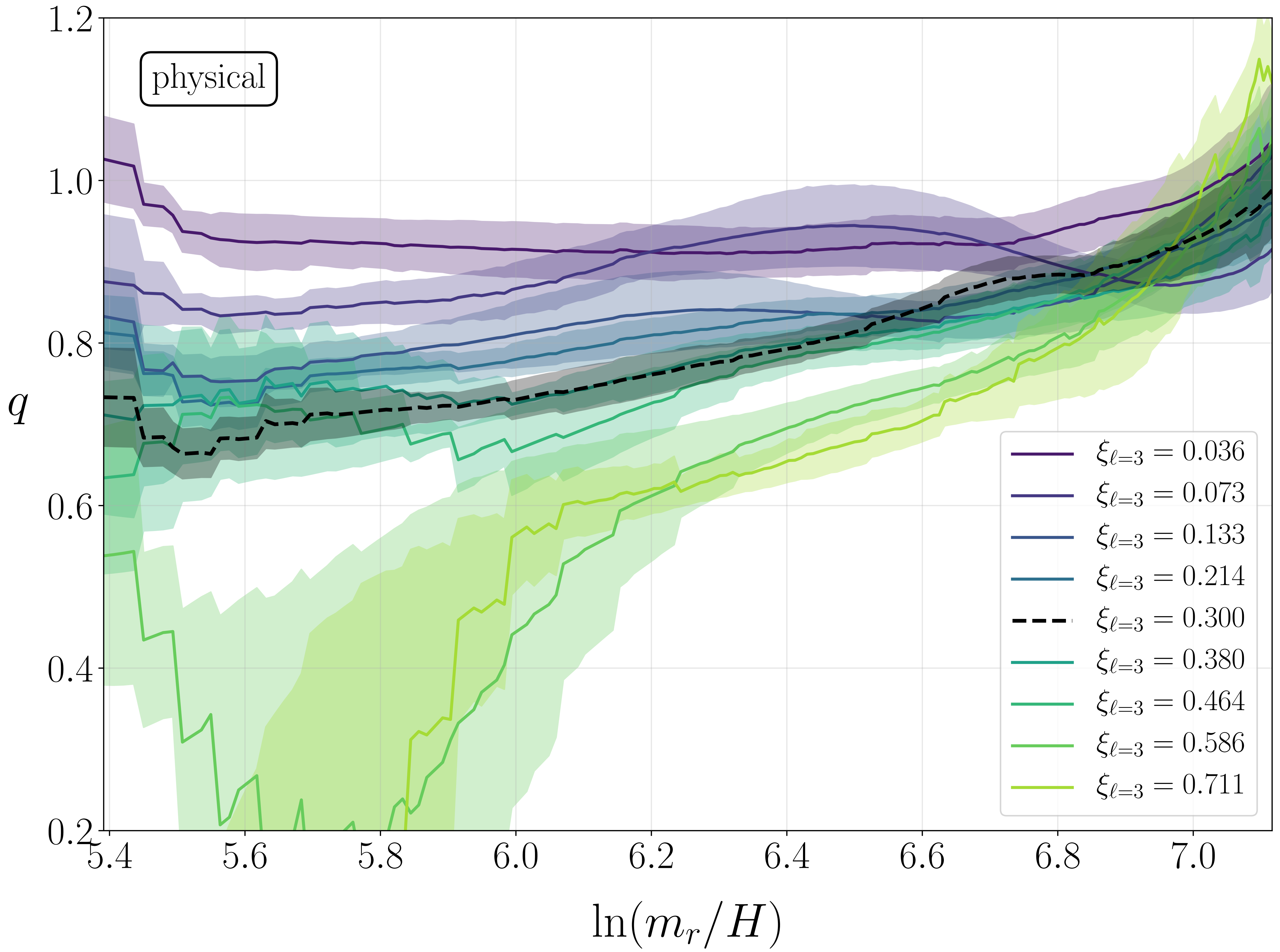


Calculation of the Instantaneous Spectrum



- Simple finite difference leads to a lot of contaminations from axion field oscillations.
- One can reduce them by applying a filter to remove high frequency components in the mode evolution data.

Initial Conditions

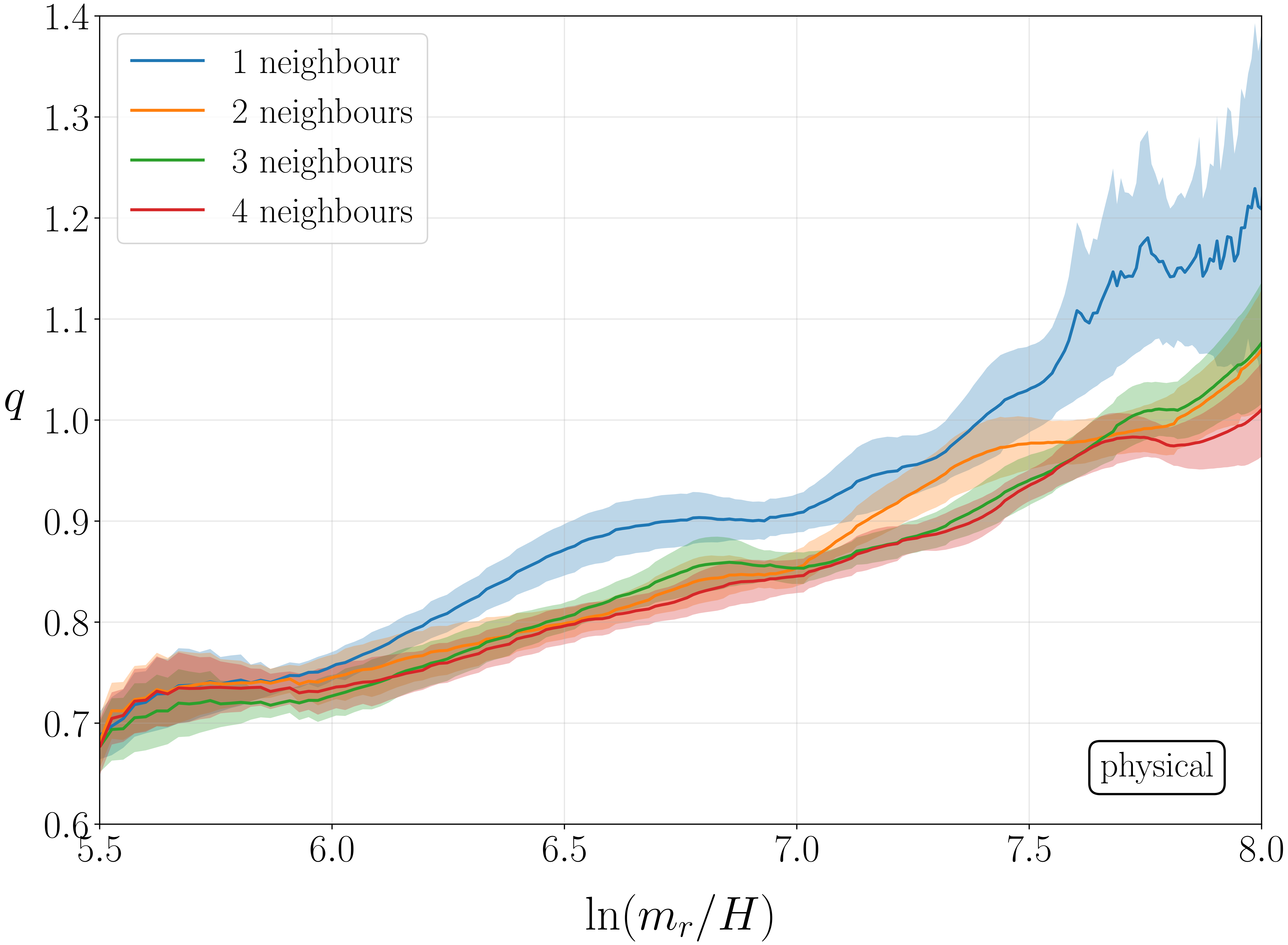


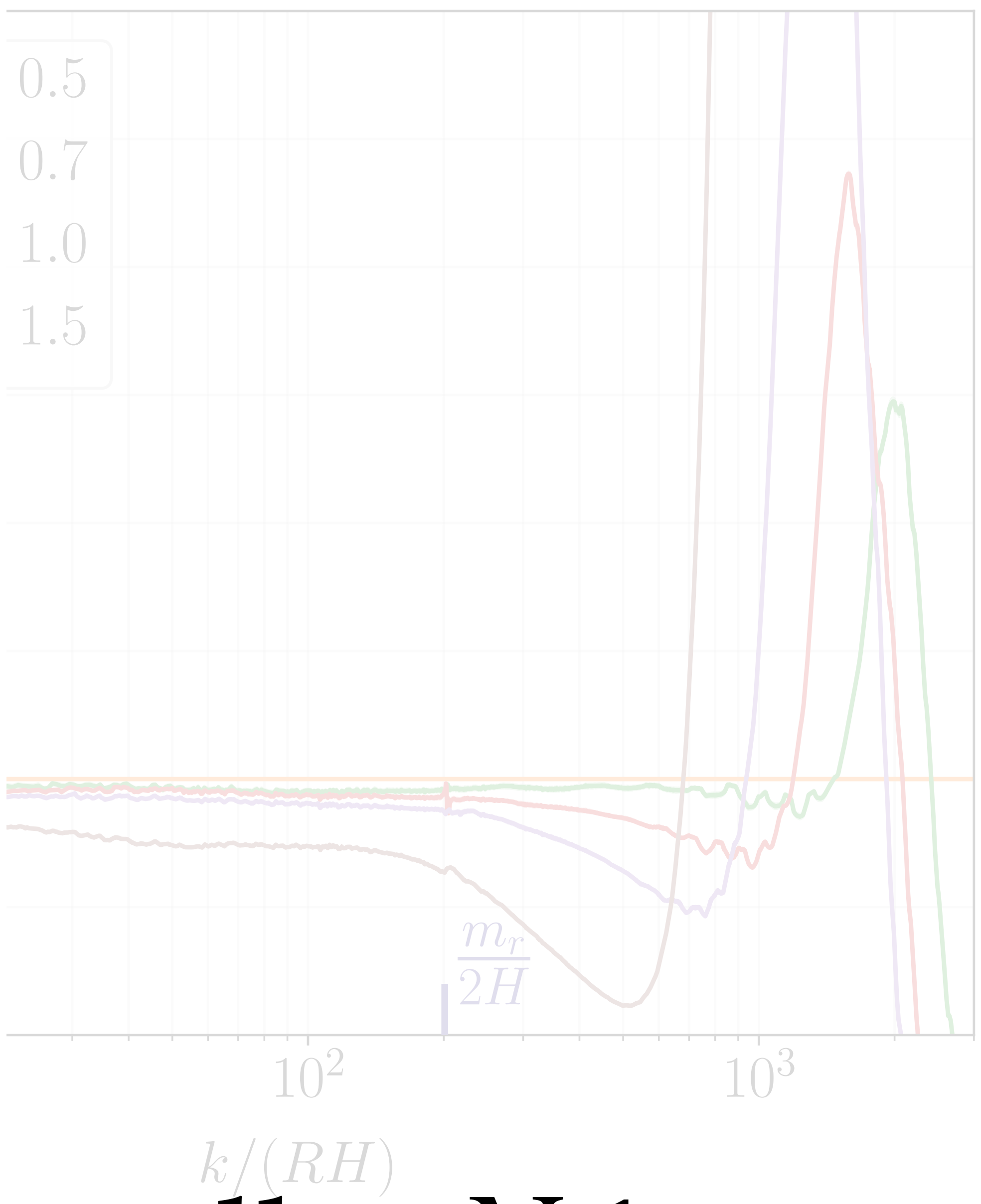
- Differences in the initial string density affect the slope of the radiation spectrum.
- Overdense (underdense) initial conditions could bias the estimation of q towards lower (higher) values.

Discretisation Effects

Effects that can b

- Discretisati
- Resolution

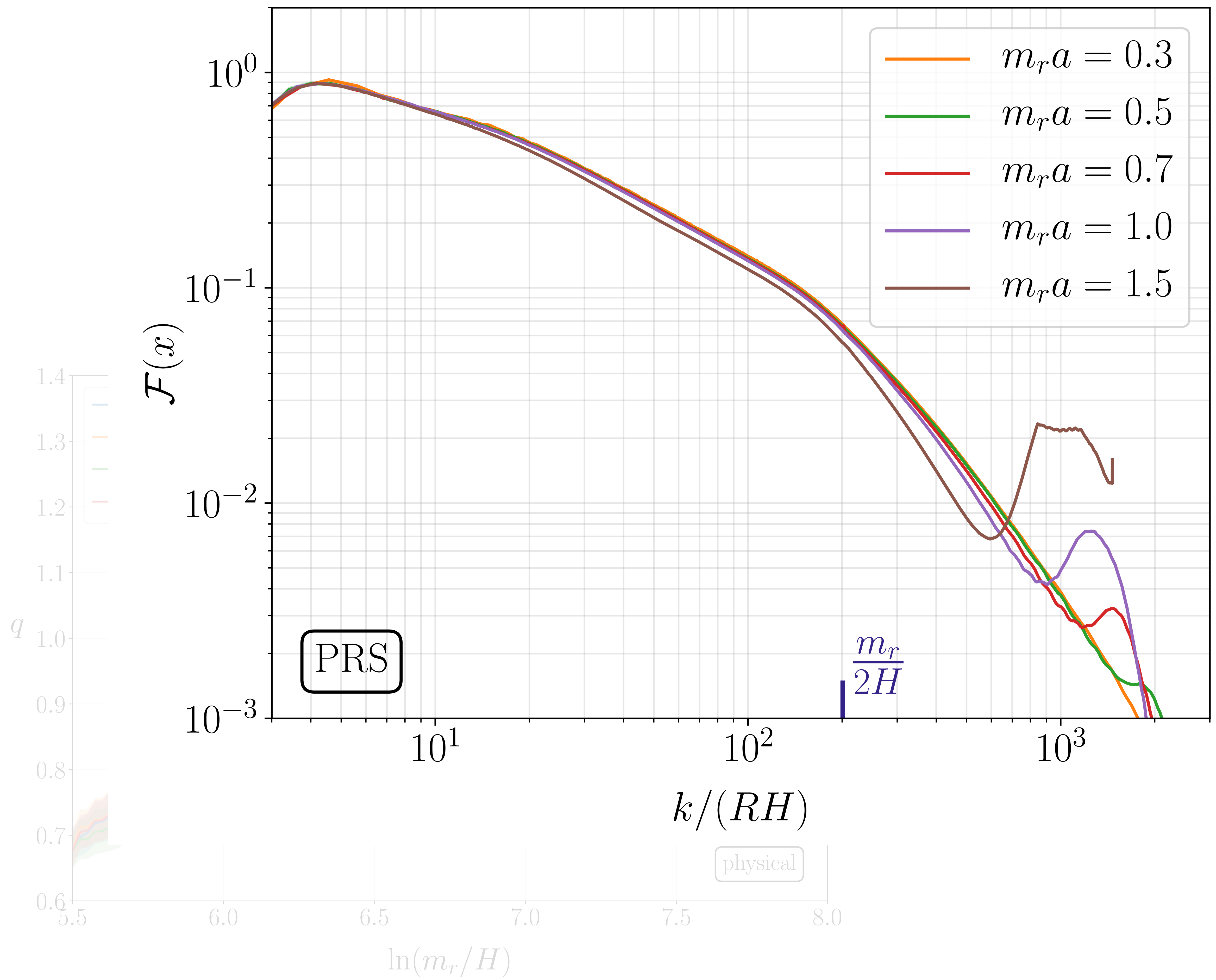


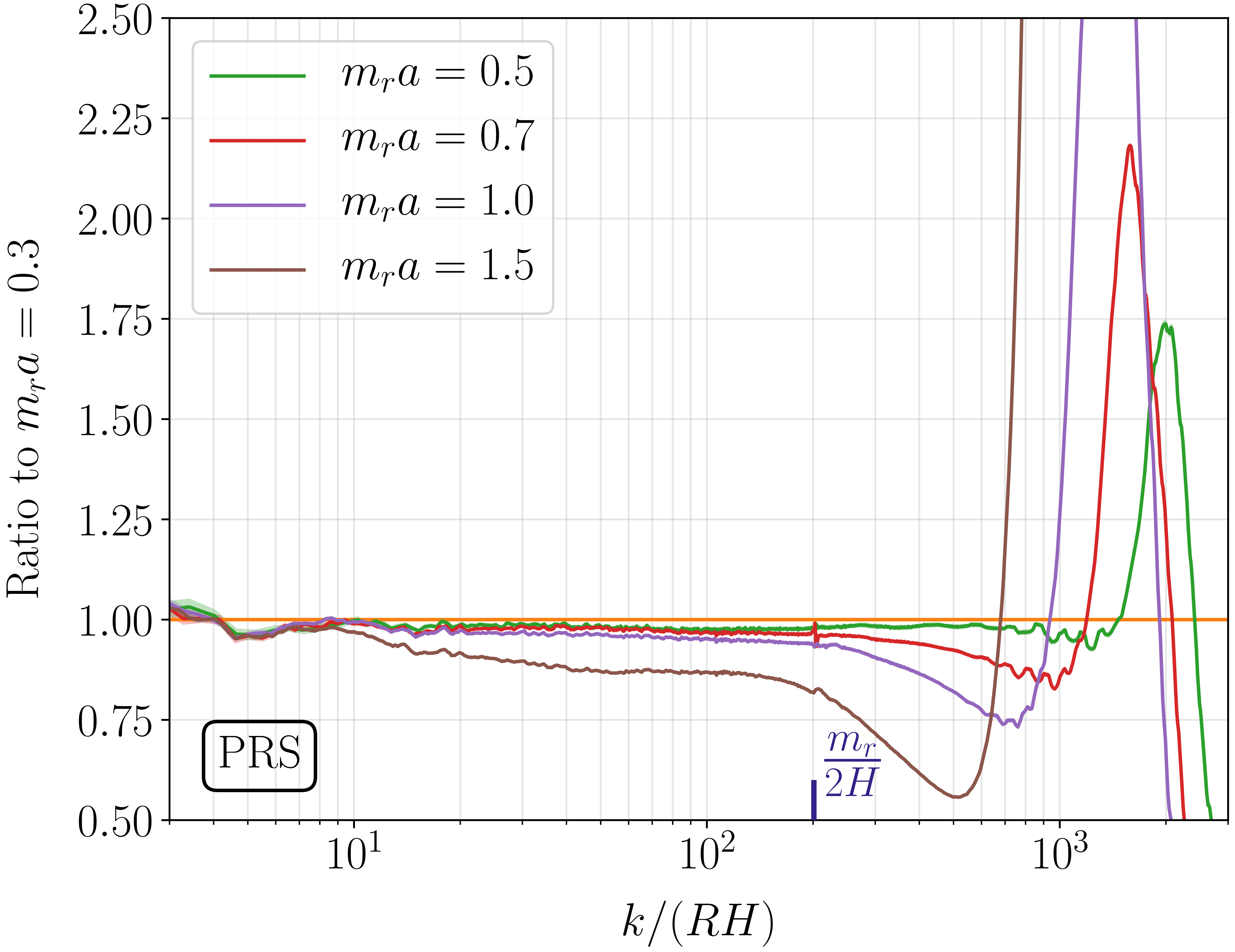


We observe that the value of q can be **overestimated** for smaller N_g!

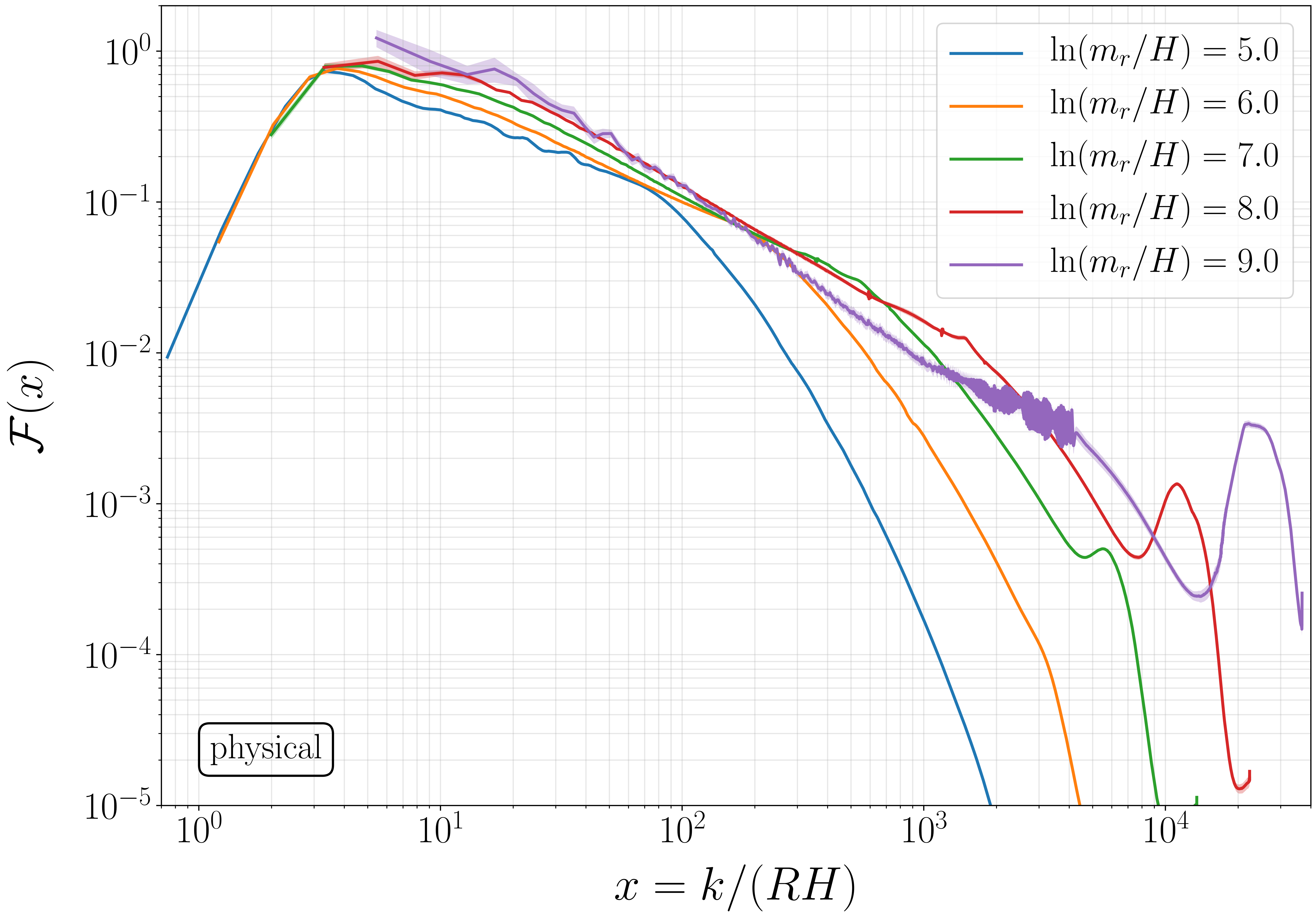
Discretisation Effects

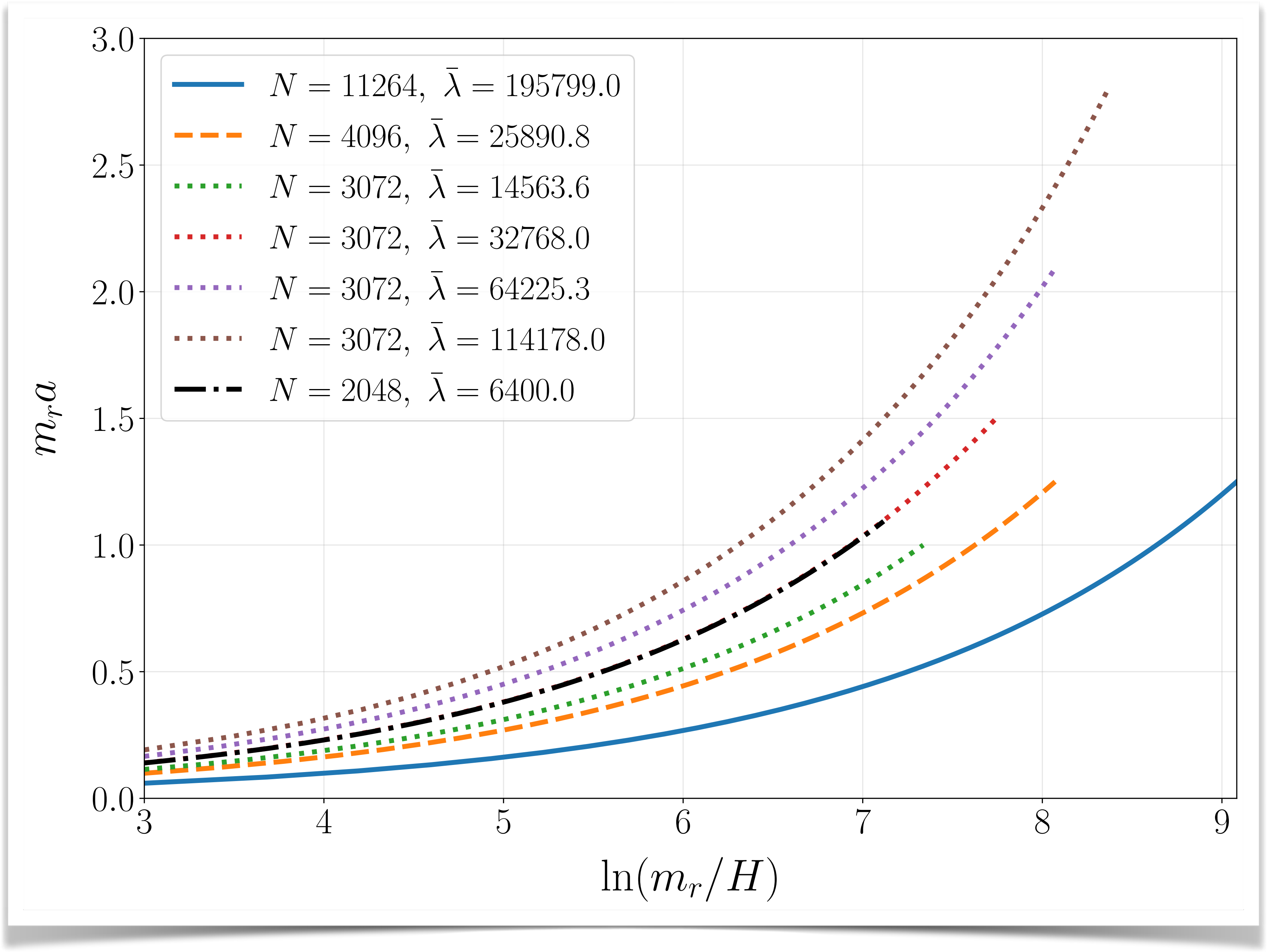
Effects that can bias q towards larger values:





Discretisation Effects



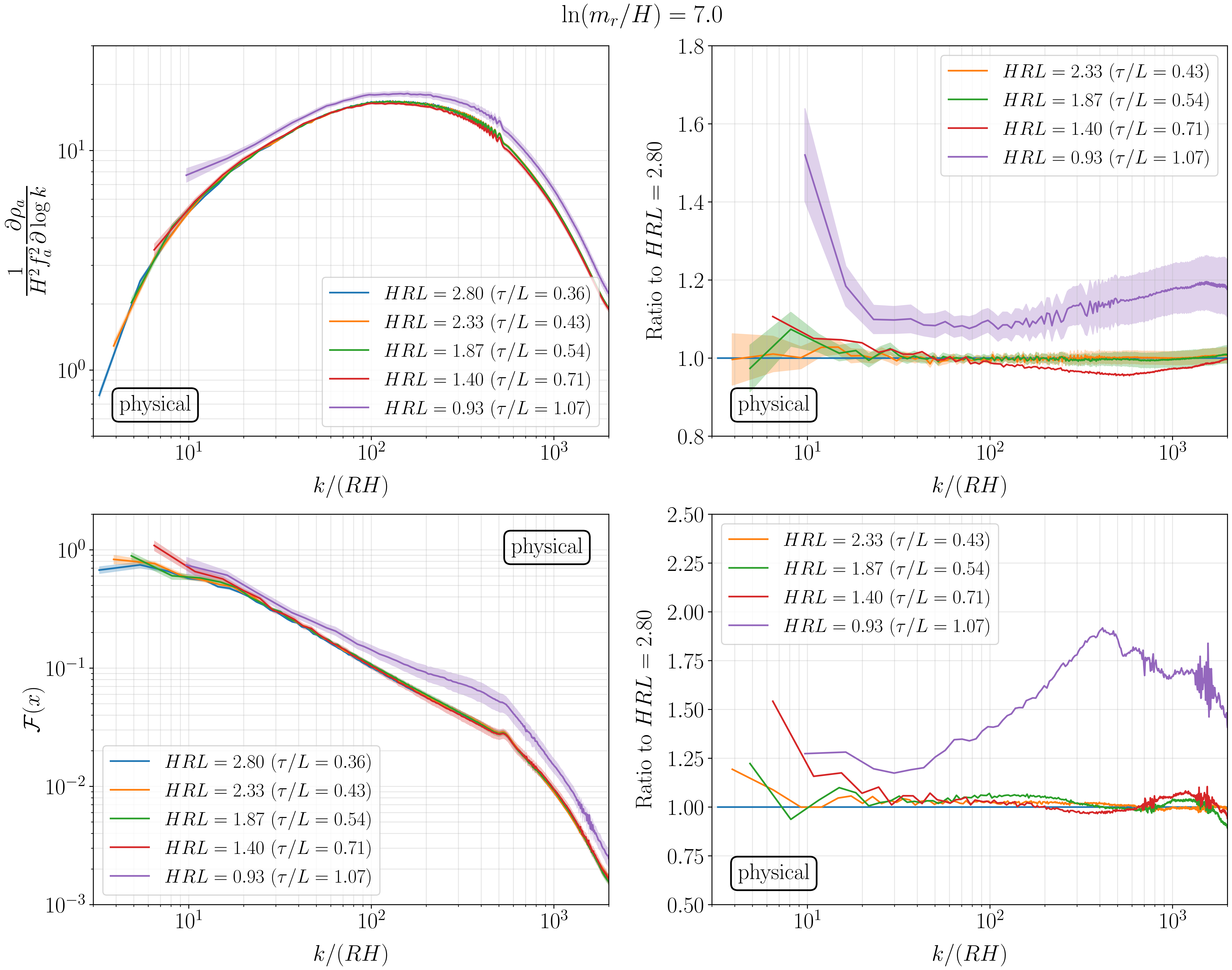


Effects increase drastically at larger $\log(m_r/H)$, leading to a significant distortion of the spectrum.

Finite Volume Effects

- Fix $m_r a = 1.0$ and vary ratio of phys. box size RL to Hubble radius H^{-1} at $\ln(m_r/H) = 7$
- Results converge for $HRL \gtrsim 1.4$ (or $\tau/L \lesssim 0.7$)
- We terminate the simulations at $\tau/L \leq 0.625$

Should **not** be a problem!



Technicalities

Masking the Spectrum

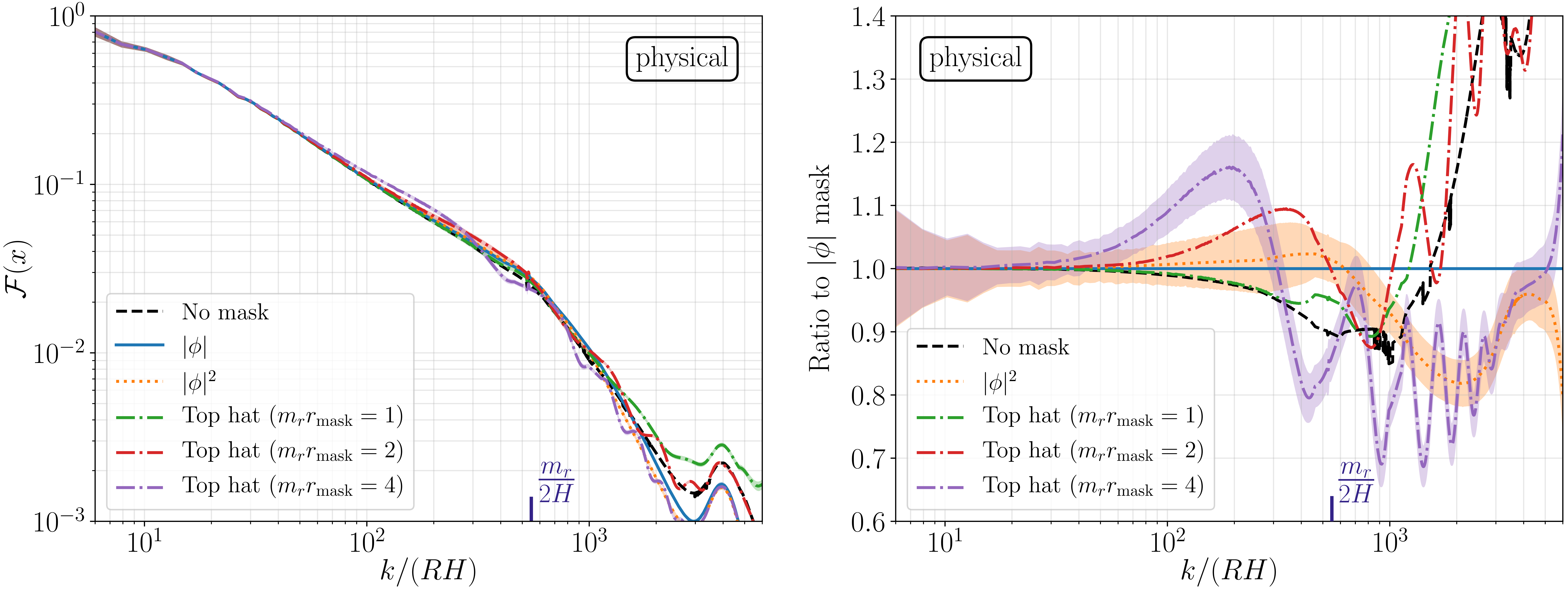
- To try to mitigate the contamination from the string core, we can introduce masks to to compute derivatives:

$$\dot{X}^{\text{mask}}(\boldsymbol{x}) = M(\boldsymbol{x})\dot{X}(\boldsymbol{x})$$

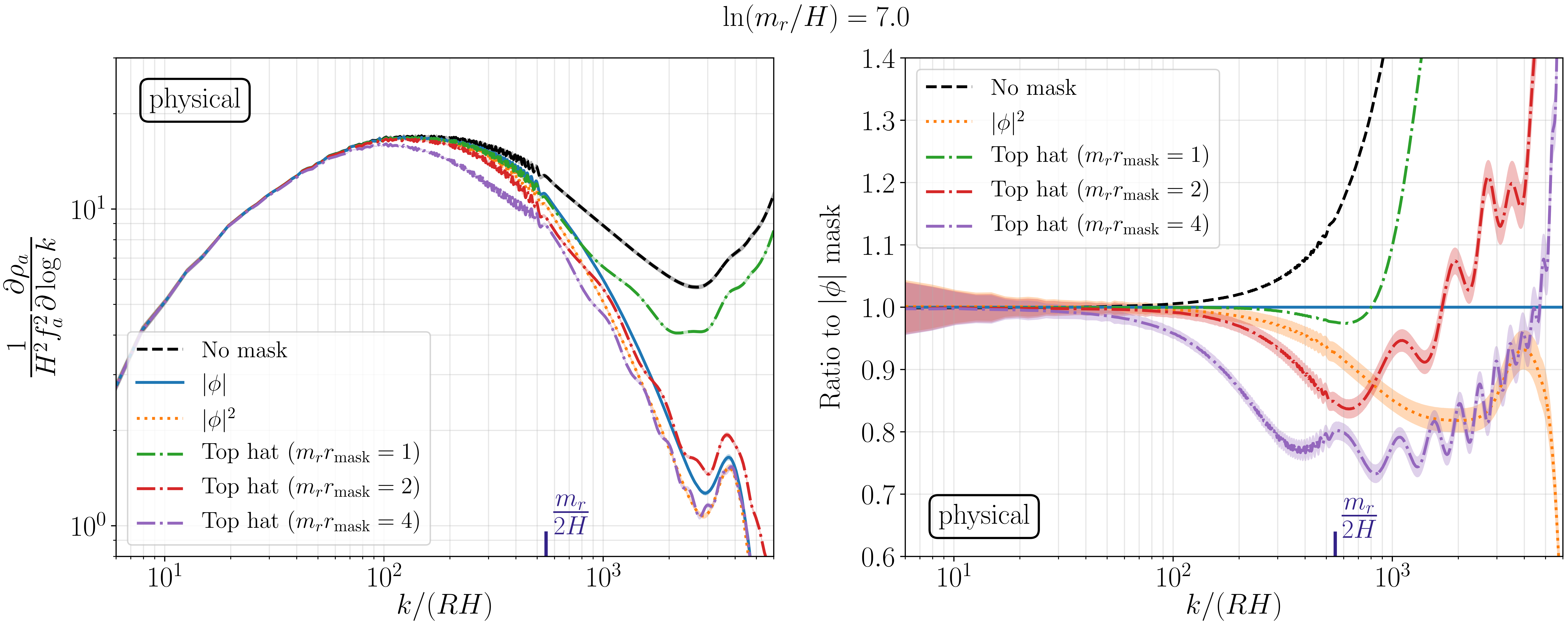
- Simple choice is to use the fact that the value of the radial field $|\phi|$ is zero inside the core.

$$M(x) = \left(\frac{|\phi(x)|}{f_a}\right)^k$$

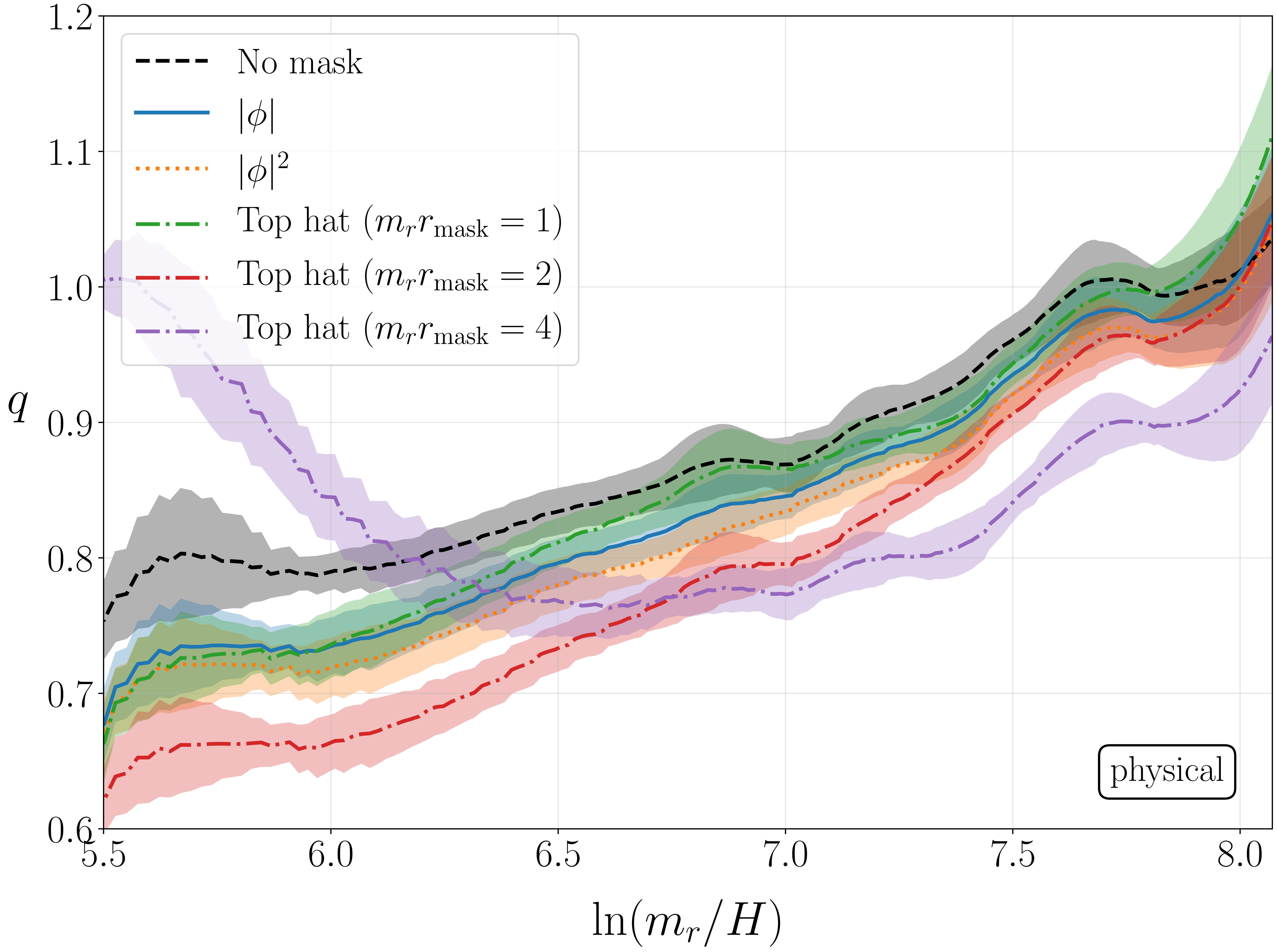
Masking the Spectrum



Masking the Spectrum



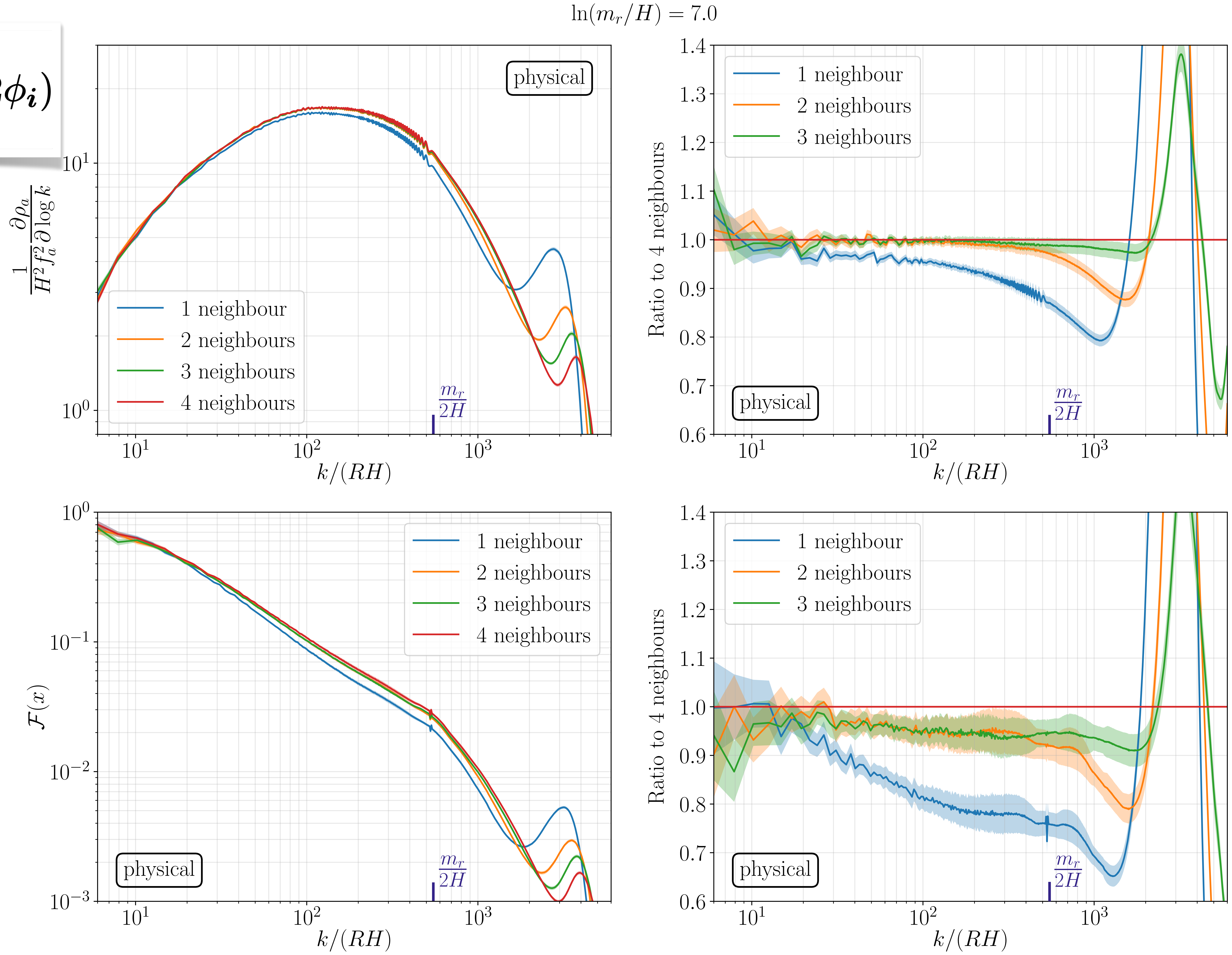
Masking the Spectrum



Discretisation of the Laplacian

$$(\nabla^2\phi)_{\boldsymbol{i}} = \frac{1}{\delta^2}\sum_{u=x,y,z}\sum_{n=1}^{N_g} C_n(\phi_{\boldsymbol{i}+n\boldsymbol{n}_u} + \phi_{\boldsymbol{i}-n\boldsymbol{n}_u} - 2\phi_{\boldsymbol{i}})$$

- Spectrum **underestimated** at intermediate momenta for smaller N_g
- Observation of peak-like structure in the UV, height related to N_g



Dynamical Range + AMR

Constructing the Axion Field around Strings

- Contribution of a s

B-field:

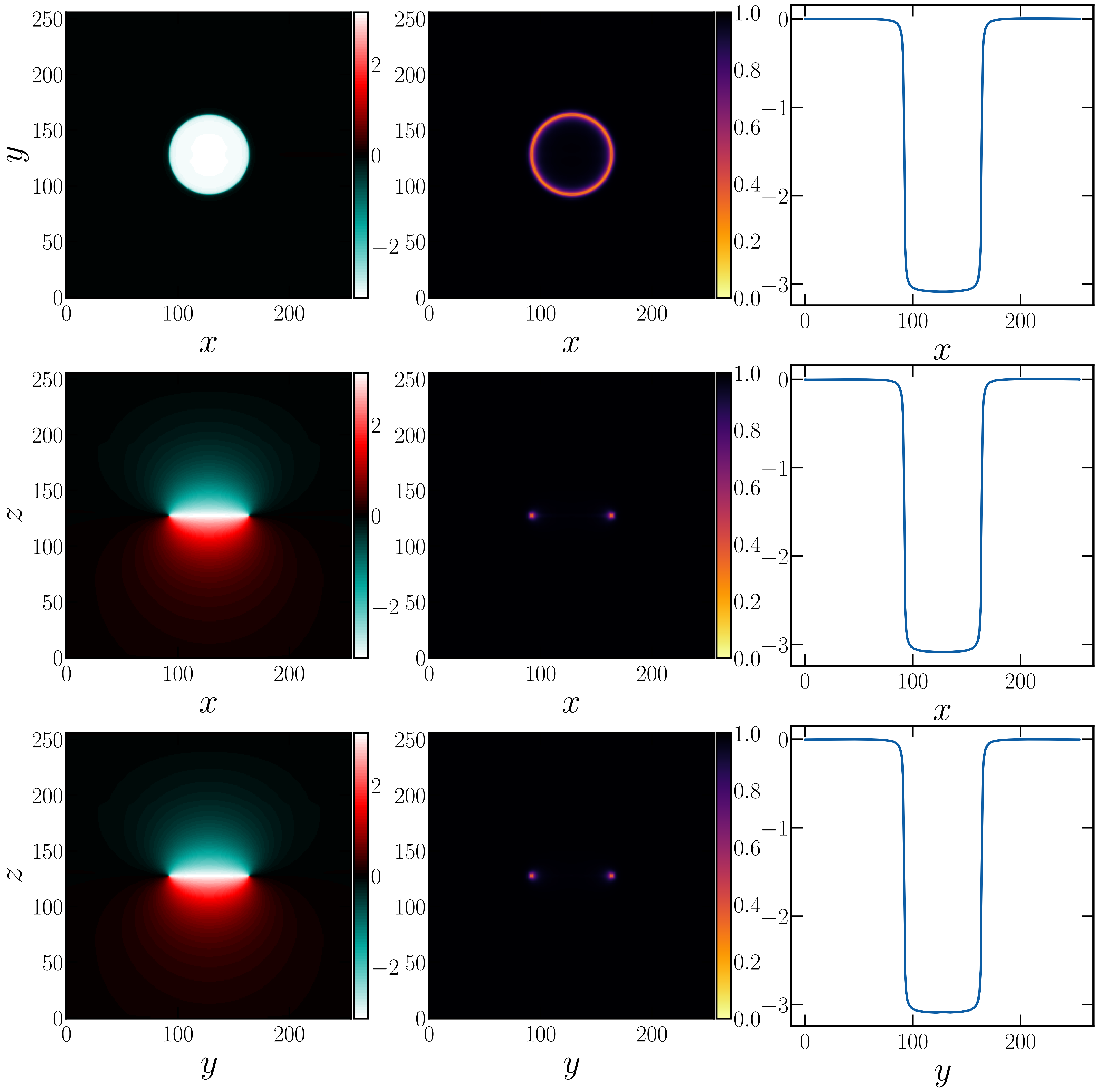
Biot-Savard law (Link)

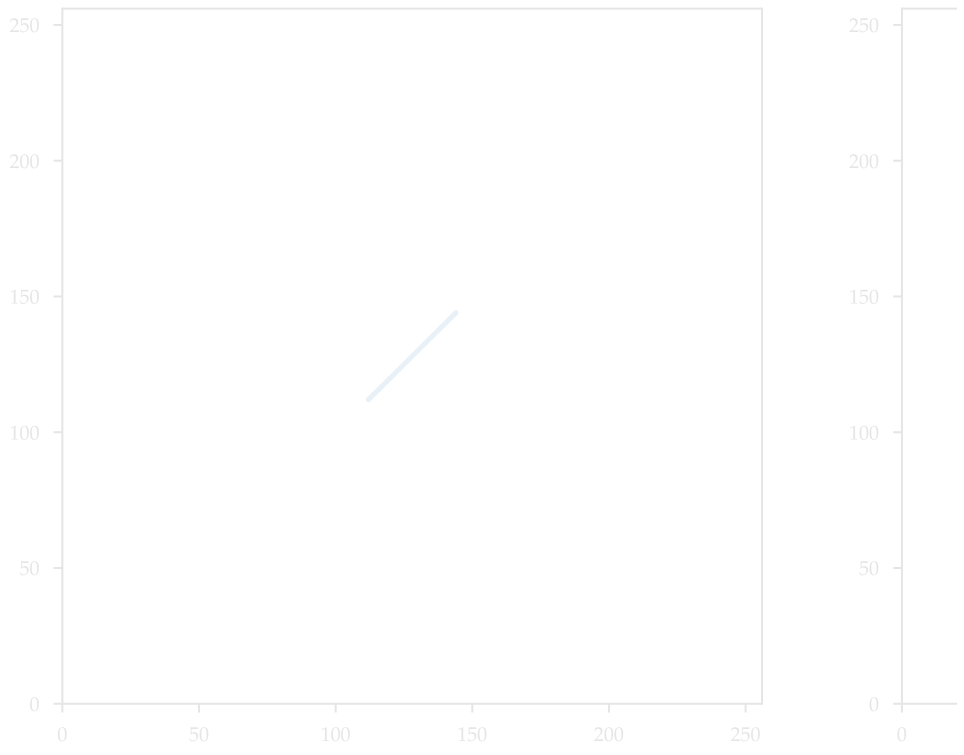
- Calculate links to c

$\theta_{\mathbf{x}+\mathrm{d}x} - \theta_{\mathbf{x}} =$

$- \mathbf{X}(\sigma)) \times \mathbf{X}'$

$\mathbf{x} - \mathbf{X}(\sigma)|^3$

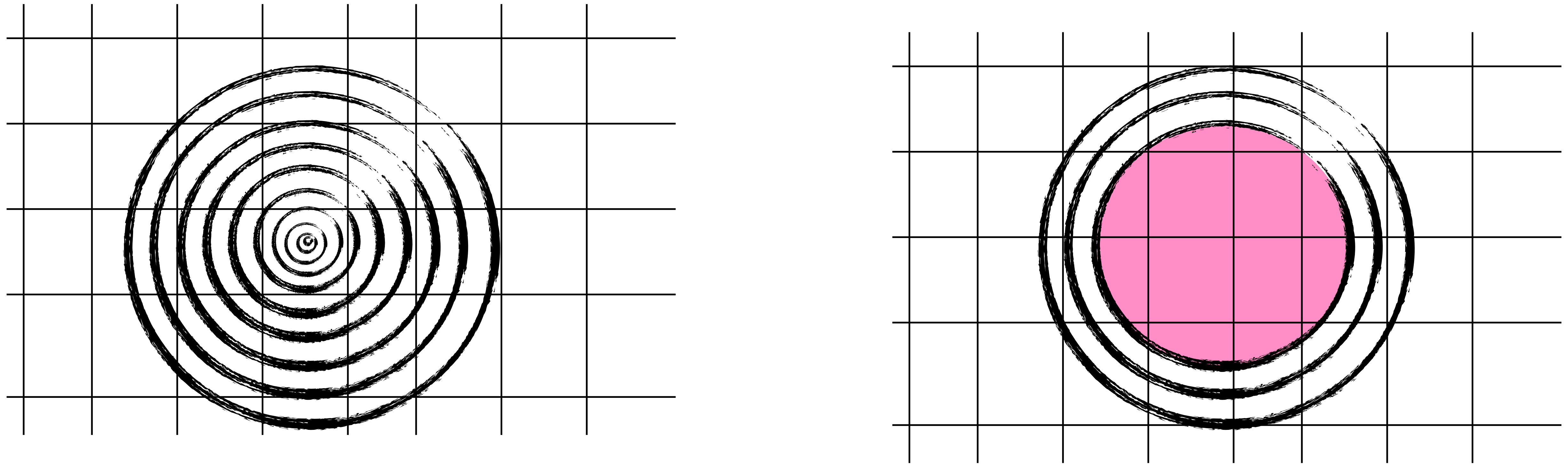




How can we reach a larger dynamical range?

- **Brute Force**: Larger simulations on more powerful supercomputers
- **Better:** Use the given computational power more efficiently: **AMR!**
- **In addition:** Study effective models that allow us to study the network dynamics at high tension (**Moore strings**) with 2+3 extra degrees of freedom (two additional complex scalars + one vector field)

Klaer, Moore [1707.05566, 1708.07521, 1912.08058]



$$\log \sim 2(q_1^2 + q_2^2)$$

Potential Improvement with AMR

- We can estimate the RAM needed to perform an AMR complex scalar simulation:

$$\mathrm{RAM} = 2 \times 2 \times 4\ \mathrm{bytes}\ \times \left(N_0^3 + \frac{\pi n_c n_r^2}{4} \frac{r^\ell - 1}{r - 1} N_p \right)$$

$N_p = \xi \times 6(L/(N_0\tau))^2 \times N_0^3$

Fleury & Moore [1509.00026]

- This takes into account, that we refine only around the strings and that we want to balance the RAM between the root and the refined grids

- Suggests time-dependent number of refinement levels:.

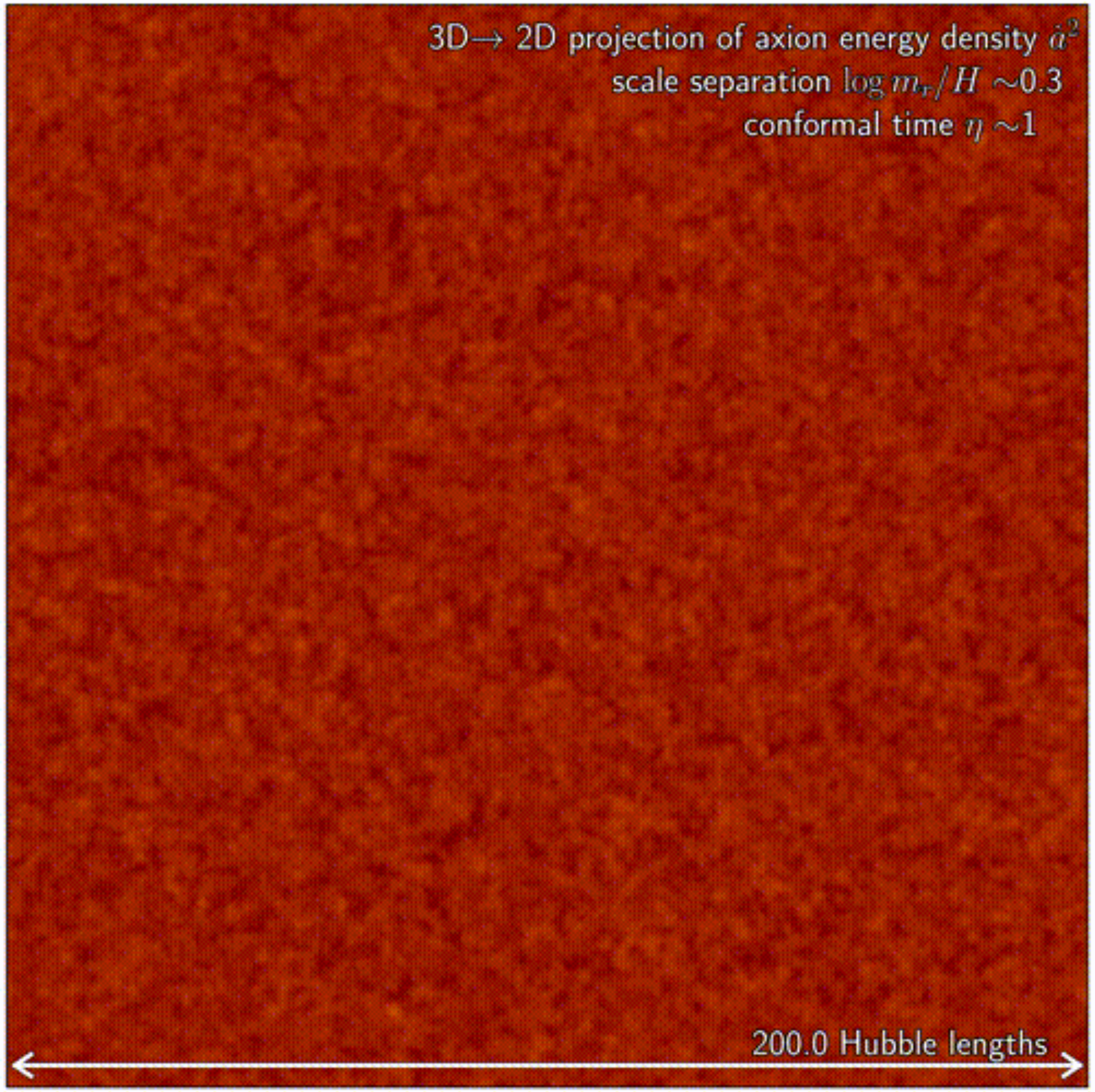
$$\ell + 7 \simeq \log_2\left(N_0^3/(\pi N_p)\right) = \log_2\left(N_0^2 \tau^2/(\pi 6 \xi L^2)\right)$$

- Results in log ~ 13,16,18 for base grids of $N_0 = 2048,\ 4096,\ 8192$ with $\ell = 9,\ 11,\ 13.$ In practice not so trivial ..

Adaptive Mesh Refinement (AMR)

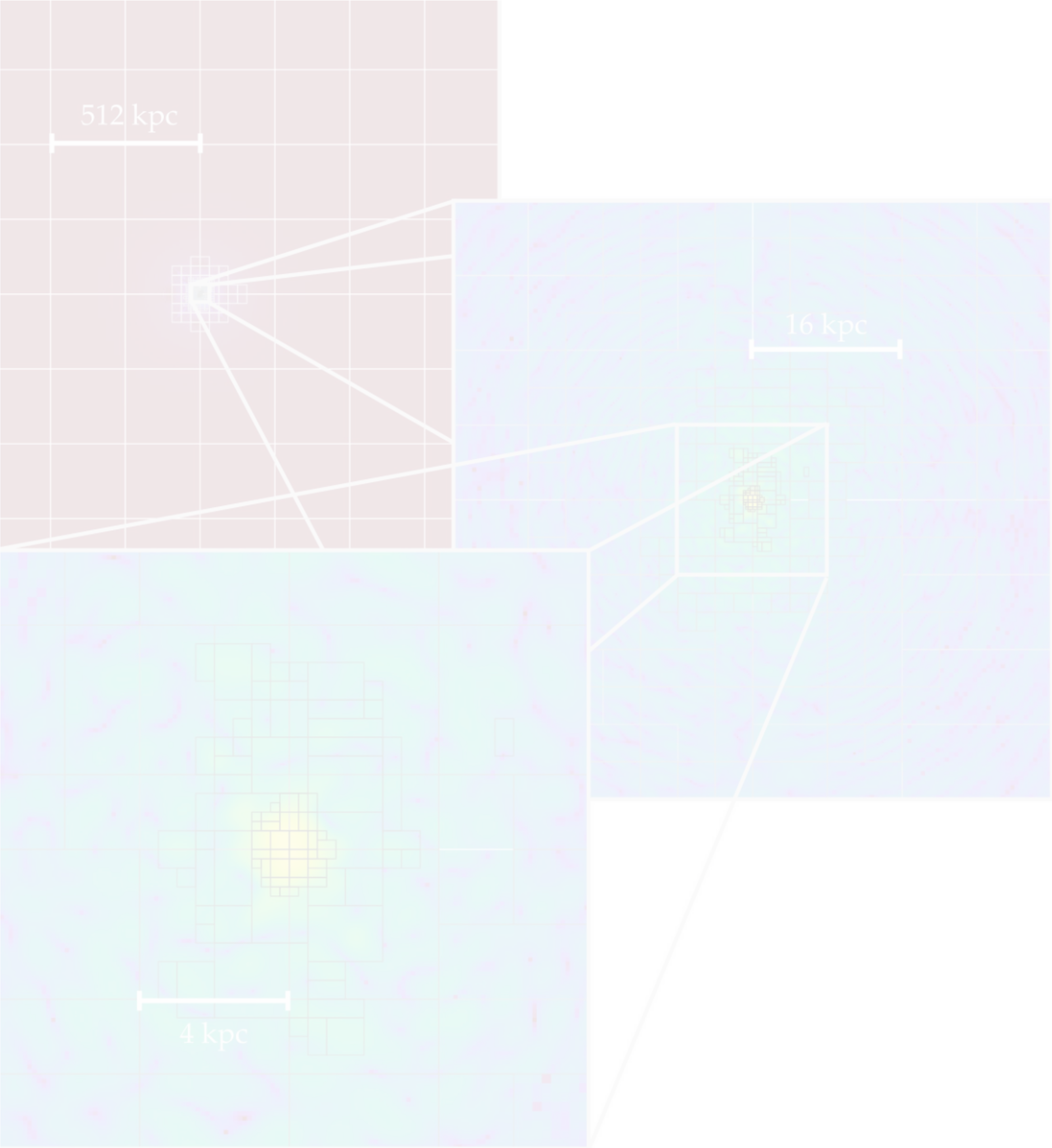
- **Idea:** Focus cor … the grid
- Nowadays wid … des, numerical relat
- Current codes r

Drew & Shellard [1910.01718]
"GRChombo"



"sledgehamr"

Buschmann [2404.02950]



Schwabe+ [2007.08256]
"axioNyx"

Potential Improvement with AMR

- W… …ulation:

$\times 6(L/(N_0\tau))^2 \times N_0^3$

& Moore [1509.00026]

- Th… …e want to…
- Su…
- R… ℓ